Algorithms for Optimal Decisions
Tutorial 7
Questions

Exercise 1

1. *An electric company needs to determine its pricing structure according to demand during peak and off–peak times. The prices and demands are as follows:*

	price()	*demand (KWh)*
peak–time	p_1	$(80 - 0.7p_1)$
off peak	p_2	$(50 - p_2)$

The company must have sufficient capacity to meet demand at all times. The capacity must not be less than $5kWh$ at any time. It costs 12 to maintain each kWh of capacity at each period. The company wishes to maximize daily revenues less operating costs. Formulate this problem.

2. *A mobile telephone company produces brands A and B. If the charge per unit is p_1 for brand A and p_2 for brand B it can sell q_1 of A and q_2 of B, where*

$$\begin{aligned} q_1 &= 5500 - 10p_1 + p_2 \\ q_2 &= 3700 - 9p_1 + 0.8p_2. \end{aligned}$$

The manufacturing requirements are:

	labor(min)	*chips*
Brand A	*20*	*3*
Brand B	*30*	*1*

At present, 1000 *hours of labor and* 5000 *chips are available. The company wants to maximize its revenue. Formulate the problem.*

Exercise 2

1. *Consider the problem:*

$$\begin{aligned} \min_x \quad & x_1^4 + 2x_1^2 + 2x_1x_2 + 4x_2^2 \\ s.t. \quad & 2x_1 + x_2 = 10, \\ & x_1 + 2x_2 \geq 10, \\ & x_1, x_2 \geq 0. \end{aligned} \qquad (1)$$

   Write the KKT conditions for this problem.

   If the SUMT algorithm were to be applied directly to this problem what would be the unconstrained function $l(x, \eta)$ to be minimized at each iteration?

2. *Consider the Frank-Wolfe algorithm for solving the nonlinear program:*

$$\min_x f(x), Ax \leq b,$$

   using the following linear–programming (LP) subproblem at every iterate x_k, $Ax_k \leq b$:

$$\begin{aligned} \min_x \quad & \nabla f(x_k)^t x \\ & Ax \leq b. \end{aligned}$$

   Let x_{LP} denote the solution of this LP. Suggest an amendment to the above subproblem that will ensure the following descent condition:

$$\nabla f(x_k)^t (x_{QP} - x_k) \leq -c(x_{QP} - x_k)^t (x_{LP} - x_k),$$

   where x_{QP} denotes the solution of the amended subproblem and c a nonnegative constant. Establish this condition for the suggested amendment.

Exercise 3 *We are considering investing in three stocks. The random variable S_i represents the annual return on 1 pound invested in stock i. We are* *given the expected values of these returns to be* $E(S_1) = 0.15$, $E(S_2) = 0.21$ *and* $E(S_3) = 0.09$. *The uncertainties are as follows:*

$$\begin{bmatrix} varS_1 & covS_1S_2 & covS_1S_3 \\ covS_1S_2 & varS_2 & covS_2S_3 \\ covS_1S_3 & covS_2S_3 & varS_3 \end{bmatrix} = \begin{bmatrix} 0.09 & 0.06 & -0.04 \\ 0.06 & 0.04 & 0.05 \\ -0.04 & 0.05 & 0.01 \end{bmatrix}. \tag{2}$$

We have 1000 pounds to invest and wish to have an expected return of at least 15%. *Formulate a quadratic programming problem to find the portfolio of minimum variance that attains an expected return of at least* 15%.