Exercise 1

1. An electric company needs to determine its pricing structure according to demand during peak and off-peak times. The prices and demands are as follows:

<table>
<thead>
<tr>
<th></th>
<th>price($)</th>
<th>demand (KWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>peak-time</td>
<td>$p_1$</td>
<td>$(80 - 0.7p_1)$</td>
</tr>
<tr>
<td>off peak</td>
<td>$p_2$</td>
<td>$(50 - p_2)$</td>
</tr>
</tbody>
</table>

The company must have sufficient capacity to meet demand at all times. The capacity must not be less than 5kWh at any time. It costs 12 to maintain each kWh of capacity at each period. The company wishes to maximize daily revenues less operating costs. Formulate this problem.

2. A mobile telephone company produces brands A and B. If the charge per unit is $p_1$ for brand A and $p_2$ for brand B it can sell $q_1$ of A and $q_2$ of B, where

$$q_1 = 5500 - 10p_1 + p_2$$
$$q_2 = 3700 - 9p_1 + 0.8p_2.$$

The manufacturing requirements are:

<table>
<thead>
<tr>
<th></th>
<th>labor(min)</th>
<th>chips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brand A</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>Brand B</td>
<td>30</td>
<td>1</td>
</tr>
</tbody>
</table>
At present, 1000 hours of labor and 5000 chips are available. The company wants to maximize its revenue. Formulate the problem.

Exercise 2

1. Consider the problem:

   $$\min_x x^4 + 2x_1^2 + 2x_1x_2 + 4x_2^2$$

   s.t. $$2x_1 + x_2 = 10,$$
   $$x_1 + 2x_2 \geq 10,$$
   $$x_1, x_2 \geq 0.$$  \hspace{1cm} (1)

   Write the KKT conditions for this problem.

   If the SUMT algorithm were to be applied directly to this problem what would be the unconstrained function $$l(x, \eta)$$ to be minimized at each iteration?

2. Consider the Frank-Wolfe algorithm for solving the nonlinear program:

   $$\min_x f(x), Ax \leq b,$$

   using the following linear-programming (LP) subproblem at every iterate $$x_k, Ax_k \leq b$$:

   $$\min_x \nabla f(x_k)^t x$$

   $$Ax \leq b.$$

   Let $$x_{LP}$$ denote the solution of this LP. Suggest an amendment to the above subproblem that will ensure the following descent condition:

   $$\nabla f(x_k)^t (x_{QP} - x_k) \leq -c(x_{QP} - x_k)^t (x_{LP} - x_k),$$

   where $$x_{QP}$$ denotes the solution of the amended subproblem and $$c$$ a nonnegative constant. Establish this condition for the suggested amendment.

Exercise 3 We are considering investing in three stocks. The random variable $$S_i$$ represents the annual return on 1 pound invested in stock $$i$$. We are
given the expected values of these returns to be \( E(S_1) = 0.15 \), \( E(S_2) = 0.21 \) and \( E(S_3) = 0.09 \). The uncertainties are as follows:

\[
\begin{bmatrix}
\text{var}S_1 & \text{cov}S_1S_2 & \text{cov}S_1S_3 \\
\text{cov}S_1S_2 & \text{var}S_2 & \text{cov}S_2S_3 \\
\text{cov}S_1S_3 & \text{cov}S_2S_3 & \text{var}S_3
\end{bmatrix} =
\begin{bmatrix}
0.09 & 0.06 & -0.04 \\
0.06 & 0.04 & 0.05 \\
-0.04 & 0.05 & 0.01
\end{bmatrix}.
\] (2)

We have 1000 pounds to invest and wish to have an expected return of at least 15%. Formulate a quadratic programming problem to find the portfolio of minimum variance that attains an expected return of at least 15%.