

Game Theory

Tutorial 2

Questions

Exercise 1 (Minimax problem) *Three linear functions y_1, y_2 and y_3 are defined as follows:*

$$\begin{aligned}y_1 &= 2 - x_1, \\y_2 &= x_1 - 1, \\y_3 &= 2x_1 - 6.\end{aligned}$$

Find

$$\min_x \max_{i=1,2,3} \{y_i\}. \quad (1)$$

Exercise 2 (Minimax problem again) *Find x_1, x_2 satisfying*

$$\begin{aligned}x_1 + x_2 &\leq 2, \\x_1, x_2 &\geq 0,\end{aligned} \quad (2)$$

and having the maximum of

$$\begin{aligned}3x_1 - x_2 \\-x_1 + x_2\end{aligned} \quad (3)$$

as small as possible.

Exercise 3 (Duality Theory 1) *Given the primal L.P. problem:*

$$\begin{aligned} \max_x \quad & c^t x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0, \end{aligned} \tag{4}$$

and its dual pair:

$$\begin{aligned} \min_y \quad & b^t y \\ \text{s.t.} \quad & A^t y \geq c \\ & y \geq 0, \end{aligned} \tag{5}$$

show that the dual of (5) is (4).

Exercise 4 (Duality Theory 2) *Find the dual problem of the following L.P. problem:*

$$\begin{aligned} \max \quad & x_0 = 3x_1 + 2x_2 \\ \text{s.t.} \quad & 5x_1 + 2x_2 \leq 0 \\ & 4x_1 + 6x_2 \leq 24 \\ & x_1 + x_2 \geq 1 \\ & x_1 + 3x_2 = 1 \\ & x_1 \geq 0. \end{aligned} \tag{6}$$