

# Game Theory

## Tutorial 3

### Questions

**Exercise 1 (Duality Theory)** Find the dual problem of the following L.P. problem:

$$\begin{aligned}
 \max \quad & x_0 = 3x_1 + 2x_2 \\
 \text{s.t.} \quad & 5x_1 + 2x_2 \leq 10 \\
 & 4x_1 + 6x_2 \leq 24 \\
 & x_1 + x_2 \geq 1 \\
 & x_1 + 3x_2 = 9 \\
 & x_1 \geq 0.
 \end{aligned} \tag{1}$$

**Exercise 2 (Free Variables)** Solve the following problem:

$$\begin{aligned}
 \min \quad & x_0 = x_1 + 2x_2 - x_3 \\
 \text{s.t.} \quad & x_1 - x_2 + x_3 \leq 1 \\
 & x_1 + x_2 - 2x_3 \leq 4 \\
 & x_1 \geq 0.
 \end{aligned} \tag{2}$$

**Exercise 3 (Game Theory)** Consider the following reward matrix:

<i>Player I</i>	<i>Player II</i>		
	<i>1</i>	<i>2</i>	<i>3</i>
<i>1</i>	<i>17</i>	<i>23</i>	<i>48</i>
<i>2</i>	<i>17</i>	<i>3</i>	<i>51</i>
<i>3</i>	<i>3</i>	<i>17</i>	<i>-2</i>

Which strategy should each of the two players choose? One answer must be obtained by applying the concept of dominated strategies to rule out a succession of inferior strategies until only one choice remains.

**Exercise 4 (Game Theory)** The manager of a multinational company and the union of workers are preparing to sit down at the bargaining table to work out the details of a new contract for the workers. Each side has developed certain proposals for the contents of the new contract. Let us call union proposals “Proposal 1”, “Proposal 2” and “Proposal 3”, and manager’s proposals “Contract A”, “Contract B” and “Contract C”. Both parties are aware of the financial aspects of each proposal–contract combination. The reward matrix is:

Proposal	Contract		
	A	B	C
1	9.5	12.0	7.0
2	7.0	8.5	6.5
3	6.0	9.0	10.0

- Is there an equilibrium point?
- Find the mixed strategies for the union and the manager.
- Formulate the LP problem to determine the optimum strategy for the union and the optimum strategy of the manager.

**Exercise 5 (Game Theory)** Consider the previous example, but with the following reward matrix:

Proposal	Contract		
	A	B	C
1	8.5	7.0	7.5
2	12.0	9.5	9.0
3	9.0	11.0	8.0

- *Is there an equilibrium point?*
- *Find the strategies which are dominated by other strategies, and reduce the size of the reward matrix.*
- *Formulate the LP problem to determine the optimum strategy for the union and the optimum strategy of the manager.*