Exercise 1  Olympic Airways Wants to load \( n \) containers on one of its cargo air planes. Container \( j \) weighs \( a_j \) tons and its value is \( c_j \) dollars. The maximum capacity of the air plane is \( b \) tons. The airline wants to load the air plane in such a way that the value of its cargo is as large as possible. Formulate the problem as an integer programming problem.

Exercise 2  The owner of a big motor company wants to build \( k = 10 \) new factories in different areas. All factories make the same product. The owner has \( n = 15 \) customers. Customer \( i \) demands \( d_i \) units of the product. The operating cost of the factory \( j \) is \( f_j \geq 0 \) and the maximum number of units it can make is \( M_j \). The cost of delivering 1 unit from factory \( i \) to customer \( j \) is \( c_{i,j} \).

Where should the owner build his new factories in order to minimise the delivery cost? Formulate the above problem as an I.P. programming problem.

Exercise 3  Reformulate as IP problem the following problem:

\[
\begin{align*}
\text{minimize} & \quad 2x_1 - 7x_2 \\
\text{subject to} & \quad 0 \leq x_1 \leq 10 \\
& \quad 0 \leq x_2 \leq 10, \\
\end{align*}
\]

and at least one of the following holds:

\[-2x_1 + 3x_2 \geq 0 \\
5x_1 - 4x_2 \geq 0.\]

1
Exercise 4 Solve the following problem:

\[
\begin{align*}
\min_x & \quad c^T x \\
\text{s.t.} & \quad Ax = b \\
& \quad x \geq 0 \\
& \quad x_1 \in \{r_1, r_2, ..., r_q\}.
\end{align*}
\] (2)

Exercise 5 Formulate the following model as a mixed integer programming problem:

\[
\begin{align*}
\min_x & \quad \sum_{j=1}^{n} C_j(x_j) \\
\text{s.t.} & \quad Ax \leq b \\
& \quad x \geq 0 \\
\end{align*}
\]

\[C_j(x_j) = \begin{cases} 0 & x_j = 0 \\ k_j + c_j x_j & x_j > 0 \end{cases}\] (3)

where \(c_j, k_j > 0\) and \(k_j\) are called fixed changes.