Integer Programming
Tutorial 2
Questions

The Cutting Plane Algorithm Let:

$$a_1 + x_1 + a_2x_2 + ... + a_nx_n = b, \tag{1}$$

be an equation which is to be satisfied for integers $x_1, x_2, ..., x_n \leq 0$ and let S be a set of possible solutions.

Now let $a_j = [a_j] + f_j$ and $b = [b] + f$ so (1) becomes:

$$\begin{aligned} \textstyle\sum_{j=1}^{n}([a_j] + f_j)x_j &= [b] + f \Rightarrow \\ \textstyle\sum_{j=1}^{n} f_jx_j - f &= [b] - \textstyle\sum_{j=1}^{n}[a_j]x_j. \end{aligned} \tag{2}$$

For $x \in S$ the right hand side of (2) is integer, so

$$\varsigma = \sum_{j=1}^{n} f_jx_j - f.$$

Also $x \geq 0, x \in S$ so $\varsigma \geq 0$ and

$$\sum_{j=1}^{n} f_jx_j \geq f.$$

If we solved the continuous problem in step 1 and the solution is not an integer. Then there exists a basic variable x_i such that:

$$x_i + \sum_{j \notin I} b_{ij}x_j = b_{i0},$$

where b_{i0} is not an integer. Putting $f_j = b_{ij} - [b_{ij}]$ and $f = b_{i0} - [b_{i0}]$ we deduce that:

$$\sum_{j \notin I} f_j x_j \geq f. \tag{3}$$

Since b_{i0} is not an integer $\Rightarrow f > 0$, then (3) is not satisfied by the current solution so it is a new cut.

Exercise 1 *Solve:*

$$\begin{array}{rl} \max & x_1 + 4x_2 \\ s.t. & 2x_1 + 4x_2 \leq 7 \\ & 10x_1 + 3x_2 \leq 14 \\ & x_1, x_2 \geq 0, integers \end{array} \tag{4}$$

Exercise 2 *Solve:*

$$\begin{array}{rl} \max & 3x_1 + 4x_2 \\ s.t. & \frac{2}{5}x_1 + x_2 \leq 3 \\ & \frac{2}{5}x_1 - \frac{2}{5}x_2 \leq 1 \\ & x_1, x_2 \geq 0, integers \end{array} \tag{5}$$

Exercise 3 *Solve the following IP problem:*

$$\begin{array}{rl} \max & 5x_1 + 6x_2 \\ s.t. & 0.2x_1 + 0.3x_2 \leq 1.8 \\ & 0.2x_1 + 0.1x_2 \leq 1.2 \\ & 0.3x_1 + 0.3x_2 \leq 2.4 \\ & x_1, x_2 \geq 0, integers \end{array} \tag{6}$$

Exercise 4 (Branch and Bound Method – 1) *Solve the following problem using branch and bound method:*

$$\begin{array}{rl} \max & x_1 + 2x_2 \\ s.t. & 2x_1 + x_2 \leq 7 \\ & -x_1 + x_2 \leq 3 \\ & x_1, x_2 \geq 0, integers \end{array} \tag{7}$$

Exercise 5 (Branch and Bound – 2) *Consider the following problem:*

$$\begin{array}{rl} \max & x_0 = 5x_1 + x_2 \\ s.t. & -x_1 + 2x_2 \leq 4 \\ & x_1 - x_2 \leq 1 \\ & 4x_1 + x_2 \leq 12 \\ & x_1, x_2 \geq 0, integers; \end{array} \tag{8}$$

- *Solve this problem graphically;*
- *Solve LP relaxation. Round this solution to the nearest integer solution and check whether it is feasible. Then enumerate all the rounded solutions, check them for feasibility and calculate x_0 for those that are feasible. Are any of these feasible rounded solutions optimal for the IP problem?*