Appendix C: Constraint Logic Programming in Computational Finance

1 Constraint Logic Programming and Integer Programming

Constraint Logic Programming (CLP) is a problem solving method that was developed out of Logic Programming and Artificial Intelligence. CLP merges two declarative paradigms; constraint solving and logic based search. The most important features of CLP are

- declarative problem modelling allowing a clean separation between the problem statement (variables and constraints) and the resolution of the problem (the constraint solving algorithms),
- propagation of the effects of decisions and
- efficient search for feasible solutions.

CLP starts the solution process by developing a model for the problem. Although the model will use variables, constraints and an objective, these are commonly not the same as that for the corresponding integer programming (IP) model. The reason for this is that CLP provides a richer modelling environment allowing to describe basic domain, arithmetic, symbolic constraints, logical operations stating truth of constraints, as well as global constraints (for instance an "alldifferent" relation on a set of variables replaces pairwise disequality constraints) [10]. The availability of non-linear and higher level constraints within CLP gives greater scope to express the model. In CLP, modelling the problem as well as modelling the search take place within the same computer language and hence environment. This allows the user to easily make problem-specific searches as well as allowing conditional constraints to be imposed during the search whenever it is necessary.

Although each technique can model the problem in different ways and extract different information, from the solving processes point of view, both IP and CLP employ tree-based search techniques, but are quite different in how they carry out the search, the models that they adopt and the search reduction algorithms that they use.

Anecdotal evidence suggests that there are performance differences between the technologies and certainly some problem types have been identified as having a greater likelihood of success using CLP technology than another. One of these types is discrete optimisation problems especially binary integer programming. CLP and IP have many applications in diverse areas of optimisation. Examples of successful CLP and IP approaches to common application types include, production scheduling cutting stock, workforce and crew scheduling, allocation and assignment, rostering and timetabling. For efficient performance on larger problems, CLP relies on the search space being significantly reduced during the search. This occurs often as a result of complex and /or numerous constraints within the problem which are described in the model.

2 Application of CLP in Computational Finance

Stochastic constraint programming has been recently developed to model combinatorial decision problems involving uncertainty and applied to a production planning problem [18]. Although some researches such as Jaffar and Maher [6], Broek and Daniels [1], Lassez and McAloon [7], and Huynh and Lassez [8] have applied CLP approach to model finance problems, an application of CLP to multi stage stochastic finance problems is quite novel.

CLP(R) which is the most natural language of expressing both numeric and non-numeric information declaratively have been mainly used to model particularly option trading and asset allocation problems. The main reason of CLP's success in option trading is that trading in options involves complex mechanisms necessitating considerable numerical computation, and sophisticated combination strategies requiring the expert knowledge.

The complex option combinations can be described by logic programming based language. The complicated combination of strategies in finance and their multi-faceted analysis are an ideal testing ground for the unique features of CLP(R) as a general purpose programming language. It is true that the modelling capabilities of CLP often present a clear, succinct representation of the problem and therefore provide a richer environment in which the modeller can operate.

The application of CLP in the worst-case design is completely novel and desires to be explored further since the natural way of expressing and searching the minimax strategies exist in CLP technology. In this project, we will not only use CLP technology to solve large scale post tax optimisation problem but also model and solve multistage stochastic finance problems. The other advantage of using CLP is that it has facilities to accommodate the non-linear constraints and objective functions and the parallel solution strategies.

The current state of art to enable the manipulation of constraints is C++ based Ilog Solver. The integration of CLP technology to model and solve the problem with the component architecture has advantages and will be one of the unique software developments in this market.

Most success in using CLP has been with the finite domain solver. Other solvers such as the rational and boolean ones have had success in limited areas. We will look at either financial problems which have significant integrality in their models or ways of decomposing the problem into a search based on a set of logical decisions leading to running current mathematical approaches on smaller models.

3 Hybrid Solving Technology

The recognition that both IP and CLP have something to offer has led to much recent research in developing hybrid approaches that attempt to combine the best of both worlds, for example see [15] and [16]. The research area of hybrid solving has now gone beyond individual papers appearing at constraint or OR conferences. It has reached the stage of having conferences (CP-AI-OR' 01 and CP-AI-OR' 02) and workshops (COSOLV'01 and COSOLV'02) in its own right, [2, 3, 4, 5], and a special volume of Annals of OR (Vol. 118, no. 1-4) with selected papers CPAIOR'01 was issued in 2003.

In favour of bringing these two technologies together is that they are both complete tree search based solvers, commonly having closely related models of the same problem. Yet the possible differences of these models together with the different information extracted about the problem, holds out the possibility that they would be effective partners in a hybrid solver. However, despite reports of some successful individual cases, no general-purpose hybrid system has emerged to date. Van Hentenryck [17] confirms this ideal in stating that there is a strong need for new theoretical and experimental results on co-operation between independent solvers. Single environments currently exist which allow multiple co-operating solvers to be run. However, it is left to the user to determine how they are configured in order to achieve effective collaboration on a problem.

This is a novel research area we will investigate to solve the exact mean variance optimisation and worst case design problems. We will introduce new hybrid solution strategies by incorporating the CLP technology with mathematical programming.

4 Application of CLP in Post-Tax optimisation

Post-tax optimisation finds an optimal investment strategy, after incorporating specific tax rules, using differing asset allocations in wrappers, over a given finite investment horizon. A wrapper is a set of assets (equities, bond, cash and properties) with a specific set of rules for taxation on regular basis and in different investor scenarios. An investor may face different life events such as withdrawals, gifts, or emigration [11] impacting on his or her investment plan. The tax rules also address these eventualities.

Although an investor usually prefers or is advised to take money obtained from the income or the dividend rather than the original capital, the net growth does not always cover the amount of withdrawal specified. In this case, the original capital needs to be taken into consideration. The withdrawal is composed of funds obtained from the income or dividends, the capital gains and the original capital. So an optimisation model would naturally attempt to take withdrawal from capital rather than paying immediate taxes [12].

Introducing the withdrawals from original capital requires the integer binary variables. Therefore, the original asset reallocation problem with specific tax rules becomes a mixed integer quadratic programming problem [13, 14]. In multistage stochastic programming framework, the problem becomes very big according to depth and branching structure of the scenario tree as well as the investment horizon and the number of assets and wrappers. The current operational research technology needs efficient decomposition and paraleliza-tion strategies to overcome difficulties occurred because of the complexity of the problem. In addition, in post tax optimisation, real life cases with "what if" can be best represented by logic based constraints and search techniques. These are natural features of modelling the problem and imposing search techniques in CLP. Therefore, we consider CLP as a new alternative technology to investigate in this project. In addition, hybridisation techniques between CLP and post-tax optimisation is very novel subject to investigate and take advantage of CLP's framework.

References

[1] J.M. Broek, H.M. Daniels, "Application of Constraint Logic Programming to Asset and Liability Management in Banks", Computer Science in Economics and Management, Volume 4, 1991, p. 107–116.

[2] CP-AI-OR01 (2001) Proceedings of the Fourth International Workshop on Integration of AI and OR techniques in Constraint Programming for Combinatorial Optimisation Problems - CP-AI-OR01, Ashford, UK, 8th - 10th April 2001.

[3] CP-AI-OR01 (2002) Proceedings of the Fourth International Workshop on Integration of AI and OR techniques in Constraint Programming for Combinatorial Optimisation Problems - CP-AI-OR02, Le Croisic, France, 25th - 27th March 2002.

[4] COSOLV01 (2001) Workshop on Cooperative Solvers in Constraint Programming, Paphos, Cyprus, 1st December 2001.

[5] COSOLV2002 (2002) Workshop on Cooperative Solvers in Constraint Programming, Ithaca, USA, 8th September 2002.

[6] J. Jaffar and M.J. Maher, "Constraint Logic Programming: A Survey", The Journal of Logic Programming, Volume 19 (20), 1994, p. 503–581

[7] C. Lassez, K. McAloon, "Constraint Logic Programming and Option Trading", IEEE Expert, 1987, p. 42–50.

[8] T. Huynh, C. Lassez, “A CLP(R) Options Trading Analysis System”, Proc. 5th International Conference on Logic Programming 1988, p 59–69.

[9] J. Little, Integer Programming, “Constraint Logic Programming and Their Collaboration in Solving Discrete Optimization Problems”, PhD Thesis, Brunel University, 2000.

[10] J. Little, K. Darby-Dowman, “The Significance of Constraint Logic Programming to Operational Research”, Operational Research Society Tutorial Papers 1995, Ed. M. Lawrence, C. Wilsdon, 1995, p. 20–45.

[11] M.A. Osorio, R., Settergren, B. Rustem, N. Gulpinar, “Post Tax Optimal Investments”, Applied optimization Vol 70, Financial Engineering, E-commerce and Supply Chain, Kluwer Academic Publishers, (2002), 153–173.

[12] M.A. Osorio, N. Gulpinar, R. Settergren, B. Rustem, “Post-Tax Optimization with Stochastic Programming”, Accepted for publication in European Journal of Operations Research. ‘

[13] M.A. Osorio, N. Gulpinar, R. Settergren, B. Rustem, “A Mixed Integer Programming Model for Multistage Mean-Variance Post-Tax Optimization”, Submitted to Operations Research.

[14] M.A. Osorio, N. Gulpinar, B. Rustem, R. Settergren, “Tax Impact on Multistage Mean-Variance Portfolio Allocation”, Submitted to International Transactions of Operations Research.

[15] Rodosek R, Wallace M G and Hajian M T, “A New Approach to Integrating Mixed Integer Programming with Constraint Logic Programming”, Annals of Operational Research, Recent Advances in Combinatorial Optimization, 1999, Vol 86, p. 63–87, 1999.

[16] J. Puget, “A Comparison Between Constraint Programming and Integer Programming”, APMOD 95 Conference, London, April 1995.

[17] P. Van Hentenryck, “Constraint Programming for Combinatorial Search Problems”, Constraints: An International Journal, 1997, 2(1), p. 99–101.

[18] T. Walsh, “Stochastic Constraint Programming ”, Proceedings of the 15th ECAI European Conference on artificial Intelligence, IOS Press, 2002.