Appendix C: Constraint Logic Programming in Computational Finance

1 Constraint Logic Programming and Integer Programming

Constraint Logic Programming (CLP) is a problem solving method that was developed out of Logic Programming and Artificial Intelligence. CLP merges two declarative paradigms; constraint solving and logic based rules. The most important features of CLP are

- declarative problem modelling allowing a clean separation between the problem statement (variables and constraints) and the resolution of the problem (the constraint solving algorithms),
- propagation of the effects of decisions and
- efficient search for feasible solutions.

CLP starts the solution process by developing a model for the problem. Although the model will use variables, constraints and an objective, these are commonly not the same as that for the corresponding integer programming (IP) model. The reason for this is that CLP provides a richer modelling environment allowing: the description of the basic domain, arithmetic, symbolic constraints, logical operations stating truth of constraints, global constraints (for instance an "alldifferent" relation on a set of variables replaces pairwise disequality constraints) [6]. The availability of non-linear and higher level constraints within CLP gives greater scope to express the model. Using CLP, modelling the problem and modelling the search is achieved using the same rule based language. This allows the user to easily make problem-specific searches as well as allowing conditional constraints to be imposed during the search whenever it is necessary.

From the solving processes point of view, both IP and CLP employ tree-based search techniques, but they are quite different in the way they carry out the search, in the models that they adopt and in the search reduction algorithms that they use.

Anecdotal evidence suggests that there are performance differences between the technologies and certainly some problem types have been identified as having a greater likelihood of success using CLP technology. One of these problem types is discrete optimisation, especially binary integer programming. CLP and IP have many applications in diverse areas of optimisation. Examples of successful CLP and IP approaches the same types of applications include: production scheduling cutting stock, workforce and crew scheduling, allocation and assignment, rostering and timetabling.

2 Application of CLP in Computational Finance

Some researches such as Jaffar and Maher [1], Broek and Daniels [2], Lassez and McAloon [3], and Huynh and Lassez [4] have applied CLP approach to model finance problems. Nevertheless, the application of CLP to multi stage stochastic finance problems is quite novel.

CLP(R), which is the most natural language for expressing both numeric and non-numeric information declaratively, has been mainly used to model option trading and asset allocation problems. The main reason for CLP's success in option trading is that trading in options involves complex mechanisms necessitating considerable numerical computation, and sophisticated combination strategies requiring expert knowledge. The complex option combinations can be described by logic programming rules. The complicated combination of strategies in finance and their multi-faceted analysis are an ideal testing ground for the unique features of CLP(R) as a general purpose programming language.

The application of CLP in worst-case design is novel and deserves to be explored further as a way of expressing and searching worst-case robust strategies. In this project, we will not only use CLP technology to solve large scale post tax optimisation problem but also model and solve multistage stochastic finance problems. The other advantage of using CLP is that it has facilities to accommodate non-linear constraints and objective functions and parallel solution strategies. The current state of the art constraint solver is the C++ based Ilog Solver.

3 Hybrid Solving Technology

The recognition that both IP and CLP have something to offer has led to research in hybrid approaches that attempt to combine the best of both worlds, for example see [10] and [11]. In favour of bringing these two technologies together is that they are both complete tree search based solvers, commonly having closely related models of the same problem. Yet the possible differences of these models together with the different information extracted about the problem, holds out the possibility that they would be effective partners in a hybrid solver.

However, despite reports of some successful individual cases, no general-purpose hybrid system has emerged to date. Van Hentenryck [12] confirms this ideal in stating that there is a strong need for new theoretical and experimental results on co-operation between independent solvers. Single environments currently exist which allow multiple co-operating solvers to be run. However, it is left to the user to determine how they are configured in order to achieve effective collaboration on a problem.

This is a novel research area we will investigate to solve the exact mean variance optimisation and worst case design problems. We will introduce new hybrid solution strategies by incorporating the CLP technology with mathematical programming.

4 Application of CLP in Post-Tax optimisation

Post-tax optimisation computes an optimal investment strategy, after incorporating specific tax rules, using differing asset allocations in wrappers, over a given finite investment horizon. A wrapper is a set of assets (e.g. equity, bond, cash and property) with a specific set of rules for taxation, with different investor scenarios. An investor may face different life events such as withdrawals, gifts, or emigration [7] impacting on his or her investment plan. The tax rules also address these eventualities.

Although an investor usually prefers, or is advised, to withdraw from income or dividend rather than the original capital, better net growth may be achieved with alternative strategies. In this case, the original capital needs to be taken into consideration. The withdrawal is composed of funds obtained from income or dividends, the capital gains and the original capital. So an optimization model may take the withdrawal from capital, rather than paying immediate taxes [8].

Introducing the withdrawals from original capital requires integer binary variables. Therefore, the original asset reallocation problem with specific tax rules becomes a mixed integer quadratic programming problem. In multistage stochastic programming framework, the problem becomes very large, depending on the depth and branching structure of the scenario tree, the investment horizon and the number of assets and wrappers. Current operational research technology needs efficient decomposition and paralelization strategies to overcome difficulties occurred because of the complexity of the problem. In addition, in post tax optimisation, real life cases with “what if” can be best represented by logic based rules and search techniques. These are natural features of modelling the problem and imposing search techniques, using CLP.

References

[1] J. Jaffar and M.J. Maher, “Constraint Logic Programming: A Survey”, The Journal of Logic Programming, Volume 19 (20), 1994, p. 503–581

[2] J.M. Broek, H.M. Daniels, “Application of Constraint Logic Programming to Asset and Liability Management in Banks”, Computer Science in Economics and Management, Volume 4, 1991, p. 107–116.

[3] C. Lassez, K. McAloon, “Constraint Logic Programming and Option Trading”, IEEE Expert, 1987, p. 42–50.

[4] T. Huynh, C. Lassez, “A CLP(R) Options Trading Analysis System”, Proc. 5th International Conferance on Logic Programming 1988, p 59–69.

[5] J. Little, Integer Programming, “Constraint Logic Programming and Their Collaboration in Solving Discrete Optimization Problems”, PhD Thesis, Brunel University, 2000.

[6] J. Little, K. Darby-Dowman, “The Significance of Constraint Logic Programming to Operational Research”, Operational Research Society Tutorial Papers 1995, Ed. M. Lawrence, C. Wilsdon, 1995, p. 20–45.

[7] M.A. Osorio, R. Settergren, B. Rustem, N. Gulpinar, “Post Tax Optimal Investments”, Submitted to Special Issue for FEES 2001, Baltzer Publishers.

[8] M.A. Osorio, N. Gulpinar, R. Settergren, B. Rustem, “Post Tax Optimisation with Stochastic Programming”, Submitted to European Journal of Operational Research.

[9] M.A. Osorio, R. Settergren, B. Rustem, N. Gulpinar, “Multistage Mean-Variance Post Tax Optimisation”, Submitted to Management Science.

[10] R. Rodosek, M.G. Wallace and M.T. Hajian , “A New Approach to Integrate Mixed Integer Programming with CLP”, Proceedings of the CP96 Workshop on Constraint Programming Applications, 1996, p. 45–54, Cambridge Massachusetts.

[11] J. Puget, “A Comparison Between Constraint Programming and Integer Programming”, APMOD 95 Conference, London, April 1995.

[12] P. Van Hentenryck, “Constraint Programming for Combinatorial Search Problems”, Constraints: An International Journal, 1997, 2(1), p. 99–101.