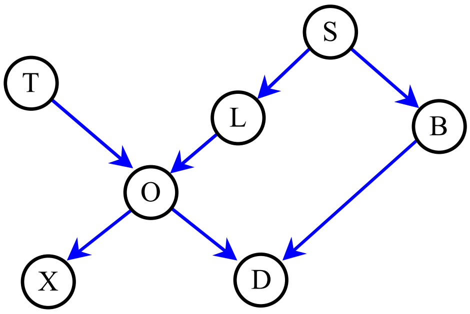
1 Probability Propagation

The following network is used for reasoning about patients with suspected lung disease



B	Bronchitis
D	Dyspnea
L	Lung Cancer
O	Reduced Lung Capacity
S	Smoker
T	Tuberculosis
X	Positive XRay

All the nodes are binary, and the prior and conditional probabilities are as follows:

$$P(O|T\&L) = \begin{pmatrix} P(O1|T1\&L1) & P(O1|T1\&L2) & P(O1|T2\&L1) & P(O1|T2\&L2) \\ P(O2|T1\&L1) & P(O2|T1\&L2) & P(O2|T2\&L1) & P(O2|T2\&L2) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P(D|O\&B) = \begin{pmatrix} P(D1|O1\&B1) & P(D1|O1\&B2) & P(D1|O2\&B1) & P(D1|O2\&B2) \\ P(D2|O1\&B1) & P(D2|O1\&B2) & P(D2|O2\&B1) & P(D2|O2\&B2) \end{pmatrix} = \begin{pmatrix} 0.9 & 0.8 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.3 & 0.9 \end{pmatrix}$$

$$P(L|S) = \begin{pmatrix} P(L1|S1) & P(L1|S2) \\ P(L2|S1) & P(L2|S2) \end{pmatrix} = \begin{pmatrix} 0.2 & 0.1 \\ 0.8 & 0.9 \end{pmatrix}$$

$P(T) = (0.1,0.9)$

$$P(B|S) = \begin{pmatrix} P(B1|S1) & P(B1|S2) \\ P(B2|S1) & P(B2|S2) \end{pmatrix} = \begin{pmatrix} 0.1 & 0.1 \\ 0.9 & 0.9 \end{pmatrix}$$

$P(S) = (0.3,0.7)$

$$P(X|O) = \begin{pmatrix} P(X1|O1) & P(X1|O2) \\ P(X2|O1) & P(X2|O2) \end{pmatrix} = \begin{pmatrix} 0.4 & 0.1 \\ 0.6 & 0.9 \end{pmatrix}$$

Given that the evidence propagated between nodes is defined by the following equations:

For one parent only

$$\lambda_c(a_k) = \sum_{j=1}^{m} P(c_j \mid a_k)\, \lambda(c_j)$$

For two parents

$$\lambda_c(a_k) = \sum_{i=1}^{n} \pi_C(b_i) \sum_{j=1}^{m} P(c_j \mid a_k \& b_i)\, \lambda(c_j)$$

$$\pi(c_i) = \sum_{j=1}^{n} \sum_{k=1}^{m} P(c_i \mid a_j \& b_k)\, \pi_C(a_j)\, \pi_C(b_k)$$

a Calculate the π evidence for the nodes L and O before any measurements are made.

b A new patient arrives and has an X-Ray taken. Fortunately for him, it is negative (state X2). Given just this evidence, calculate the probability of his suffering from Lung cancer (ie the probability distribution over L).

c He is now examined and found to be suffering from Dyspnea (D is in state D1). Explain why it is no longer possible to compute a probability of his suffering from lung cancer.

d He now admits to being a smoker (S is in state s1). Calculate the probability of his suffering from Lung Cancer.

e Explain briefly the advantages and disadvantages of using a join tree for calculating probabilities rather than simple λ and π messages.

The five parts carry equal marks

2 The maximum Weighted Spanning Tree

A data warehouse contains the following vast data set connecting three variables A B and C:

A	B	C
a1	b1	c1
a1	b1	c2
a1	b1	c2
a1	b2	c1
a2	b2	c2
a2	b2	c1
a2	b2	c2
a2	b2	c1

a Construct co-occurrence matrices for the three possible pairings AB, BC and AC:

	a1	a2
b1		
b2		

etc

b From the co-occurrences construct the joint probability table for each pair, and the marginalisations, using the following format:

	a1	a2	P(B)
b1			
b2			
P(A)			

etc.

c Calculate the L1 metric for each possible pair of nodes

$$\text{Dep}(A,B) = \sum_{A \times B} |P(ai\&bj) - P(ai)P(bj)|$$

d Given that A is the root node construct the tree.

e Calculate the prior probability distribution P(A).

f Calculate the two conditional probability matrices for the tree found in part 2d.

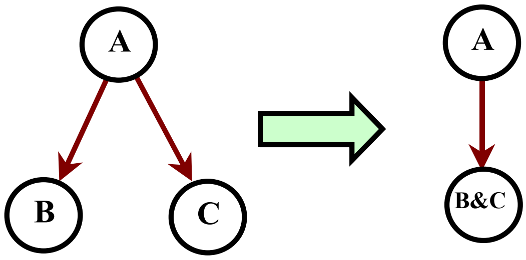
g The tree of part (d) is being used in the case where it is not possible to measure node B. The state of C is found to be c2. Estimate the probabilities of nodes A and B.

The seven parts carry, respectively, 15%,20%,15%,10%,10%,15% and 15% of the marks

3 Join Trees

Joining two variables is one technique that can be used to improve the accuracy of a network in cases where there are dependencies that cannot be accommodated in a singly connected graph. Consider the following simple network and data set:

a0	b0	c1
a0	b0	c1
a0	b1	c0
a0	b1	c0
a1	b0	c0
a1	b0	c0
a1	b1	c0
a1	b1	c1

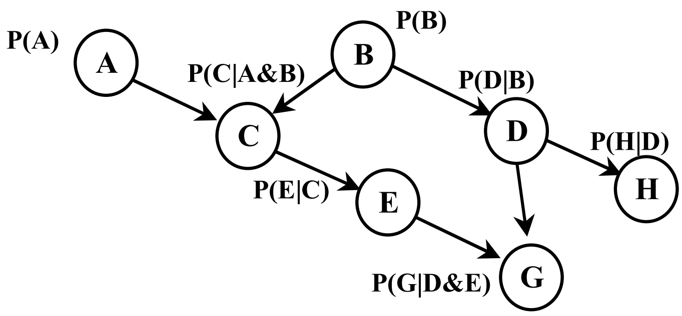


a Find the conditional probability matrices for:
   i) the original network (P(B|A) and P(C|A))
   ii) the joined network (P(B&C|A)).

b The network is being initialised and the prior probability of A is known to be P(A) = {0.3,0.7}. This prior probability is treated as π evidence and propagated. What are the posterior distributions over B and C in:
   i) the original network
   ii) the joined network

c The node B is instantiated to state b1, and node C is instantiated to c0. Calculate the λ evidence at node A using
   i) the original network
   ii) the joined network

d Given the network:



find a join tree using the following steps from the algorithm by Lauritzen and Spiegelhalter:
   i) Find the moral graph
   ii) Triangulate the moral graph
   iii) Identify the cliques of the resulting triangulated moral graph
   iv) Define a join tree in which the ordering of the cliques satisfies the running intersection property.
   v) Draw up a table showing the potential functions and the R and S sets for each node

The four parts carry, respectively, 20%, 20%, 20% and 40% of the marks

4 Co-variance and Principal Component Analysis

The following artificial problem has three variables and just two data points set out in the table below

x	y	z
1	0	3
5	2	1

a Find the mean-centred data matrix $U = [\mathbf{x}\text{-}\mathbf{x_m}, \mathbf{y}\text{-}\mathbf{y_m}, \mathbf{z}\text{-}\mathbf{z_m}]$, hence, find the co-variance matrix using U^TU

b For real problems, where sample sizes are much smaller than the number of variables, it is common to use the Karhunen-Lowe transformation to find the principal components. Using the answer to part a, find the reduced matrix $A = UU^T$ and calculate its principal eigenvector. First calculate the eigenvalues using $\det(A-\lambda I)=0$, then solve $(A-\lambda I)\, e = 0$ to find the eigenvectors.

c Find the principal eigenvector of the covariance matrix of part a by multiplying the eigenvector found in part b by the data matrix U. Check that this is indeed an eigenvector of the covariance matrix, and find its corresponding eigenvalue.

d Explain what is meant by an eigenface in face recognition. How many eigenfaces could be calculated from a face data base in which the images are 128 by 128 resolution and there are three images of twenty subjects in the data base.

e The Mahalanobis distance between two two-dimensional points (x_1,y_1) (x_2,y_2) is defined by the formula:

$$\sqrt{((x_2-x_1,y_2-y_1)\ \Sigma^{-1}\ (x_2-x_1,y_2-y_1)^T)}$$

where Σ is the covariance matrix.

Explain briefly why the Mahalanobis distance might be used in preference to the Euclidian distance in classification problems.

The five parts carry equal marks