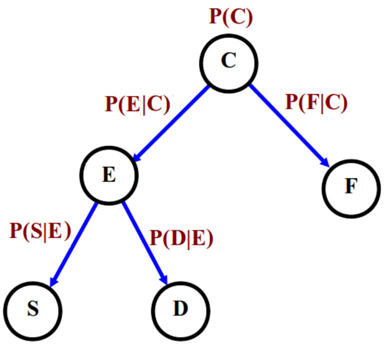
Tutorial 2: Causal Networks with multiple parents

1. Tuberculosis and lung cancer can cause shortness of breath (dyspnea) with equal likelihood. A positive chest XRay is also equally likely given tuberculosis or lung cancer. Bronchitis is another cause of dyspnea. A recent visit to Asia increases the likelihood of tuberculosis, while smoking is a possible cause of both lung cancer and bronchitis. (From Neopolitan p183)

   (a) Construct a Bayesian Network to model the causes and effects described in this scenario. Note that this will not be a singly connected network.

   (b) Since the resulting network is not singly connected, propagation may not terminate in all cases. Identify which single node when instantiated will ensure that probability propagation will terminate.

2. Returning to the network introduced in tutorial 1, with the given conditional probability distributions:



Variable	Interpretation	Value
C	Cat	c_1, c_2
E	Eyes	e_1, e_2, e_3
S	Separation of the eyes	$s_1, s_2, s_3, s_4, s_5, s_6, s_7$
D	Difference in eye size	d_1, d_2, d_3, d_4
F	Fur colour	$f_1, f_2, \cdots f_{10}$

Suppose that for a given data set we obtain the following link matrices:

$$P(D|E) = \begin{bmatrix} P(d_1|e_1) & P(d_1|e_2) & P(d_1|e_3) \\ P(d_2|e_1) & P(d_2|e_2) & P(d_2|e_3) \\ P(d_3|e_1) & P(d_3|e_2) & P(d_3|e_3) \\ P(d_4|e_1) & P(d_4|e_2) & P(d_4|e_3) \end{bmatrix} = \begin{bmatrix} 0.4 & 0.33 & 0.29 \\ 0.4 & 0.33 & 0.14 \\ 0.2 & 0.34 & 0.14 \\ 0 & 0.33 & 0.43 \end{bmatrix}$$

$$P(S|E) = \begin{bmatrix} P(s_1|e_1) & P(s_1|e_2) & P(s_1|e_3) \\ P(s_2|e_1) & P(s_2|e_2) & P(s_2|e_3) \\ P(s_3|e_1) & P(s_3|e_2) & P(s_3|e_3) \\ P(s_4|e_1) & P(s_4|e_2) & P(s_4|e_3) \\ P(s_5|e_1) & P(s_5|e_2) & P(s_5|e_3) \\ P(s_6|e_1) & P(s_6|e_2) & P(s_6|e_3) \\ P(s_7|e_1) & P(s_7|e_2) & P(s_7|e_3) \end{bmatrix} = \begin{bmatrix} 0 & 0.33 & 0.14 \\ 0.6 & 0 & 0.14 \\ 0.4 & 0.34 & 0 \\ 0 & 0.33 & 0.14 \\ 0 & 0 & 0.14 \\ 0 & 0 & 0.14 \\ 0 & 0 & 0.28 \end{bmatrix}$$

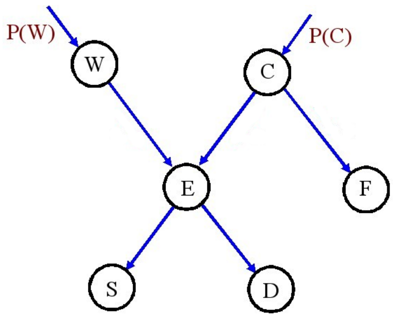
$$P(F|C) = \begin{bmatrix} P(f_1|c_1) & P(f_1|c_2) \\ P(f_2|c_1) & P(f_2|c_2) \\ P(f_3|c_1) & P(f_3|c_2) \\ P(f_4|c_1) & P(f_4|c_2) \\ P(f_5|c_1) & P(f_5|c_2) \\ P(f_6|c_1) & P(f_6|c_2) \\ P(f_7|c_1) & P(f_7|c_2) \\ P(f_8|c_1) & P(f_8|c_2) \\ P(f_9|c_1) & P(f_9|c_2) \\ P(f_{10}|c_1) & P(f_{10}|c_2) \end{bmatrix} = \begin{bmatrix} 0 & 0.3 \\ 0.125 & 0 \\ 0.125 & 0.14 \\ 0.25 & 0.14 \\ 0.125 & 0 \\ 0.125 & 0 \\ 0 & 0.14 \\ 0.125 & 0.14 \\ 0 & 0.14 \\ 0.125 & 0 \end{bmatrix}$$

$$P(E|C) = \begin{bmatrix} P(e_1|c_1) & P(e_1|c_2) \\ P(e_2|c_1) & P(e_2|c_2) \\ P(e_3|c_1) & P(e_3|c_2) \end{bmatrix} = \begin{bmatrix} 0.5 & 0.14 \\ 0.25 & 0.14 \\ 0.25 & 0.72 \end{bmatrix}$$

   (a) Given that C is instantiated and found to be in state c_2, what π evidence will be propagated to node E?

   (b) Given that C is not instantiated but has a prior probability distribution: $[P(c_1) = 0.6, P(c_2) = 0.4]$ and F is instantiated and found to be in state f_4, what π evidence will be propagated to node E?

3. The network is now changed to include the W node with two states w_1 meaning the picture shows an Owl, w_2 the picture does not show an owl. The same data base of pictures is used, and the state of w_2 is determined for each. Thus only the link matrix between E and W and C changes. Calculate this matrix. (For the case where you have no data assume an equi-probable distribution.)



w_2	c_1	e_1	s_2	d_1	f_5
w_2	c_1	e_1	s_2	d_2	f_6
w_1	c_2	e_1	s_3	d_2	f_4
w_2	c_1	e_1	s_2	d_3	f_3
w_2	c_1	e_1	s_3	d_1	f_4
w_1	c_2	e_2	s_1	d_1	f_7
w_2	c_1	e_2	s_3	d_2	f_{10}
w_2	c_1	e_2	s_4	d_3	f_4
w_2	c_1	e_3	s_5	d_1	f_8
w_2	c_2	e_3	s_4	d_2	f_3
w_2	c_2	e_3	s_6	d_3	f_1
w_2	c_1	e_3	s_7	d_1	f_2
w_2	c_2	e_3	s_1	d_4	f_9
w_2	c_2	e_3	s_2	d_4	f_8
w_1	c_2	e_3	s_7	d_4	f_1

4. Assuming that the λ evidence for E from nodes S and D is $\lambda(e_1) = 0.5, \lambda(e_2) = 0.3, \lambda(e_3) = 0.2$ and that the state of F and C are as defined in question 2b above, and that the W node is instantiated and is in state w_2, calculate the posterior probability distribution of E and C.