Tutorial 9 solution

1. In the original data space the pooled covariance estimate $S_p$ has at most a rank $N$-$g$. Therefore the scatter matrix $S_w = (N\text{-}g)\ S_p$ similarly has maximum rank of $N$-$g$, thus the PCA projection can have at most $N$-$g$ non-zero eigenvectors. Therefore the dimension of P is (at best) ($n$ x $N - g$).

2. $\mathrm{Xp} = \mathrm{X}_{\mathrm{Nxn}} \cdot \mathrm{P}_{\mathrm{nx(N-g)}}$. Xp therefore has dimension $N$x$(N$-$g)$.The LDA transformation matrix is limited by the number of groups because the rank of $S_b$ is $g-1$. Therefore the dimension of L is ($N-g$ x $g-1$).

3. $\mathrm{Xf} = \mathrm{Xp}_{\mathrm{Nx(N\text{-}g)}} \cdot \mathrm{L}_{\mathrm{(N-g)x(g\text{-}1)}}$. The final dimension of the data matrix is the dimension of Xf, that is, ($N$ x $g-1$).

4. $\mathrm{Y} = \mathrm{Xf}_{\mathrm{Nx(g\text{-}1)}} \cdot \mathrm{L}^{\mathrm{T}}_{\mathrm{(g\text{-}1)x(N\text{-}g)}} \cdot \mathrm{P}^{\mathrm{T}}_{\mathrm{(N\text{-}g)x(n)}}$. No, Y is an approximation of X because in the PCA step we have had to discard some principal components to overcome the singularity of $S_w$. We have used ($N - g$) principal components rather than ($N - 1$).