Tutorial 9: Linear Discriminant Analysis (LDA)

As we have seen in the lecture, the standard LDA can be seriously degraded if there are only a limited number of observations *N* compared to the dimension of the feature space *n*. One possible way to overcome this problem is to use a two-stage feature extraction method such as PCA followed by LDA (PCA+LDA). We will analyse this two-stage technique in more details and check that whether we lose some information compared to the standard LDA.

1. Let an *N* x *n* training set matrix X be composed of *N* observations with *n* variables. Assume that X is defined with n columns and N rows (the transpose of the U matrix defined in the lectures), and is mean adjusted so that all the columns sum to zero. We write the dimensions as subscripts thus: X_{Nxn}.
If the PCA projection matrix is P what is the maximum number of non-zero eigenvectors that P can have? What is the corresponding dimension of P? Assume that the columns of P are its corresponding eigenvectors, there are *g* groups to discriminate (*g* > 1), and all *N* observations are linearly independent.

2. The most expressive features matrix Xp is found by projecting the training set X on P. What dimension does Xp have (assuming the PCA projection is maximum)? What is the maximum dimension of the linear discriminant transformation L for discriminating the *g* groups?

3. The most discriminant features matrix Xf is found by projecting Xp on L. Define the final dimension of the data matrix (Xf) in the PCA+LDA space.

4. Calculate matrix Y which is a reconstruction of the original data by projecting Xf back to the original *n*-dimensional space. Can we say that Y = X? Explain your answer.