Tutorial 8: Solution

1. A simple procedure that contains the main steps in calculating P can be described as follows:

   (a) Calculate the covariance matrix S of X

   (b) Find the eigenvectors Φ and eigenvaluesΛ of S;

   (c) Order the eigenvector-eigenvalue pairs (λ_1, ϕ_1), $(\lambda_2, \phi_2) \cdots$ (λ_n, ϕ_n) such that $\lambda_1 \geq \lambda_2 \cdots \geq \lambda_n$

   (d) Form the matrix P = $P = [\phi_1, \phi_2, \cdots \phi_k]$ where:

   $$\sum_{j=1}^{k} \lambda_j / \sum_{i=1}^{n} \lambda_i \geq \nu$$

2. Assuming that the correlation matrix is written as:

   $$S = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

   (a) The correlation matrix can be written:

   $$R = \begin{bmatrix} \sigma_{11}/(\sqrt{\sigma}_{11}\sqrt{\sigma}_{11}) & \sigma_{12}/(\sqrt{\sigma}_{11}\sqrt{\sigma}_{22}) \\ \sigma_{21}/(\sqrt{\sigma}_{22}\sqrt{\sigma}_{11}) & \sigma_{22}/(\sqrt{\sigma}_{22}\sqrt{\sigma}_{22}) \end{bmatrix} = \begin{bmatrix} 1 & 0.4 \\ 0.4 & 1 \end{bmatrix}$$

   (b) $\lambda_1 = 100.16 \qquad \lambda_2 = 0.84$

   (c) $\phi_1 = \begin{bmatrix} 0.040 \\ 0.999 \end{bmatrix} \qquad \phi_2 = \begin{bmatrix} 0.999 \\ -0.04 \end{bmatrix}$

   (d) $\lambda_1/(\lambda_1 + \lambda_2) = 100.16/101 = 0.992$
   x_2 completely dominates ϕ_1 because of its large variance compared to x_2.

   (e) $\lambda_1 = 1.4 \qquad \lambda_2 = 0.6 \qquad \phi_1 = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix} \qquad \phi_2 = \begin{bmatrix} 0.707 \\ -0.707 \end{bmatrix}$

   (f) $\lambda_1/(\lambda_1 + \lambda_2) = 1.4/2 = 0.7$
   When the covariance matrix is normalised into the correlation matrix the resulting variables x_1 and x_2 contribute equally to the principal components determined from R.

   (g) This exercise demonstrates that the normalisation is not inconsequential. Variables should be normalised if they are measured on scales with widely differing ranges. Otherwise, we should expect a principal component with a heavy weighting of the variable with the relatively large variance.