Tutorial 9 solution

1. In the original data space the pooled covariance estimate Σ_p has at most a rank *N-g*. Therefore the scatter matrix $S_w = (N\text{-}g)\ \Sigma_p$ similarly has maximum rank of *N-g,* thus the PCA projection can have at most *N-g* non-zero eigenvectors. Therefore the dimension of P is (at best) (n x $N-g$).

2. $\mathrm{Xp} = \mathrm{X}_{\mathrm{Nxn}} \cdot \mathrm{P}_{\mathrm{nx(N-g)}}$. Xp therefore has dimension *N*x*(N-g)*.The LDA transformation matrix is limited by the number of groups because the rank of S_b is $g-1$. Therefore the dimension of L is ($N-g$ x $g-1$).

3. $\mathrm{Xf} = \mathrm{Xp}_{\mathrm{Nx(N-g)}} \cdot \mathrm{L}_{\mathrm{(N-g)x(g-1)}}$. The final dimension of the data matrix is the dimension of Xf, that is, (N x $g-1$).

4. $\mathrm{Y} = \mathrm{Xf}_{\mathrm{Nx(g-1)}} \cdot \mathrm{L}^{\mathrm{T}}_{\mathrm{(g-1)x(N-g)}} \cdot \mathrm{P}^{\mathrm{T}}_{\mathrm{(N-g)x(n)}}$. No, Y is an approximation of X because in the PCA step we have had to discard some principal components to overcome the singularity of S_w. We have used (*N* – *g*) principal components rather than (*N* – 1).