Tutorial 2: Analysis of three dimensional space.

This tutorial is about the use of vector algebra in the analysis of three dimensional scenes used in computer graphics system. The following notation is used:

Position vectors are denoted by boldface capital letters: **P**, **Q**, **V** etc. Position vectors are the same as cartesian coordinates, and represent position relative to the origin.

Direction vectors are indicated by boldface lowercase letters **d**, **n** etc. Direction vectors are independent of any origin.

A plane is an object that is only defined in Cartesian space, however, each plane has a normal vector, whose size is non zero, and whose direction is at right angles to that plane. We can find a normal vector by taking the cross product of any two direction vectors which are parallel to the plane.

1. Given three points **P1**=[10,20,5], **P2**=[15,10,10], **P3**=[25,20,10], find two direction vectors which are parallel to the plane defined by **P1**, **P2** and **P3,** and hence a normal vector to the plane.

2. A plane is defined in vector terms by the equation:

$$\mathbf{n}.(\mathbf{P}-\mathbf{P1})=0$$

where **P**=[x,y,z] is the locus of a point on the plane, and **P1** is any known point on the plane. For the points given in part 1, expand the vector plane equation to find the Cartesian form of the plane equation, (ie ax+by+cz+d=0). Verify that you get the same result using **P2** rather than **P1**.

3. Write a procedure in any programming language you like which takes as input three points and returns the coefficients of the Cartesian plane equation (a,b,c and d).

4. Starting from any point on a face of a polyhedron, an inner surface normal is a normal vector to the plane of the face whose direction points into the polyhedron A tetrahedron is defined by the three points of part 1, and a fourth point **P4**=[30,20,10]. Determine whether the normal vector that you calculated in part 1 is an inner surface normal, and if not find the inner surface normal.

5. Two lines intersect at a point **P1**, and are in the directions defined by **d1** and **d2**. Provided that **d1** and **d2** are different directions, the two lines define a plane. Any point on the plane can be reached by travelling from **P1** in direction **d1** by some distance μ and then in direction **d2** by a distance ν. Using this fact construct the parametric equation of any point on the plane of part 1 in terms of μ,ν, **P1**, **P2** and **P3**. By taking the dot product with a normal vector to the plane **n**, show that the parametric equation is equivalent to the plane equation of part 2.