Tutorial 8: Spline Curves and Surfaces

1. A four knot, two dimensional Bezier curve is defined by the following table

	x	y
$\mathbf{P_0}$	0	0
$\mathbf{P_1}$	2	3
$\mathbf{P_2}$	3	-1
$\mathbf{P_3}$	0	0

Use Casteljau's construction to sketch the curve.

Calculate the coefficients $\mathbf{a_0}$, $\mathbf{a_1}$, $\mathbf{a_2}$ and $\mathbf{a_3}$ of the corresponding cubic spline patch:
$\mathbf{P}(\mu) = \mathbf{a_3}\mu^3 + \mathbf{a_2}\mu^2 + \mathbf{a_1}\mu + \mathbf{a_0}$

Differentiate the spline patch equation to find $\mathbf{P}'(\mu)$ and hence show that the gradient at $\mathbf{P_3}$ is the same as the gradient of the line joining $\mathbf{P_3}$ to $\mathbf{P_2}$.

2. A Coons surface patch is to be drawn using the following array of points:

		μ			
		-1	0	1	2
ν	-1	(0,0,0)	(0,10,5)	(0,20,10)	(0,30,20)
	0	(10,0,5)	(10,10,20)	(10,25,30)	(15,35,40)
	1	(20,0,10)	(20,12,40)	(20,30,50)	(25,40,30)
	2	(30,0,5)	(35,15,30)	(40,35,40)	(50,50,20)

We are interested in the patch constructed on the centre knots, P[0,0], P[0,1], P[1,0] and P[1,1].

a. Find the equations of the four cubic spline patches that bound the Coon's Patch $\mathbf{P}(\mu,0)$, $\mathbf{P}(\mu,1)$, $\mathbf{P}(0,\nu)$, $\mathbf{P}(1,\nu)$. These are each parametric cubic splines of the form:

$\mathbf{P} = \mathbf{a_3}\mu^3 + \mathbf{a_2}\mu^2 + \mathbf{a_1}\mu + \mathbf{a_0}$
whose parameters are found using:

$$\begin{pmatrix} \mathbf{a_0} \\ \mathbf{a_1} \\ \mathbf{a_2} \\ \mathbf{a_3} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{bmatrix} \begin{pmatrix} \mathbf{P_i} \\ \mathbf{P'_i} \\ \mathbf{P_{i+1}} \\ \mathbf{P'_{i+1}} \end{pmatrix}$$

b. Find the point at the centre of the patch using the equation:

$$\mathbf{P}(\mu,\nu) = \mathbf{P}(\mu,0)(1-\nu) + \mathbf{P}(\mu,1)\nu + \mathbf{P}(0,\nu)(1-\mu) + \mathbf{P}(1,\nu)\mu$$
$$- \mathbf{P}(0,0)(1-\nu)(1-\mu) - \mathbf{P}(0,1)\nu(1-\mu) - \mathbf{P}(1,0)(1-\nu)\mu - \mathbf{P}(1,1)\nu\mu$$

NB: the numerical solution to this is rather tedious unless you use a programmable calculator or spreadsheet