## Tutorial 9: Warping and Morphing

1. Explain what is meant by the following equation:

$$
\text { morphing }=(\text { warping })^{2}+\text { blending }
$$

2. In the algorithm developed by Beier and Neeley pairs of lines are used to specify the warping. In a concrete example two pairs of lines specify a 2 D warping: In the source image the line $\mathrm{L}_{1}$ starts at $(1,1)$ and ends at $(1,9)$. The line $L_{2}$ starts at $(9,2)$ and ends at $(9,8)$. In the target image the corresponding line $L_{1}$ starts at $(1,1)$ and ends at $(1,9)$ while $L_{2}$ starts at $(3,2)$ and ends at $(9,2)$. Calculate where the pixel $\mathbf{p}=(5,5)$ in the source image would map to in the target image. Assume that the constants controlling the warping are $a=b=p=1$.
3. An image with $300 \times 175$ pixels is warped using a two-dimensional free-form deformation based on linear B-splines defined by a $6 \times 6$ mesh of control points.
a. Calculate the spacing between control points in pixels.
b. Calculate the pixel coordinates for the following B-spline integer lattice coordinates $i, j$ and the fractional lattice coordinates $u, v$ :
i. $(\mathrm{i}, \mathrm{j})=(1,1)$ and $(\mathrm{u}, \mathrm{v})=(0,0)$
ii. $(\mathrm{i}, \mathrm{j})=(1,1)$ and $(\mathrm{u}, \mathrm{v})=(0.5,0.5)$
iii. $(\mathrm{i}, \mathrm{j})=(1,3)$ and $(\mathrm{u}, \mathrm{v})=(0.75,0.2857)$
c. Calculate the B-spline integer lattice coordinates $\mathrm{i}, \mathrm{j}$ and the fractional lattice coordinates $\mathrm{u}, \mathrm{v}$ for the following pixels:
i. $(x, y)=(120,140)$
ii. $(x, y)=(100,100)$
iii. $(x, y)=(150,130)$
d. Calculate the new location of a pixel $(x, y)=(135,122.5)$ after warping. The matrix of control points looks as follows:

| $(1,4)$ | $(-3,7)$ | $(3,8)$ | $(4,7)$ | $(0,1)$ | $(2,3)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $(-3,7)$ | $(-2,9)$ | $(2,7)$ | $(3,1)$ | $(2,-2)$ | $(2,2)$ |
| $(4,2)$ | $(3,8)$ | $(2,1)$ | $(4,2)$ | $(2,1)$ | $(3,1)$ |
| $(3,2)$ | $(2,9)$ | $(-3,8)$ | $(6,8)$ | $(3,4)$ | $(3,5)$ |
| $(-1,3)$ | $(-2,3)$ | $(1,3)$ | $(2,3)$ | $(8,3)$ | $(-4,2)$ |
| $(0,0)$ | $(-2,1)$ | $(1,1)$ | $(-2,2)$ | $(1,2)$ | $(0,0)$ |

