UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2006

BEng Honours Degree in Computing Part III
MSc in Computing Science
MEng Honours Degree in Electrical Engineering Part IV
MSc in Computing for Industry
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
MSci Honours Degree in Mathematics and Computer Science Part IV
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C317=I3.16=E4.32

GRAPHICS

Monday 8 May 2006, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 Transformations of Graphics Scenes

a In a computer graphics animation scene an object is defined as a planar polyhedron. The centre of the object is located at position P = [10,0,10], and the scene is drawn, as usual, in perspective projection with the viewpoint at the origin and the view direction along the z-axis. Calculate the transformation matrix that will shrink the object in size by a factor of 0.9 towards its centre point.

b In a different animation, the object, defined in part a is required to rotate clockwise (viewed from the origin) while shrinking. In each successive frame it is to rotate by 10° while shrinking to 0.9 of its original size. The rotation axis is to be the z axis, and the shrinkage is, as before, towards the object's centre. Given that $\text{Cos}(10^{\circ}) = .98$ and $\text{Sin}(10^{\circ}) = .17$, what is the transformation matrix that will achieve this animation?

c For another sequence the object is to shrink, as defined in part a, and to drop vertically downwards by 1 unit each frame. If the animation sequence is made up of a number of frames, numbered consecutively from zero, what is the transformation matrix that should be applied at frame n?

d The scene is to be viewed from a moving viewpoint specified by its position **C** and a left handed viewing coordinate system [***u***,***v***,***w***]. At one point in the animation the view direction is ***w*** = [-1, 0, 0], and the viewpoint is given by **C** = [50,10,-10]. Given that the view is in the horizontal plane (***v*** = [0,1,0]) find the value of ***u***.

e Hence, or otherwise, find the viewing transformation matrix.

The five parts carry equal marks

2 A cubic spline patch is defined by the parametric equation:

$$\mathbf{P}(\mu) = \mathbf{a_3}\,\mu^3 + \mathbf{a_2}\,\mu^2 + \mathbf{a_1}\,\mu + \mathbf{a_0}$$

where $\mathbf{P}(\mu)$ is a point that traces the locus of the curve with $0<\mu<1$ and $\mathbf{a_3}\ \mathbf{a_2}\ \mathbf{a_1}\ \mathbf{a_0}$ are vector constants that define the shape of the curve.

a Given that the patch is to be drawn between two points $\mathbf{P_i}$ and $\mathbf{P_{i+1}}$ and the gradients at the ends are to be $\mathbf{P'_i}$ and $\mathbf{P'_{i+1}}$ respectively, write down four equations connecting $\mathbf{P_i}\ \mathbf{P_{i+1}}\ \mathbf{P'_i}\ \mathbf{P'_{i+1}}$ and $\mathbf{a_3}\ \mathbf{a_2}\ \mathbf{a_1}\ \mathbf{a_0}$.

b Solve the above equations to find the values of matrix A in the equation:

$$\begin{pmatrix} \mathbf{a_0} \\ \mathbf{a_1} \\ \mathbf{a_2} \\ \mathbf{a_3} \end{pmatrix} = A \begin{pmatrix} \mathbf{P_i} \\ \mathbf{P'_i} \\ \mathbf{P_{i+1}} \\ \mathbf{P'_{i+1}} \end{pmatrix}$$

c Given that the spline patch is one of several making up a more complex curve, and the points are defined as $\mathbf{P_{i-1}} = [0,0]$, $\mathbf{P_i} = [1,3]$, $\mathbf{P_{i+1}} = [5,3]$and $\mathbf{P_{i+2}} = [4,2]$, calculate the values of the gradients $\mathbf{P'_i}\ \mathbf{P'_{i+1}}$ using the central difference approximation and hence calculate the values of $\mathbf{a_3}\ \mathbf{a_2}\ \mathbf{a_1}\ \mathbf{a_0}$.

d Calculate the co-ordinate and the gradient direction of the mid point of the spline patch.

The four parts carry, respectively, 20%, 30%, 30% and 20% of the marks.

3 Illumination and ray tracing

a Explain in detail how to calculate ambient, diffuse and specular illumination at a surface point **P** as seen from a viewpoint **V**. Assume a single light source.

b Explain how this illumination model can be modified to account for effects such as reflections and transparency.

c Show in detail how you can calculate whether a ray has any intersections with a sphere with radius r centered at $\mathbf{p}_s$.

d A scene contains the following three geometric primitives:

A sphere **A** with centre $\mathbf{p}_s = (10, 5, 0)$ and radius $r = 6$
A sphere **B** with centre $\mathbf{p}_s = (32, 1, 1)$ and radius $r = 3$
A sphere **C** with centre $\mathbf{p}_s = (0, -3, 8)$ and radius $r = 2$

A ray originates at the viewpoint $\mathbf{p}_v = (-5, 0, 0)$ and passes through the viewing plane at $\mathbf{p}_i = (0, 0, 0)$. Calculate the intersections of the ray with the spheres above.

e Calculate the intersections of the ray for the following objects, assuming the spheres defined above.

$A + B + C$
$A - B$
$(B - A) \cap C$
$A + (B \cap C)$

The five parts carry, respectively, 20%, 10%, 30%, 20% and 20% of the marks.

4 Warping and Morphing

a Briefly describe the Beier-Neely algorithm for warping.

b A particular Beier-Neely warping is defined by two lines: in the source image line $\mathbf{l}_1$ goes from (0, 0) to (20, 0) and line $\mathbf{l}_2$ goes from (0, 0) to (0, 20). In the destination image line $\mathbf{l}_1$ has been changed so that the line goes from (0, 0) to (20, 5) while line $\mathbf{l}_2$ has not been changed. Given three points $\mathbf{p}_1$ = (5, 15), $\mathbf{p}_2$ = (10, 10) and $\mathbf{p}_3$ = (15, 5) calculate their position in the warped image. You can assume that the contribution of each line pair to the warping is equal.

c The convex hull of a set of 2D control points is partitioned into a set of triangles. Derive an expression for the position of a point $\mathbf{p}$ after a piecewise affine warping.

d Using a sketch, show which problem may arise using piecewise affine warping. How can this problem be overcome?

The four parts carry, respectively, 20%, 30%, 30% and 20% of the marks.