

*Interactive Computer Graphics*

Lecture 10: Ray tracing

Graphics Lecture 10: Slide 1

Some slides adopted from  
H. Pfister, Harvard



*Direct and Global Illumination*

- Direct illumination: A surface point receives light directly from all light sources in the scene.
  - Computed by the direct illumination model.
- Global illumination: A surface point receives light after the light rays interact with other objects in the scene.
  - Points may be in shadow.
  - Rays may refract through transparent material.
  - Computed by reflection and transmission rays.

Graphics Lecture 10: Slide 4

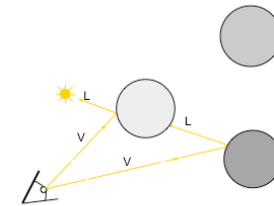
## Albrecht Dürer's Ray Casting Machine

- Albrecht Dürer, 16<sup>th</sup> century



## Arthur Appel, 1968

- On calculating the illusion of reality, 1968
- Cast one ray per pixel (ray casting).
  - For each intersection, trace one ray to the light to check for shadows
  - Only a local illumination model
- Developed for pen-plotters



Graphics Lecture 10: Slide 6

## Ray casting

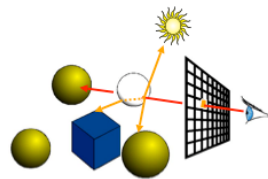
cast ray

Intersect all objects

color = ambient term

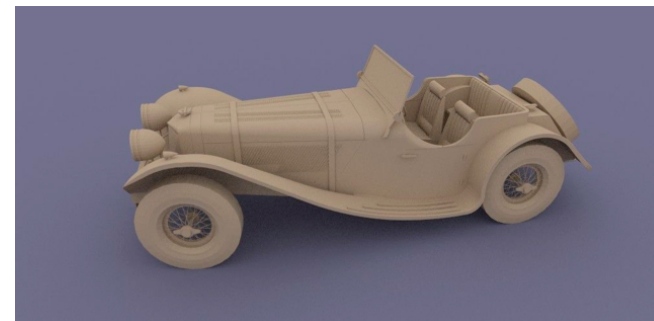
For every light cast shadow ray

col += local shading term



Graphics Lecture 10: Slide 7

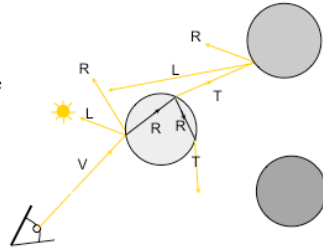
## Ray Casting



Graphics Lecture 10: Slide 8

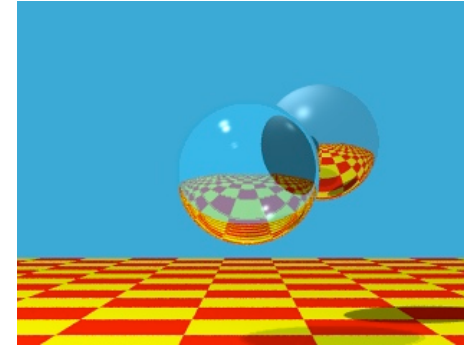
### Turner Whitted, 1980

- An Improved Illumination Model for Shaded Display, 1980
- First global illumination model:
  - An object's color is influenced by lights and other objects in the scene
  - Simulates specular reflection and refractive transmission



Graphics Lecture 10: Slide 9

### Turner Whitted, 1980

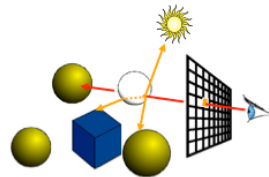


Graphics Lecture 10: Slide 10

### Recursive ray casting

```

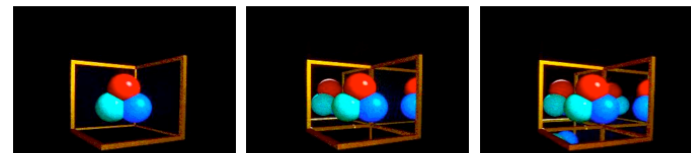
trace ray
  Intersect all objects
  color = ambient term
  For every light
    cast shadow ray
    col += local shading term
  If mirror
    col += k_refl * trace reflected ray
  If transparent
    col += k_trans * trace transmitted ray
    
```



Graphics Lecture 10: Slide 11

### Does it ever end?

- Stopping criteria:
  - Recursion depth: Stop after a number of bounces
  - Ray contribution: Stop if reflected / transmitted contribution becomes too small



0 recursion

1 recursion

2 recursions

Graphics Lecture 10: Slide 12

### Ray tracing: Primary rays

- For each ray we need to test which objects are intersecting the ray:
  - If the object has an intersection with the ray we calculate the distance between viewpoint and intersection
  - If the ray has more than one intersection, the smallest distance identifies the visible surface.
- Primary rays are rays from the view point to the nearest intersection point
- Local illumination is computed as before:

$$L = k_a + (k_d(\mathbf{n} \cdot \mathbf{l}) + k_s(\mathbf{v} \cdot \mathbf{r})^q)I_s$$

Graphics Lecture 10: Slide 13

### Ray tracing: Secondary rays

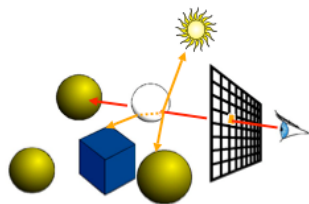
- Secondary rays are rays originating at the intersection points
- Secondary rays are caused by
  - rays reflected off the intersection point in the direction of reflection
  - rays transmitted through transparent materials in the direction of refraction
  - shadow rays

Graphics Lecture 10: Slide 14

### Recursive ray tracing: Putting it all together

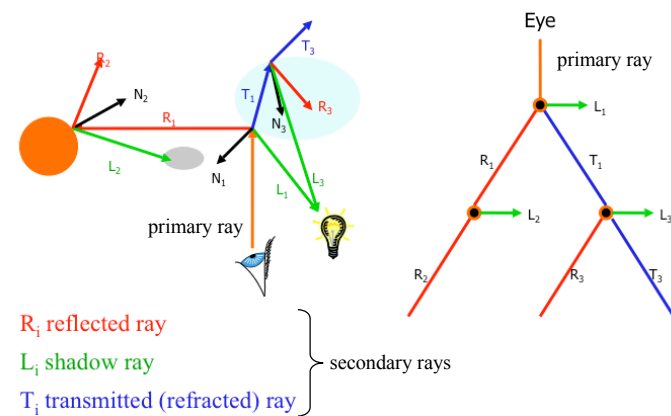
- Illumination can be expressed as

$$L = k_a + (k_d(\mathbf{n} \cdot \mathbf{l}) + k_s(\mathbf{v} \cdot \mathbf{r})^q)I_s + k_{reflected}L_{reflected} + k_{refracted}L_{refracted}$$

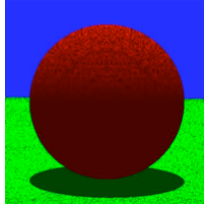


Graphics Lecture 10: Slide 15

### Recursive Ray Tracing: Ray Tree

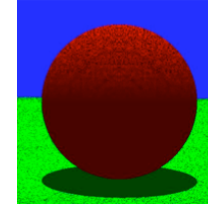


## Precision Problems



Graphics Lecture 10: Slide 17

## Precision Problems



- In ray tracing, the origin of (secondary) rays is often on the surface of objects
  - Theoretically, the intersection point should be on the surface
  - Practically, calculation imprecision creeps in, and the origin of the new ray is slightly beneath the surface
- Result: the surface area is shadowing itself

Graphics Lecture 10: Slide 18

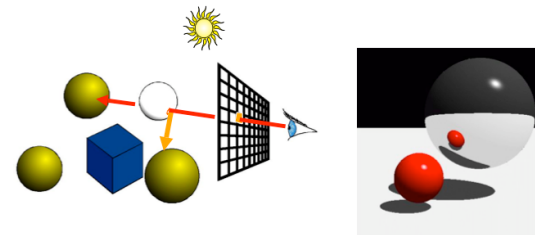
## $\epsilon$ to the rescue ...

- Check if  $t$  is within some epsilon tolerance:
  - if  $\text{abs}(\mu) < \epsilon$ 
    - point is on the surface
  - else
    - point is inside/outside
  - Choose the  $\epsilon$  tolerance empirically
- Move the intersection point by epsilon along the surface normal so it is outside of the object
- Check if point is inside/outside surface by checking the sign of the implicit (sphere etc.) equation

Graphics Lecture 10: Slide 19

## Mirror reflection

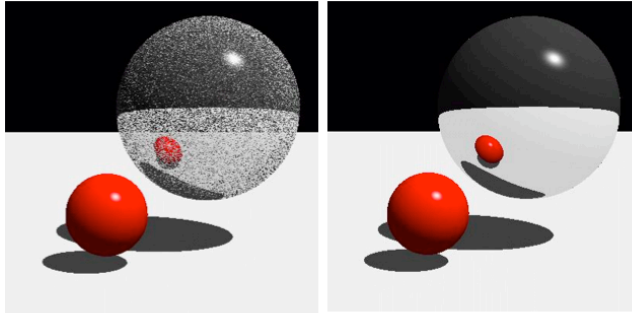
- Compute mirror contribution
- Cast ray in direction symmetric wrt. normal
- Multiply by reflection coefficient (color)



Graphics Lecture 10: Slide 20

## Mirror reflection

- Don't forget to add epsilon to the ray

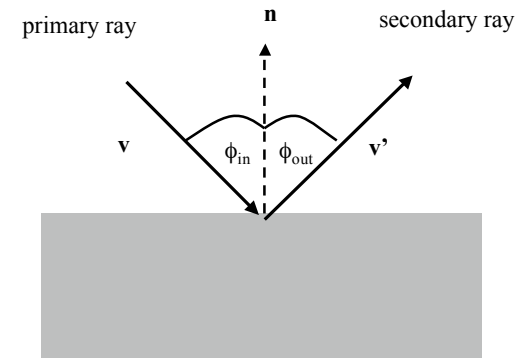


Without epsilon

With epsilon

Graphics Lecture 10: Slide 21

## Mirror reflection



Graphics Lecture 10: Slide 22

## Mirror reflection

- To calculate illumination as a result of reflections
  - calculate the direction of the secondary ray at the intersection of the primary ray with the object.
- given that
  - n is the unit surface normal
  - v is the direction of the primary ray
  - v' is the direction of the secondary ray as a result of reflections

$$\mathbf{v}' = \mathbf{v} - (2\mathbf{v} \cdot \mathbf{n})\mathbf{n}$$

Graphics Lecture 10: Slide 23

## Mirror reflection

The  $\mathbf{v}$ ,  $\mathbf{v}'$  and  $\mathbf{n}$  are unit vectors and coplanar so:

$$\mathbf{v}' = \alpha \mathbf{v} + \beta \mathbf{n}$$

Taking the dot product with  $\mathbf{n}$  yields the eq.:

$$\mathbf{n} \cdot \mathbf{v}' = \alpha \mathbf{v} \cdot \mathbf{n} + \beta = \mathbf{v} \cdot \mathbf{n}$$

Requiring  $\mathbf{v}'$  to be a unit vector yields the second eq.:

$$1 = \mathbf{v}' \cdot \mathbf{v}' = \alpha^2 + 2 \alpha \beta \mathbf{v} \cdot \mathbf{n} + \beta^2$$

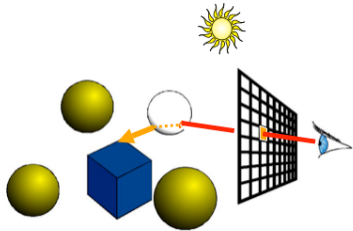
- Solving both equations yields:

$$\mathbf{v}' = \mathbf{v} - (2\mathbf{v} \cdot \mathbf{n})\mathbf{n}$$

Graphics Lecture 10: Slide 24

## Transparency

- Compute transmitted contribution
- Cast ray in refracted direction
- Multiply by transparency coefficient



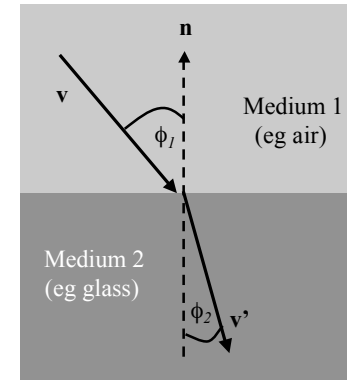
Graphics Lecture 10: Slide 25

## Refraction

- The angle of the refracted ray can be determined by Snell's law:

$$k_1 \sin(\phi_1) = k_2 \sin(\phi_2)$$

- $\eta_1$  is a constant for medium 1
- $\eta_2$  is a constant for medium 2
- $\phi_1$  is the angle between the incident ray and the surface normal
- $\phi_2$  is the angle between the refracted ray and the surface normal



Graphics Lecture 10: Slide 26

## Refraction

- In vector notation Snell's law can be written:

$$k_1 (\mathbf{v} \times \mathbf{n}) = k_2 (\mathbf{v}' \times \mathbf{n})$$

- The direction of the refracted ray is

$$\mathbf{v}' = \frac{\eta_1}{\eta_2} \left( \left[ \sqrt{(\mathbf{n} \cdot \mathbf{v})^2 + \left(\frac{\eta_2}{\eta_1}\right)^2} - 1 - \mathbf{n} \cdot \mathbf{v} \right] \cdot \mathbf{n} + \mathbf{v} \right)$$

Graphics Lecture 10: Slide 27

## Refraction

- This equation only has a solution if

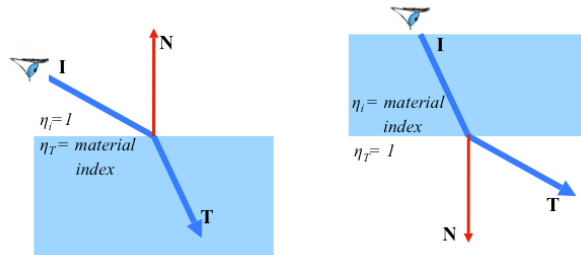
$$(\mathbf{n} \cdot \mathbf{v})^2 > 1 - \left(\frac{\eta_2}{\eta_1}\right)^2$$

- This illustrates the physical phenomenon of the limiting angle:
  - if light passes from one medium to another medium whose index of refraction is low, the angle of the refracted ray is greater than the angle of the incident ray
  - if the angle of the incident ray is large, the angle of the refracted ray is larger than  $90^\circ$ 
    - ➔ the ray is reflected rather than refracted

Graphics Lecture 10: Slide 28

## Refraction

- Make sure you know whether you are entering or leaving the transmissive material

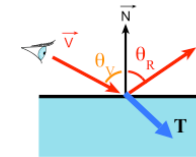


Graphics Lecture 10: Slide 29

## Amount of reflection and refraction

- Traditional (hacky) ray tracing
  - Constant coefficient reflectionColor
  - Component per component multiplication
- Better: Mix reflected and refracted light according to the Fresnel factor.

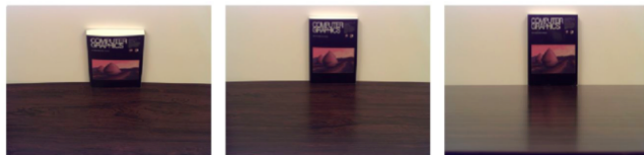
$$L = k_{fresnel} L_{reflected} + (1 - k_{fresnel}) L_{refracted}$$



Graphics Lecture 10: Slide 30

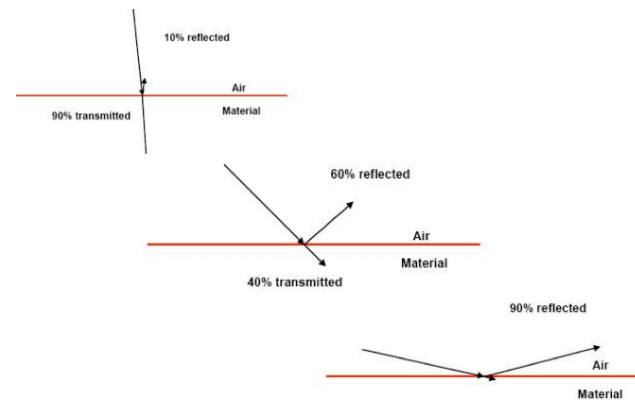
## Fresnel factor

- More reflection at grazing angle



Graphics Lecture 10: Slide 31

## Fresnel factor



Graphics Lecture 10: Slide 32



### Schlick's Approximation

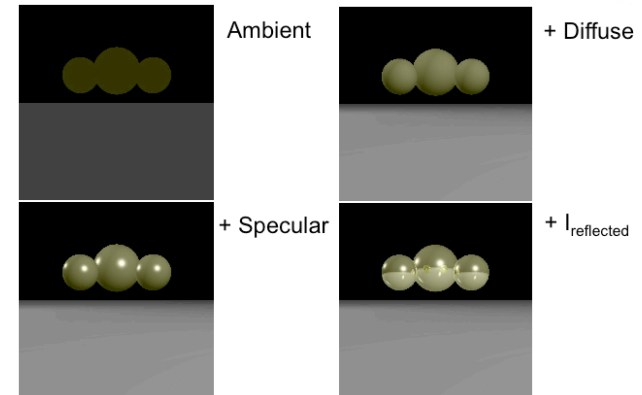
- Schlick's approximation

$$k_{fresnel}(\theta) = k_{fresnel}(0) + (1 - k_{fresnel}(0))(1 - (\mathbf{n} \cdot \mathbf{l}))^5$$

- $k_{fresnel}(0)$  = Fresnel factor at zero degrees
- Choose  $k_{fresnel}(0) = 0.8$ , this will look like stainless steel

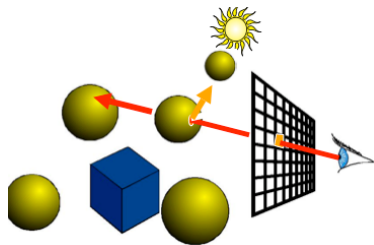
Graphics Lecture 10: Slide 33

### Example



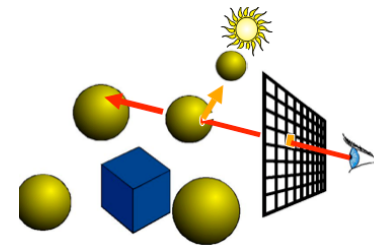
Graphics Lecture 10: Slide 34

### How do we add shadows?



Graphics Lecture 10: Slide 35

### How do we add shadows?



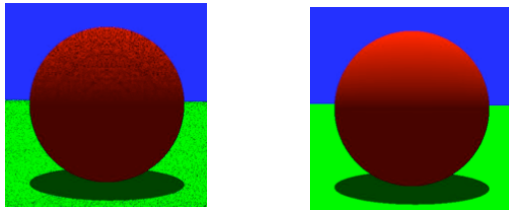
$$L = k_a + s(k_d(\mathbf{n} \cdot \mathbf{l}) + k_s(\mathbf{v} \cdot \mathbf{r})^q)I_s + k_{reflected}L_{reflected} + k_{refracted}L_{refracted}$$

$$s = \begin{cases} 0 & \text{if light source is obscured} \\ 1 & \text{if light source is not obscured} \end{cases}$$

Graphics Lecture 10: Slide 36

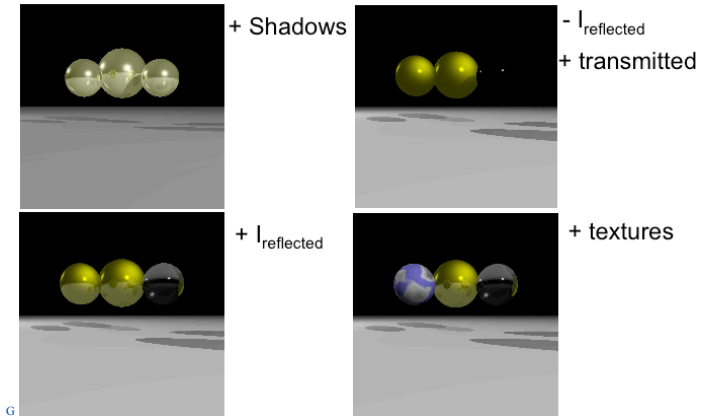
### Shadows: Problems?

- Make sure to avoid self-shadowing



Graphics Lecture 10: Slide 37

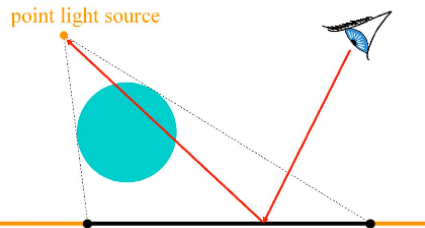
### Example



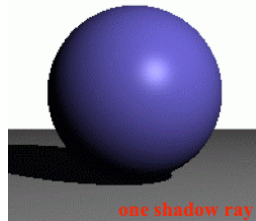
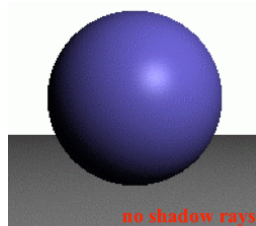
G

### Shadows

- One shadow ray per intersection per point light source

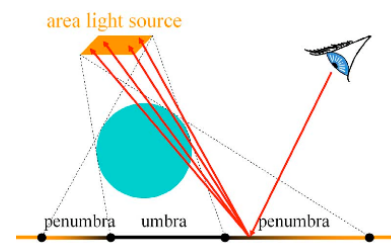


Graphics Lecture 10: Slide 39

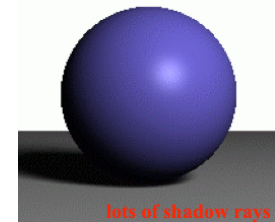
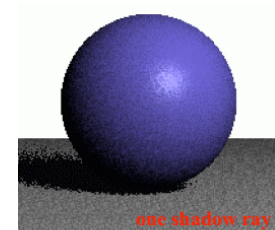


### Soft shadows

- Multiple shadow rays to sample area light source

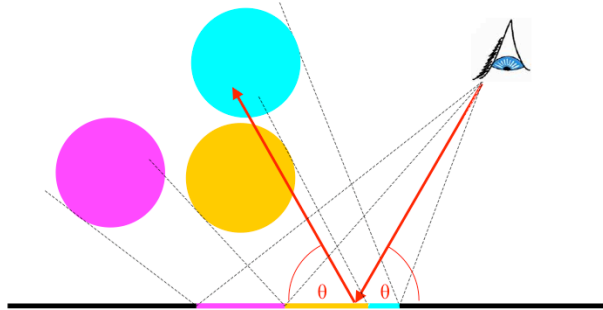


Graphics Lecture 10: Slide 40



### Reflection: Conventional ray tracing

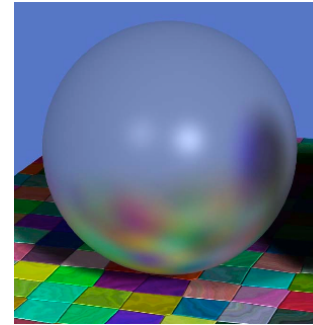
- One reflection per intersection



Graphics Lecture 10: Slide 41

### Reflection: Conventional ray tracing

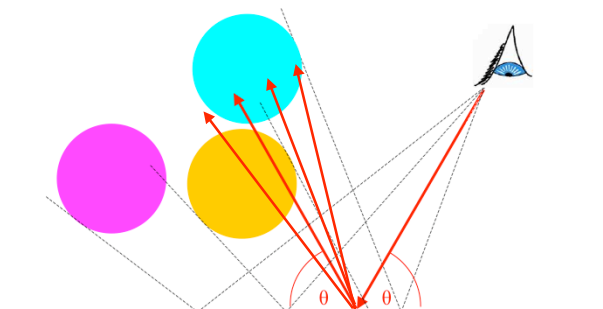
- How can we create effects like this?



Graphics Lecture 10: Slide 42

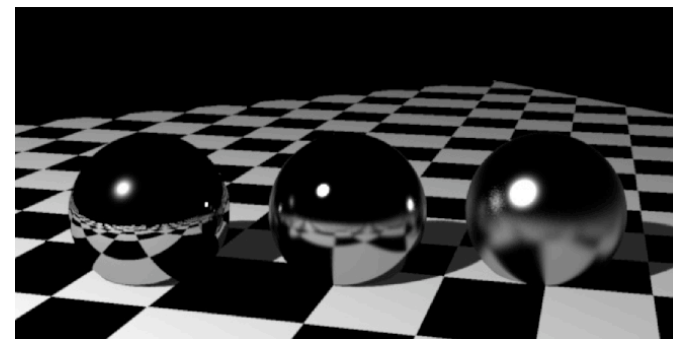
### Reflection: Monte Carlo ray tracing

- Random reflection rays around mirror direction



Graphics Lecture 10: Slide 43

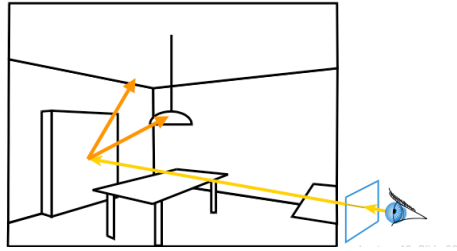
### Glossy surfaces



Graphics Lecture 10: Slide 44

### Ray tracing

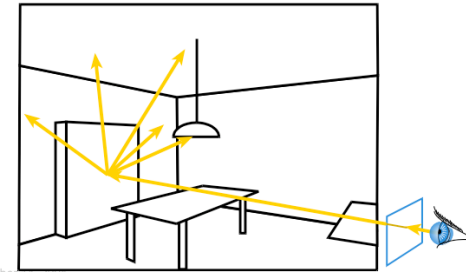
- Cast a ray from the eye through each pixel
- Trace secondary rays (light, reflection, refraction)



Graphics Lecture 10: Slide 45

### Monte-Carlo Ray Tracing

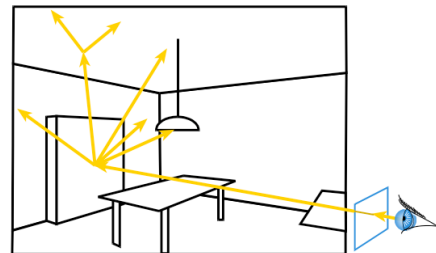
- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
  - Accumulate radiance contribution



Graphics Lecture 10: Slide 46

### Monte-Carlo Ray Tracing

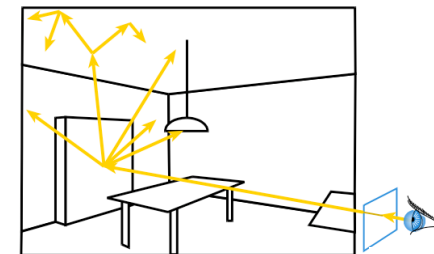
- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



Graphics Lecture 10: Slide 47

### Monte-Carlo Ray Tracing

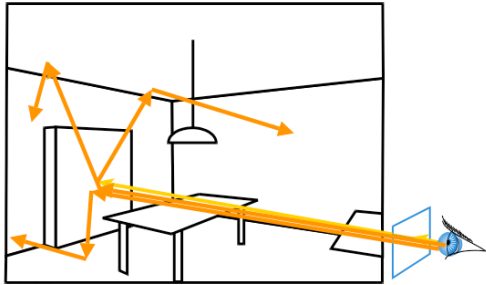
- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



Graphics Lecture 10: Slide 48

### Monte Carlo Path Tracing

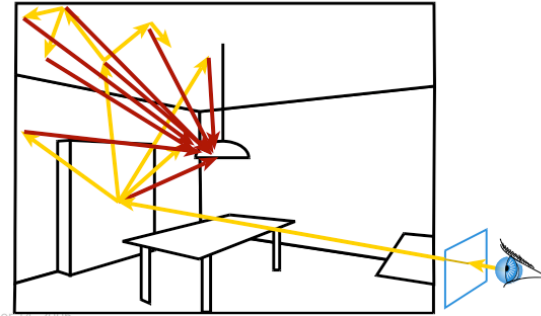
- Trace only one secondary ray per recursion
- But send many primary rays per pixel



Graphics Lecture 10: Slide 49

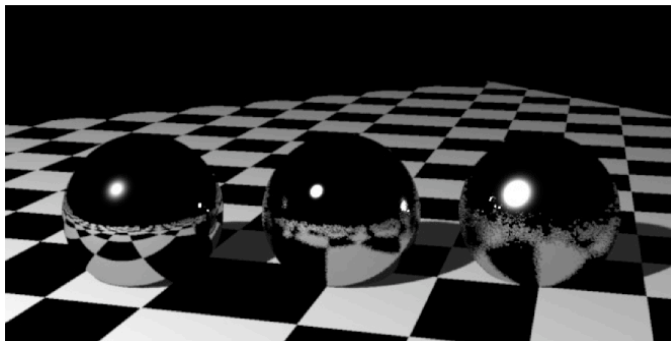
### Monte Carlo Ray Tracing

- Send rays to light



Graphics Lecture 10: Slide 50

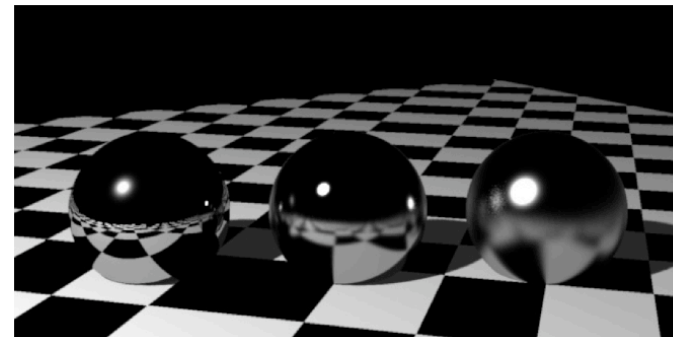
### Example



- 1 sample per pixel

Graphics Lecture 10: Slide 51

### Example



- 256 samples per pixel

Graphics Lecture 10: Slide 52

*Some cool pictures*



Graphics Lecture 10: Slide 53

*Some cool pictures*



Graphics Lecture 10: Slide 56

*Some cool pictures*



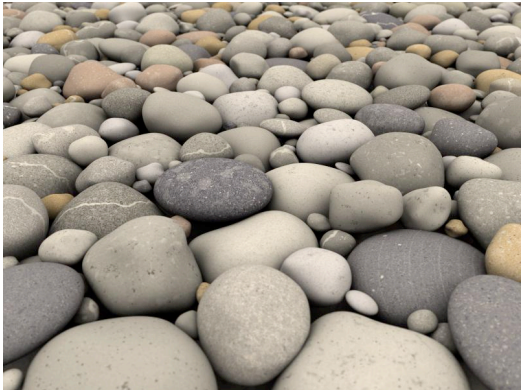
Graphics Lecture 10: Slide 55

*Some cool pictures*



Graphics Lecture 10: Slide 56

*Some cool pictures*



Graphics Lecture 10: Slide 57

took 4.5 days to render!