Interactive Computer Graphics

Lecturers: Duncan Gillies (dfg@doc.ic.ac.uk)
Daniel Rueckert (dr@doc.ic.ac.uk)

Tutors: Paul Aljabar (pa100@doc.ic.ac.uk)
David Thornley (djt@doc.ic.ac.uk)
Julien Pansiot (jpansiot@doc.ic.ac.uk)
Amardeep Singh (asingh@doc.ic.ac.uk)

Webpage:
http:/www.doc.ic.ac.uk/~dfg/graphics/graphics.html

Non-DOC Students

In order to do this course you need to get a department of computing login.

Please email dfg@doc.ic.ac.uk:

- Name
- CID
- IC Login
- Course
- Course Administrator

Interactive Computer Graphics

Lecture 1:

Three Dimensional Graphical Scenes, Projection and Transformation

Two Dimensional Graphics

The lowest level of graphics processing operates directly on the pixels in a window provided by the operating system.

Typical Primitives are:

SetPixel(int x, int y, int colour);

DrawLine(int xs, int ys, int xf, int yf);

etc.

World Coordinate Systems

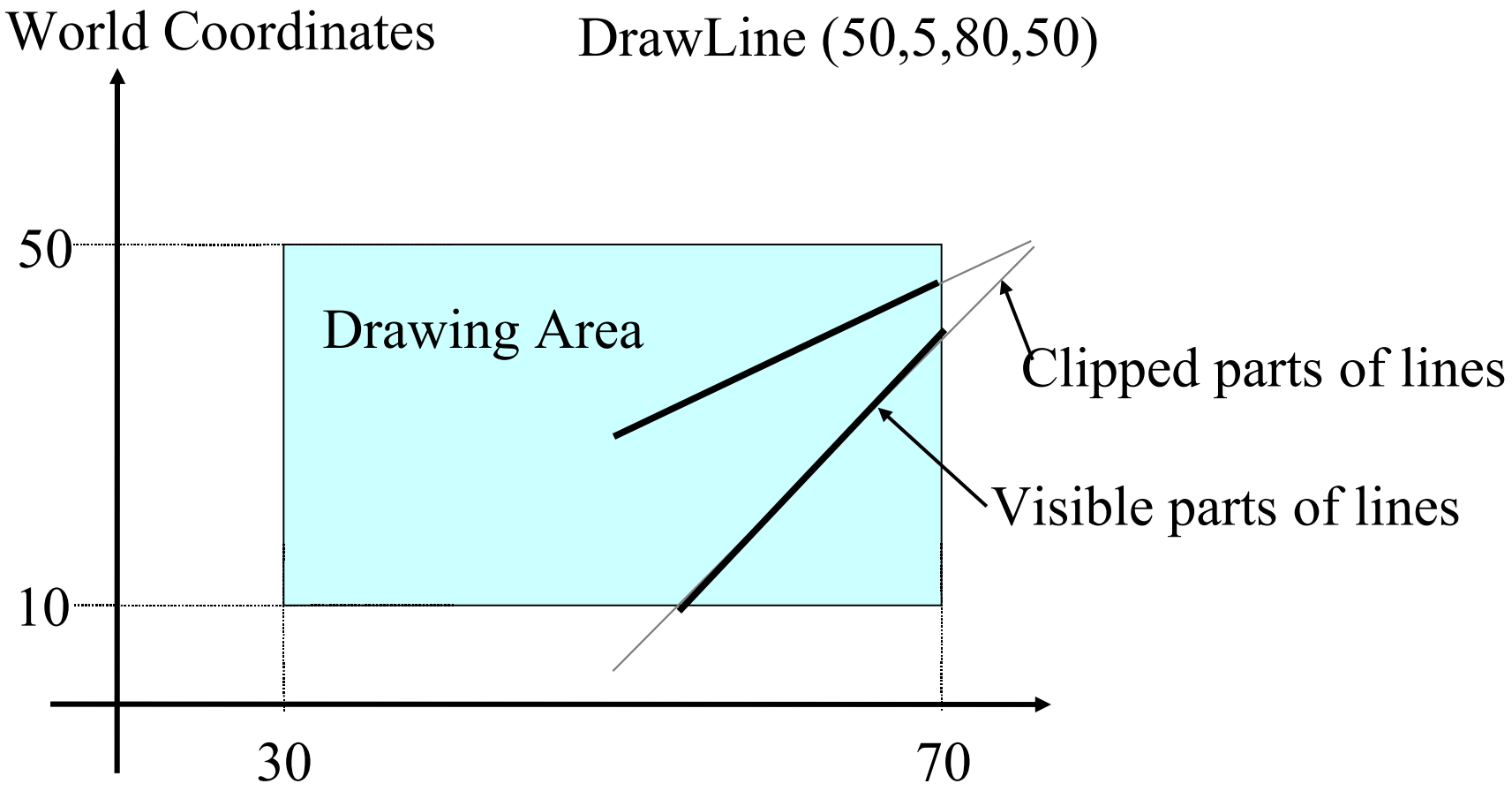
To achieve device independence when drawing objects we can define a world coordinate system.

This will define our drawing area in units that are suited to the application:

- meters
- light years
- microns
- etc

Example

SetWindow(30,10,70,50)
DrawLine (50,30,80,50)
DrawLine (50,5,80,50)



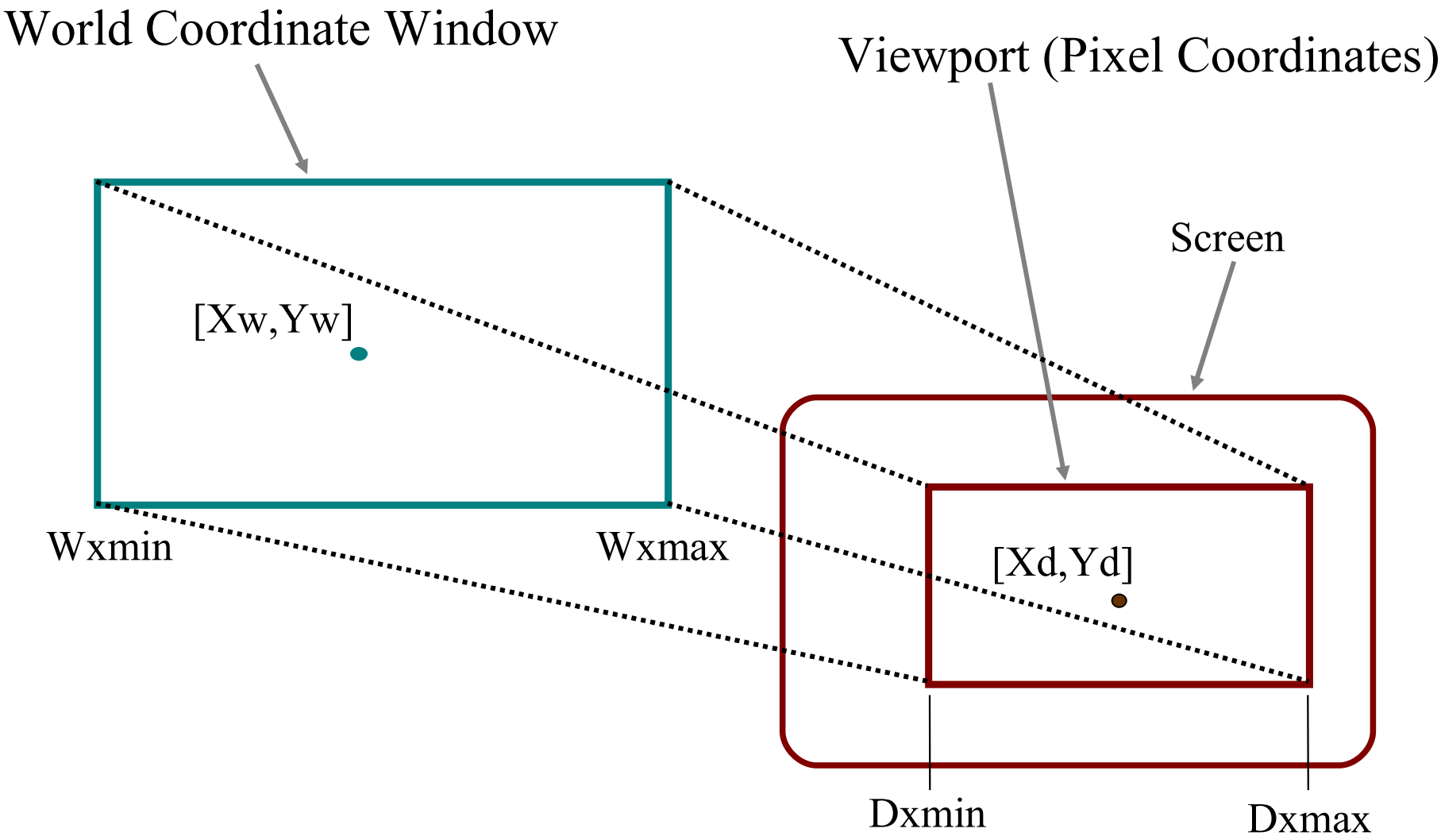
Normalisation

To map device independent graphics commands to the drawing commands using the screen pixels we need a process of normalisation.

First we must call the API to find out from the operating system the pixel addresses of the corners of the area we are using.

Then we translate the world coordinates to pixel coordinates.

Normalisation



Normalisation

Having defined our world coordinates, and obtained our device coordinates we relate the two by simple ratios:

$$\frac{X_w - WX_{min}}{WX_{max} - WX_{min}} = \frac{X_d - DX_{min}}{DX_{max} - DX_{min}}$$

rearranging we get:

$$X_d = \frac{(X_w - WX_{min})(DX_{max} - DX_{min})}{WX_{max} - WX_{min}} + DX_{min}$$

Normalisation

A similar equation allows us to calculate the Y pixel coordinate. The two form a simple pair of linear equations:

```
Xd := Xw * A + B;
Yd := Yw * C + D;
```

Where A, B, C and D are constants defining the normalisation

Input for Graphics Systems

An input event occurs when something changes, ie a mouse is moved or a button is pressed. The operating system informs the application program of events that are relevant to it.

The application program must receive this information in what is sometimes called a callback procedure (or event loop).

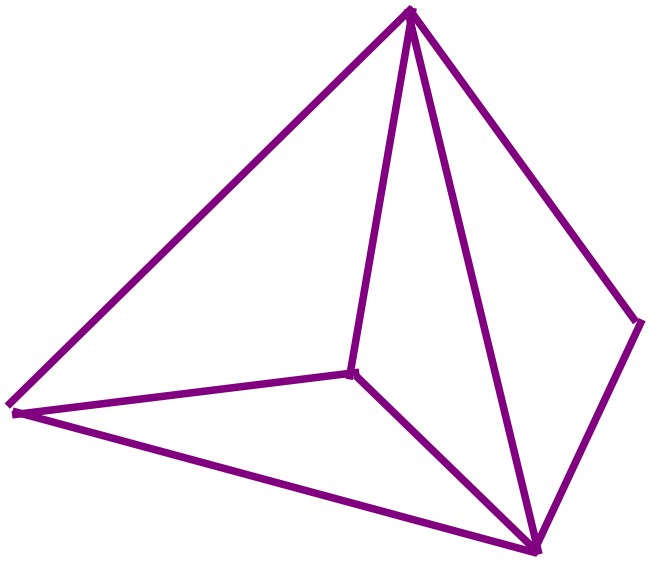
Simple Callback procedure

```
while (executing) do
{     if (menu event) ProcessMenuRequest();
      if (mouse event)
      {   GetMouseCoordinates();
          GetMouseButtons();
          PerformMouseProcess();
      }
  if (window resize event) RedrawGraphics();
}
```

Polygon Rendering

Many graphics applications use scenes built out of planar polyhedra.

These are three dimensional objects whose faces are all *planar polygons* often called *facets*.



Representing Planar Polygons

In order to represent planar polygons in the computer we will require a mixture of numerical and topological data.

Numerical Data

Actual 3D coordinates of vertices, etc.

Topological Data

Details of what is connected to what

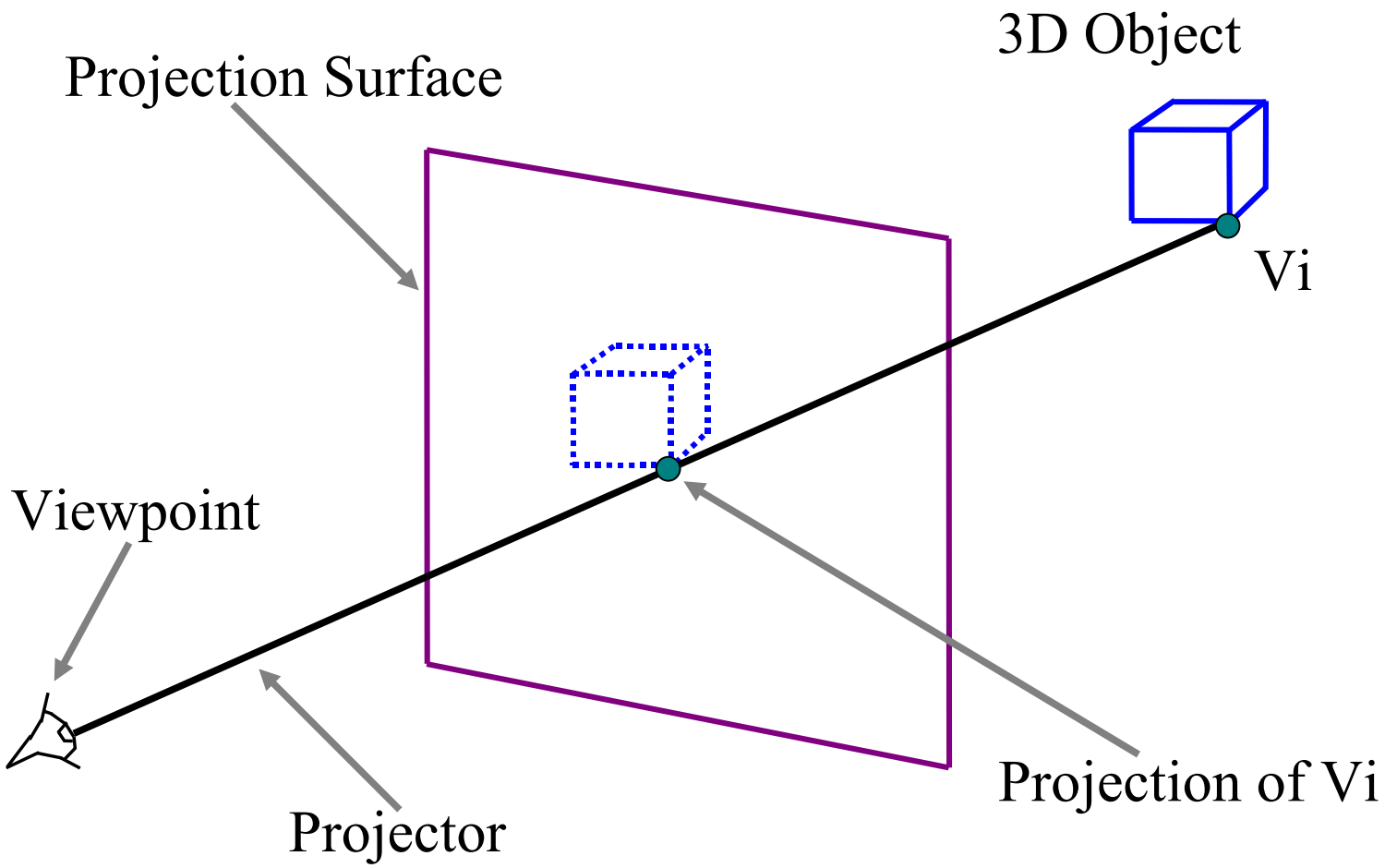
Projections of Wire Frame Models

Wire frame models simply include points and lines.

In order to draw a 3D wire frame model we must first convert the points to a 2D representation. Then we can use simple drawing primitives to draw them.

The conversion from 3D into 2D is a projection.

Projection


3D Object
Projection Surface
Vi
Viewpoint
Projection of Vi
Projector

Non Linear Projections

In general it is possible to project onto any surface:

- Sphere
- Cone
- etc

or to use curved projectors, for example to produce lens effects.

However we will only consider planar linear projections.

Normal Orthographic Projection

This is the simplest form of projection, and effective in many cases.

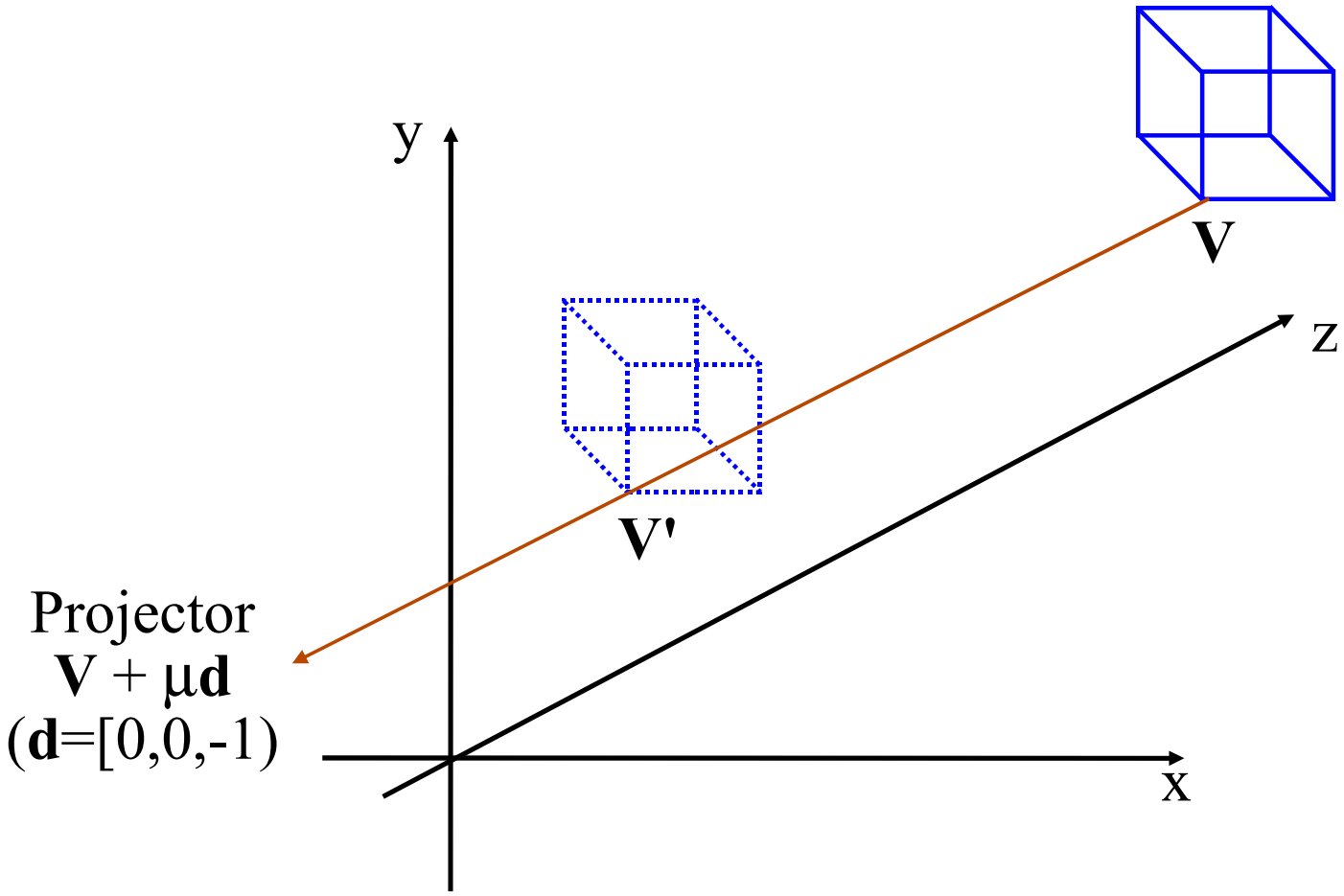
The viewpoint is at $z = -\infty$

The plane of projection is $z=0$

so

All projectors have direction $d = [0,0,-1]$

Orthographic Projection onto z=0



Calculating an Orthographic Projection

Projector Equation:

$\mathbf{P} = \mathbf{V} + \mu\mathbf{d}$ (from vertex $\mathbf{V}$)

Substitute $\mathbf{d} = [0,0,-1]$

Yields cartesian form

$Px = Vx + 0 \quad Py = Vy + 0 \quad Pz = Vz - \mu$

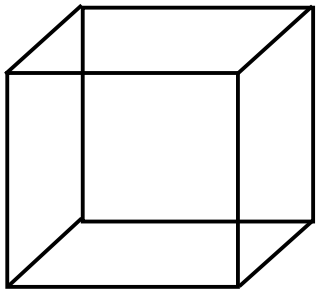
The projection plane is z=0 so the projected coordinate is

[Vx,Vy,0]

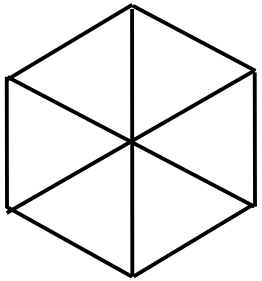
ie we simply take the 3D x and y components of the vertex

Orthographic Projection of a Cube

Looking at a Face



General View



Looking at a vertex

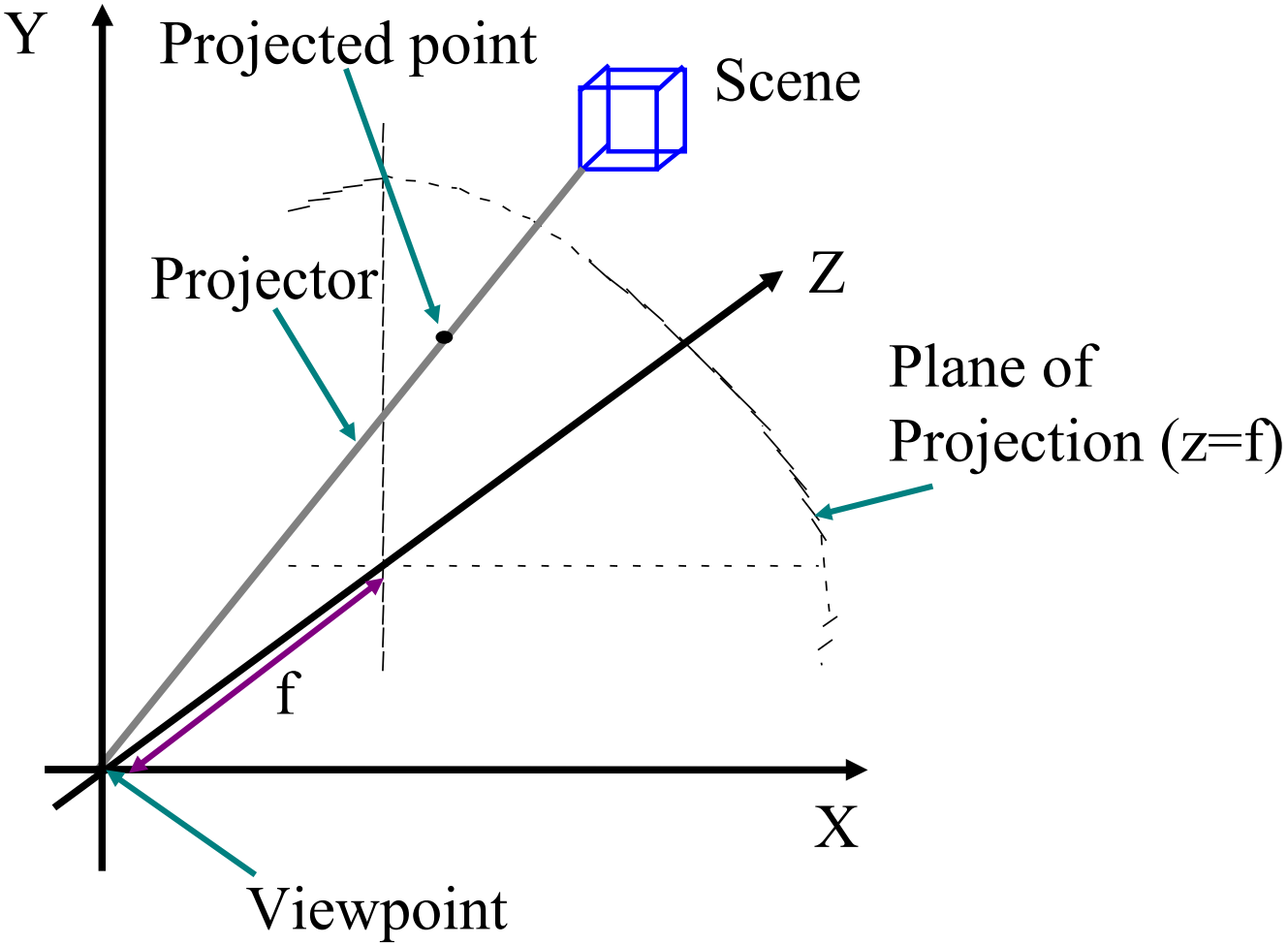
Perspective Projection

Orthographic projection is fine in cases where we are not worried about depth (ie most objects are at the same distance from the viewer).

However for close work (particularly computer games) it will not do.

Instead we use perspective projection

Canonical Form for Perspective Projection



Calculating Perspective Projection

Projector Equation (from vertex **V**):

$\mathbf{P} = \mu\mathbf{V}$ (all projectors go through the origin)

At the projected point Pz=f

$\mu_p = Pz/Vz = f/Vz$

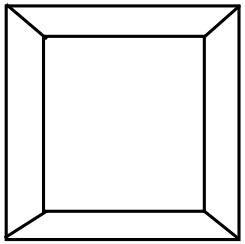
$Px = \mu_p Vx$ and $Py = \mu_p Vy$

Thus

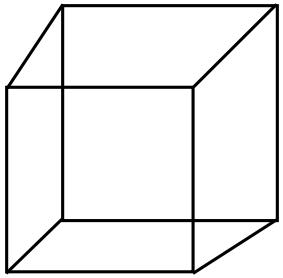
$Px = f\,Vx/Vz$ and $Py = f\,Vy/Vz$

The constant μ_p is sometimes called the fore-shortening factor

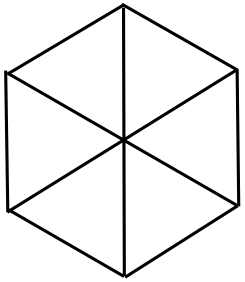
Perspective Projection of a Cube



Looking at a Face



General View



Looking at a vertex

Problem Break

Given that the viewpoint is at the origin, and the viewing plane is at z=5: What point on the viewplane corresponds to the 3D vertex {10,10,10} in

a. Perspective projection

b. Orthographic projection

Problem Break

Given that the viewpoint is at the origin, and the viewing plane is at z=5: What point on the viewplane corresponds to the 3D vertex {10,10,10} in

a. Perspective projection

b. Orthographic projection

Perspective $x' = f\,x/z = 5$ and $y' = f\,y/z = 5$

Orthographic $x' = 10$ and $y' = 10$

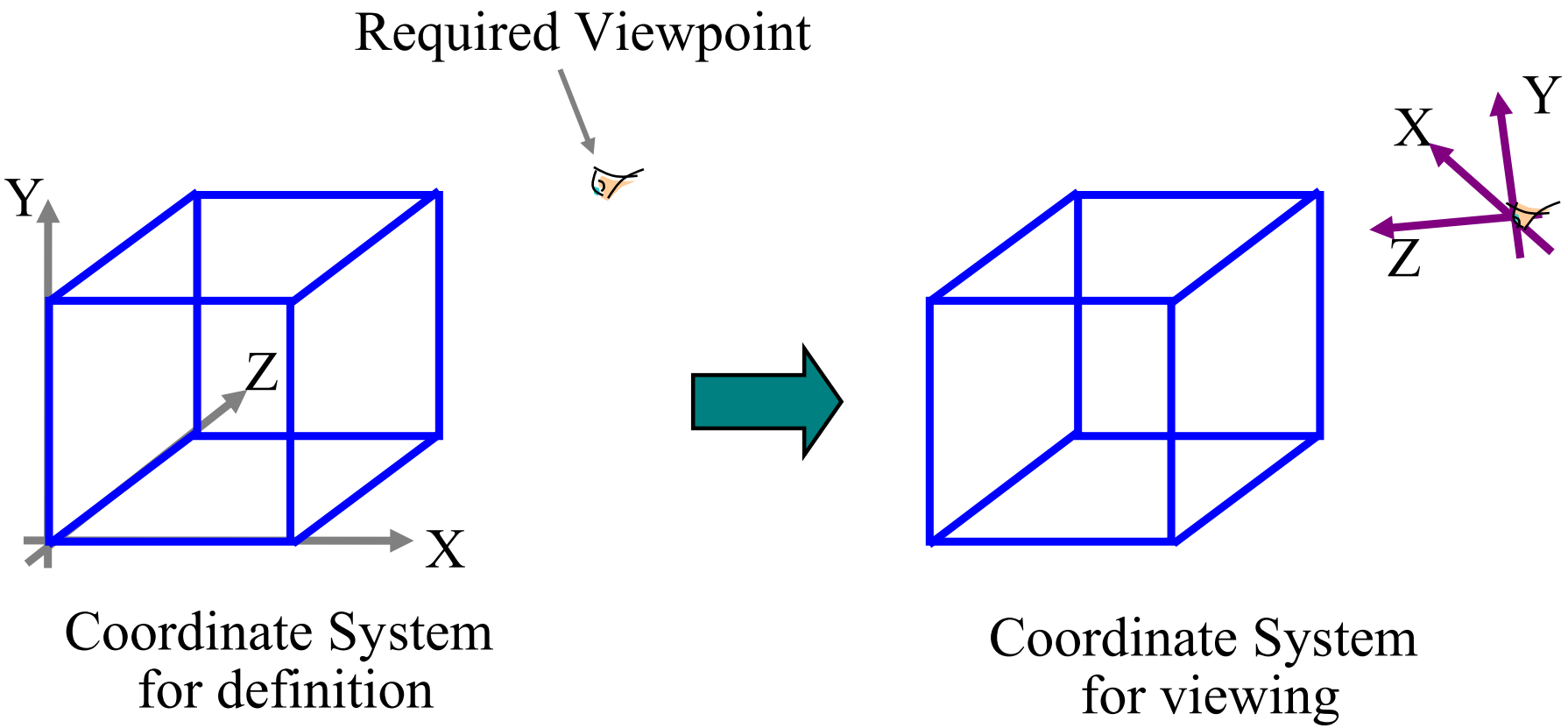
The Need for Transformations

Graphics scenes are defined in a particular co-ordinate system, however we want to be able to draw a graphics scene from any angle

To draw a graphics scene we need the viewpoint to be the origin and the z axis to be the direction of view.

Hence we need to be able to transform the coordinates of a graphics scene.

Transformation of viewpoint



Other Transformations

We also need transformations for other purposes:

Animating Objects

eg flying titles rotating shrinking etc.

Multiple Instances

the same object may appear at different places or different sizes

Reflections and other special effects

Matrix transformations of points

To transform points we use matrix multiplications, for example to make an object at the origin twice as big we could use:

$$[x', y', z'] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

which multiplied out gives:

$$x' = 2x \ , \ y' = 2y \ , \ z' = 2z$$

Translation by Matrix multiplication

Many of our transformations will require translation of the points.

For example if we want to move all the points two units along the x axis we would require:

$$x' = x + 2$$
$$y' = y$$
$$z' = z$$

But how can we do this with a matrix?

Honogenous Coordinates

The answer is to use 4D homogenous coordinates. The use of the fourth ordinate allows us to place a translation in the bottom row of the matrix.

$$[x', y', z', 1] = [x, y, z, 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

multiplying out gives:

x' = x + 2, y' = y, z' = z

General Homogenous Coordinates

In most cases the last ordinate will be 1, but in general it is a scale factor.

Thus, in the projection from 4D to 3D:

[x, y, z, s] is equivalent to [x/s, y/s, z/s]

Homogenous Cartesian

Affine Transformations

Affine transformations are those that preserve parallel lines.

Most transformations we require are affine, the most important being:

- Scaling
- Translating
- Rotating

Other more complex transforms will be built from these three.

Translation

We can apply a general translation by (tx, ty, tz) to the points of a scene by using the following matrix multiplication.

$$[x, y, z, 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix} = [x + t_x, y + t_y, x + t_z, 1]$$

Inverting a translation

Since we know what transformation matrices do, we can write down their inversions directly

For example:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix} \text{ has inversion } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -t_x & -t_y & -t_z & 1 \end{bmatrix}$$

Scaling

Scaling simply multiplies each ordinate by a scaling factor. It can be done with the following homogenous matrix:

$$[x, y, z, 1] \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [s_x x, s_y y, s_z z, 1]$$

Inverting scaling

To invert a scaling we simply divide the individual ordinates by the scale factor.

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ has inversion } \begin{bmatrix} 1/s_x & 0 & 0 & 0 \\ 0 & 1/s_y & 0 & 0 \\ 0 & 0 & 1/s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Combining transformations

Suppose we want to make an object at the origin twice as big and then move it to a point [5, 5, 20].

The transformation is a scaling followed by a translation:

$$[x', y', z', 1] = [x, y, z, 1] \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 5 & 5 & 20 & 1 \end{bmatrix}$$

Combined transformations

We multiply out the transformation matrices first, then transform the points

$$[x', y', z', 1] = [x, y, z, 1] \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 5 & 5 & 20 & 1 \end{bmatrix}$$

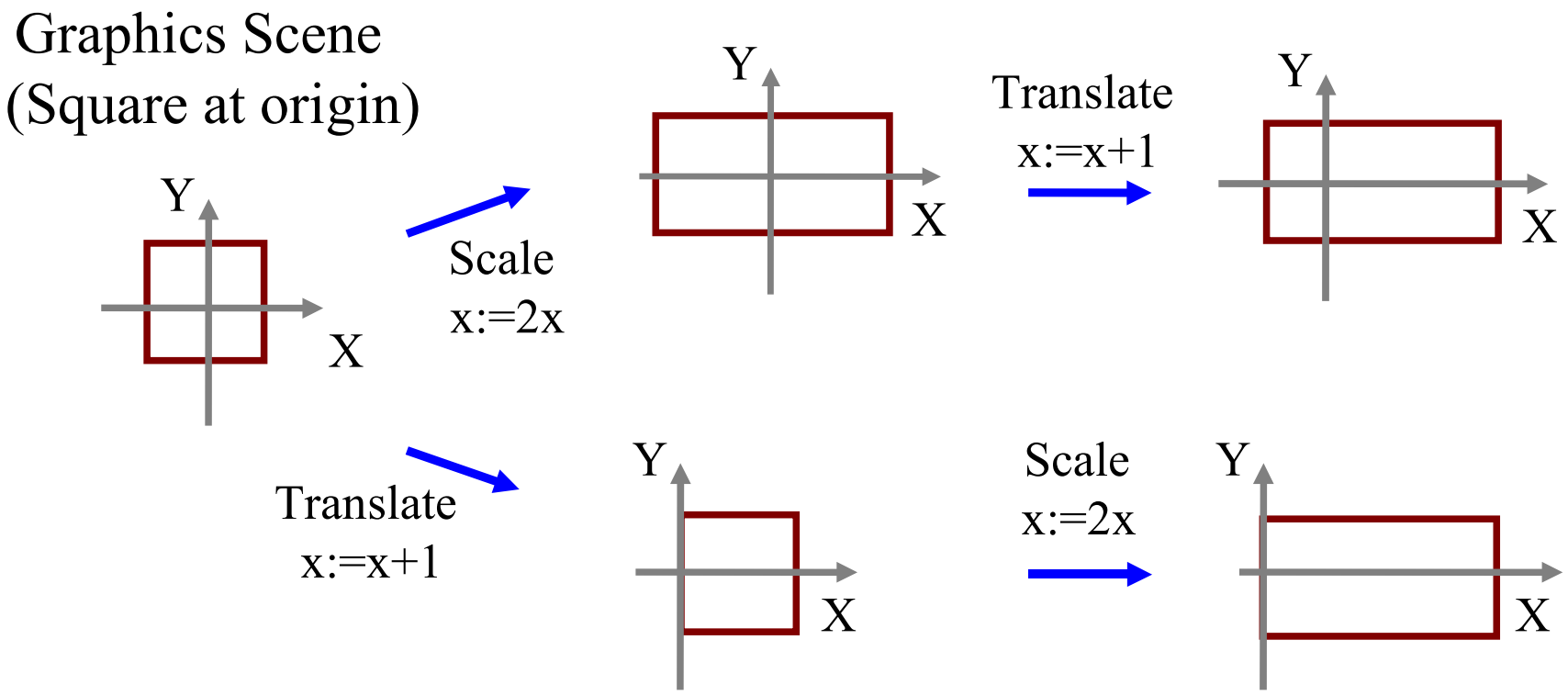
Transformations are not commutative

The order in which transformations are applied matters:

In general

T * ***S*** is not the same as ***S*** * ***T***

The order of transformations is significant



Rotation

To define a rotation we need an axis.

The simplest rotations are about the Cartesian axes

eg

Rx - Rotate about the X axis
Ry - Rotate about the Y axis
Rz - Rotate about the Z axis

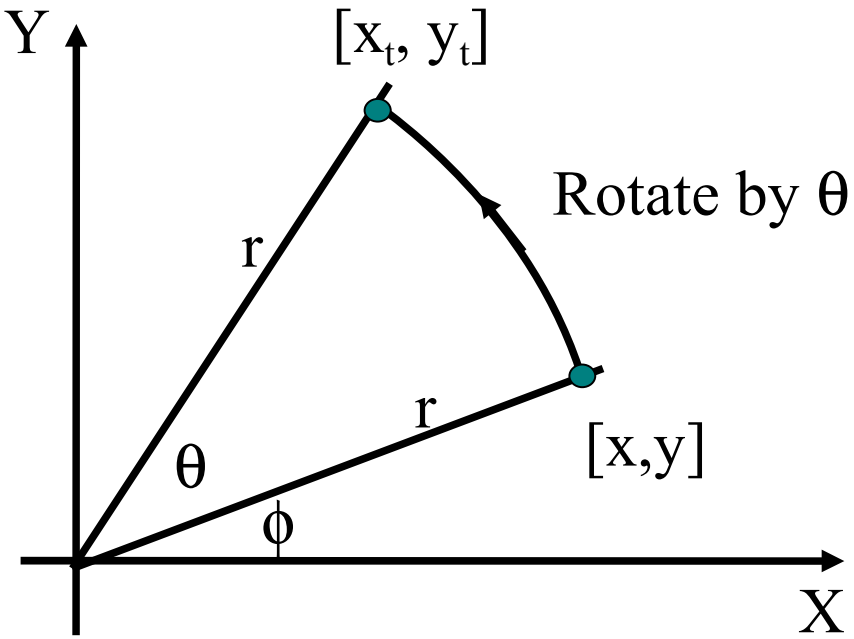
Rotation Matrices

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & Cos\theta & Sin\theta & 0 \\ 0 & -Sin\theta & Cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y = \begin{bmatrix} Cos\theta & 0 & -Sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ Sin\theta & 0 & Cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} Cos\theta & Sin\theta & 0 & 0 \\ -Sin\theta & Cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Deriving Rz

Error in Handout:
θ and ϕ are the wrong way round



$$
\begin{aligned}
[x, y] &= [\, rCos\phi \;,\; rSin\phi \,] \\
[x_t, y_t] &= [\, rCos(\phi + \theta) \;,\; rSin(\phi + \theta) \,] \\
&= [\, rCos\phi Cos\theta - rSin\phi Sin\theta \;,\; rCos\phi Sin\theta + rSin\phi Cos\theta \,] \\
&= [\, xCos\theta - ySin\theta \;,\; xSin\theta + yCos\theta \,] \\
&= [x, y] \begin{bmatrix} Cos\theta & Sin\theta \\ -Sin\theta & Cos\theta \end{bmatrix}
\end{aligned}
$$

Signs of Rotations

Rotations have a direction.

The following rule applies to the matrix formulations given in the notes:

Rotation is clockwise when viewed from the positive side of the axis

Inverting Rotation

Inverting a rotation by an angle θ is equivalent to rotating through an angle of -θ, now

$$\text{Cos}(-\theta) = \text{Cos}(\theta)$$

and

$$\text{Sin}(-\theta) = -\text{Sin}(\theta)$$

Inverting Rz

To invert a rotation matrix simply change the sign of the sin terms.

$$\begin{bmatrix} Cos\theta & Sin\theta & 0 & 0 \\ -Sin\theta & Cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ has inversion } \begin{bmatrix} Cos\theta & -Sin\theta & 0 & 0 \\ Sin\theta & Cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$