

## Tutorial 6: Spline Curves and Surfaces

1. A four knot, two dimensional Bezier curve is defined by the following table

	$(x, y)$
$\mathbf{P}_0$	$(0, 0)$
$\mathbf{P}_1$	$(2, 3)$
$\mathbf{P}_2$	$(3, -1)$
$\mathbf{P}_3$	$(0, 0)$

a. Use de Casteljau's construction to sketch the curve.

b. Calculate the coefficients  $\mathbf{a}_0$ ,  $\mathbf{a}_1$ ,  $\mathbf{a}_2$  and  $\mathbf{a}_3$  of the corresponding cubic spline patch:

$$\mathbf{P}(\mu) = \mathbf{a}_3\mu^3 + \mathbf{a}_2\mu^2 + \mathbf{a}_1\mu + \mathbf{a}_0$$

c. Differentiate the spline patch equation to find  $\mathbf{P}'(\mu)$  and hence show that the gradient at  $\mathbf{P}_3$  is the same as the gradient of the line joining  $\mathbf{P}_3$  to  $\mathbf{P}_2$ .

2. A Coons surface patch is to be drawn using the following array of points:

		$\mu$			
		-1	0	1	2
$\nu$	-1	$(0, 0, 0)$	$(0, 10, 5)$	$(0, 20, 10)$	$(0, 30, 20)$
	0	$(10, 0, 5)$	$(10, 10, 20)$	$(10, 25, 30)$	$(15, 35, 40)$
	1	$(20, 0, 10)$	$(20, 12, 40)$	$(20, 30, 50)$	$(25, 40, 30)$
	2	$(30, 0, 5)$	$(35, 15, 30)$	$(40, 35, 40)$	$(50, 50, 20)$

We are interested in the patch constructed on the centre knots,  $\mathbf{P}(0, 0)$ ,  $\mathbf{P}(0, 1)$ ,  $\mathbf{P}(1, 0)$  and  $\mathbf{P}(1, 1)$ .

a. Find the equations of the four cubic spline patches that bound the Coons Patch:

$$\mathbf{P}(\mu, 0), \mathbf{P}(\mu, 1), \mathbf{P}(0, \nu), \mathbf{P}(1, \nu)$$

These are each parametric cubic splines of the form:

$$\mathbf{P} = \mathbf{a}_3\mu^3 + \mathbf{a}_2\mu^2 + \mathbf{a}_1\mu + \mathbf{a}_0 \quad \text{or} \quad \mathbf{P} = \mathbf{a}_3\nu^3 + \mathbf{a}_2\nu^2 + \mathbf{a}_1\nu + \mathbf{a}_0$$

The parameters for either form can be found using:

$$\begin{pmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{P}_i \\ \mathbf{P}'_i \\ \mathbf{P}_{i+1} \\ \mathbf{P}'_{i+1} \end{pmatrix}$$

b. Find the point at the centre of the patch using the equation:

$$\begin{aligned} \mathbf{P}(\mu, \nu) = & \mathbf{P}(\mu, 0)(1-\nu) + \mathbf{P}(\mu, 1)\nu + \mathbf{P}(0, \nu)(1-\mu) + \mathbf{P}(1, \nu)\mu \\ & - \mathbf{P}(0, 0)(1-\mu)(1-\nu) - \mathbf{P}(0, 1)(1-\mu)\nu - \mathbf{P}(1, 0)\mu(1-\nu) - \mathbf{P}(1, 1)\mu\nu \end{aligned}$$

NB: This numerical calculation is rather tedious unless you use a programmable calculator, spreadsheet or software such as MatLab (which is available on the lab machines).