

Tutorial Solution: Warping and Morphing

1. Explain what is meant by the following equation:

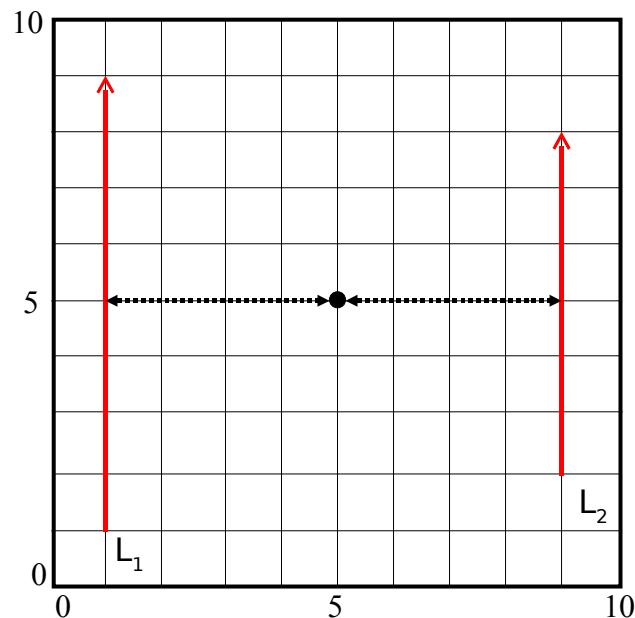
$$\text{morphing} = (\text{warping})^2 + \text{blending}$$

The equation above refers to the fact that morphing is a two-stage process which involves coupling image warping with color interpolation. As the morphing proceeds, the first image (source) is gradually warped towards the second image (target) while fading out. At same time the second image starts warping towards the first image and is faded in. Thus, the early images in the sequence are much like the first image. The middle image of the sequence is the average of the first image distorted halfway towards the second one and the second image distorted halfway back towards the first one. The last images in the sequence are similar to the second one. Then, the whole process consists of warping two images so that they have the same "shape" and then cross dissolving the resulting images.

2. In the algorithm developed by Beier and Neeley pairs of lines are used to specify the warping. In a concrete example two pairs of lines specify a 2D warping: In the source image the line L_1 starts at (1, 1) and ends at (1, 9). The line L_2 starts at (9, 2) and ends at (9, 8). In the target image the corresponding line L_1' starts at (1, 1) and ends at (1, 9) while L_2' starts at (3, 2) and ends at (9, 2). Calculate where the pixel $\mathbf{p} = (5, 5)$ in the source image would map to in the target image. Assume that the constants controlling the warping are $a = b = p = 1$.

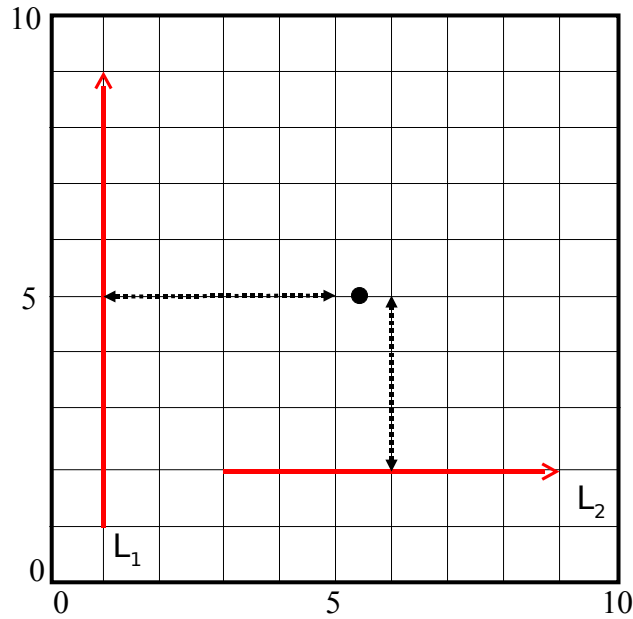
Most of the times it is best to start visualizing the warping with a sketch:

The configuration of the source looks as follows:



1.

The configuration of the target looks as follows:



For line L_1 the point p has the following u and v values:

$$u_1 = 0.5 \text{ and } v_1 = 4$$

Similarly for line L_2 :

$$u_2 = 0.5 \text{ and } v_2 = 4$$

Using these values in the target image produces the following points: For line L_1 the corresponding point is at (5, 5). For line L_2 the corresponding point is at (6, 5). We now need to calculate the weights of both lines L_1 and L_2 :

$$w_i = \left(\frac{\text{length}_{i,p}}{a + \text{dist}_i} \right)^b$$

For L_1 the weight $w_1 = 8 / (1 + 4) = 1.6$ and for L_2 the weight $w_2 = 6 / (1 + 3) = 1.5$. Thus the location of pixel $p = (5, 5)$ will be

$$p' = \frac{\sum_i w_i p_i}{\sum_i w_i} = \frac{1.6 \cdot \begin{pmatrix} 5 \\ 5 \end{pmatrix} + 1.5 \cdot \begin{pmatrix} 6 \\ 5 \end{pmatrix}}{3.1} = \begin{pmatrix} 5.48 \\ 5 \end{pmatrix}$$

3. An image with 300 x 175 pixels is warped using a two-dimensional free-form deformation based on linear B-splines defined by a 6 x 6 mesh of control points.

a. Calculate the spacing between control points in pixels.

The spacing between control points in the horizontal direction is 60 pixels. The spacing between control points in the vertical direction is 35 pixels.

b. Calculate the pixel coordinates for the following B-spline integer lattice coordinates i, j and the fractional lattice coordinates u, v :

- (i) $(i, j) = (1, 1)$ and $(u, v) = (0, 0)$
- (ii) $(i, j) = (1, 1)$ and $(u, v) = (0.5, 0.5)$
- (iii) $(i, j) = (1, 3)$ and $(u, v) = (0.75, 0.2857)$

The pixel coordinates for point $(i, j) = (1, 1)$ and $(u, v) = (0, 0)$ are $(60, 35)$.

The pixel coordinates for point $(i, j) = (1, 1)$ and $(u, v) = (0.5, 0.5)$ are $(90, 52.5)$.

The pixel coordinates for point $(i, j) = (1, 3)$ and $(u, v) = (0.75, 0.2857)$ are $(105, 105)$.

c. Calculate the B-spline integer lattice coordinates i, j and the fractional lattice coordinates u, v for the following pixels:

For $(x, y) = (120, 140)$:

$$\begin{aligned} i &= \text{floor}(((n_x - 1)x) / X) = \text{floor}(600/300) = 1 \\ j &= \text{floor}(((n_y - 1)y) / Y) = \text{floor}(700/175) = 4 \\ u &= \text{frac}(((n_x - 1)x) / X) = \text{frac}(600/300) = 0 \\ v &= \text{frac}(((n_y - 1)y) / Y) = \text{frac}(700/175) = 0 \end{aligned}$$

For $(x, y) = (100, 77)$:

$$\begin{aligned} i &= \text{floor}(((n_x - 1)x) / X) = \text{floor}(500/300) = 1 \\ j &= \text{floor}(((n_y - 1)y) / Y) = \text{floor}(385/175) = 2 \\ u &= \text{frac}(((n_x - 1)x) / X) = \text{frac}(500/300) = 0.6666 \\ v &= \text{frac}(((n_y - 1)y) / Y) = \text{frac}(385/175) = 0.2 \end{aligned}$$

For $(x, y) = (150, 119)$:

$$\begin{aligned} i &= \text{floor}(((n_x - 1)x) / X) = \text{floor}(750/300) = 2 \\ j &= \text{floor}(((n_y - 1)y) / Y) = \text{floor}(595/175) = 3 \\ u &= \text{frac}(((n_x - 1)x) / X) = \text{frac}(750/300) = 0.5 \\ v &= \text{frac}(((n_y - 1)y) / Y) = \text{frac}(595/175) = 0.4 \end{aligned}$$

d. Calculate the new location of a pixel $(x,y) = (135, 122.5)$ after warping. The matrix of control points looks as follows:

(1, 4)	(-3, 7)	(3, 8)	(4, 7)	(0, 1)	(2, 3)
(-3, 7)	(-2, 9)	(2, 7)	(3, 1)	(2, -2)	(2, 2)
(4, 2)	(3, 8)	(2, 1)	(4, 2)	(2, 1)	(3, 1)
(3, 2)	(2, 9)	(-3, 8)	(6, 8)	(3, 4)	(3, 5)
(-1, 3)	(-2, 3)	(1, 3)	(2, 3)	(8, 3)	(-4, 2)
(0, 0)	(-2, 1)	(1, 1)	(-2, 2)	(1, 2)	(0, 0)

First, we need to calculate the integer lattice coordinates i, j as well as the fractional lattice coordinates u, v of pixel (x, y) :

$$\begin{aligned} i &= \text{floor}(((n_x - 1)x) / X) = \text{floor}(675/300) = 2 \\ j &= \text{floor}(((n_y - 1)y) / Y) = \text{floor}(612.5/175) = 3 \\ u &= \text{frac}(((n_x - 1)x) / X) = \text{frac}(675/300) = 0.25 \\ v &= \text{frac}(((n_y - 1)y) / Y) = \text{frac}(612.5/175) = 0.5 \end{aligned}$$

$$(x', y') = (x, y) + (1-u)(1-v)\mathbf{c}(2, 3) + (1-u)v\mathbf{c}(2, 4) + u(1-v)\mathbf{c}(3, 3) + u v \mathbf{c}(3, 4)$$

In this concrete example we can write the warping equation as:

$$(x', y') = (x, y) + 0.375 \mathbf{c}(2, 3) + 0.375 \mathbf{c}(2, 4) + 0.125 \mathbf{c}(3, 3) + 0.125 \mathbf{c}(3, 4)$$

with

$$\mathbf{c}(2, 3) = (-3, 8)$$

$$\mathbf{c}(2, 4) = (1, 3)$$

$$\mathbf{c}(3, 3) = (6, 8)$$

$$\mathbf{c}(3, 4) = (2, 3)$$

Finally, we can calculate the new pixel coordinates as:

$$x' = 135 + 0.25 = 135.25$$

$$y' = 122.5 + 5.5 = 128$$