

## Lecture 5: Physical Realisation of Logic Gates

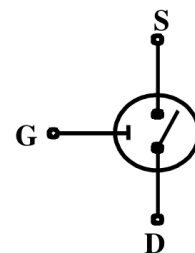
The purpose of this lecture is to present a set of models which show how logic gates are made, and how they behave in practice. We will set the scene by discussing what is meant by a physical model. It is important to realise that our understanding of the laws of nature is just an approximation. This is true for any branch of physics. For example, by means of Newton's laws we can explain to a high degree of accuracy the orbits of the planets in the solar system. Until the end of the last century it was widely believed that Newton's laws were correct, since they fitted data exactly to the precision that could be measured. However, when more accurate means of measuring the planetary motion were found, it was discovered that the orbits deviated from the predictions made from Newton's laws by very small amounts that couldn't be attributed to experimental error. Hence it had to be concluded that Newton's laws were only approximately correct, and a more complex and accurate theory was required. This was provided by quantum mechanics, but particle physics again demonstrated that this was also only approximate and worse still that it was not possible to make complete measurements. It remains a conjecture as to whether there really are laws of physics and whether we can discover them. However, the approximate theories are still of use since they provide models of the real world that we can use in design. In many instances we are interested in the simplest model that we can find that encapsulates the physical behaviour of the system we are designing. For example, it would clearly be ludicrous to use quantum mechanics if we were designing a car, since Newtonian mechanics will be more accurate than we need. Further, for the bearing parts of a machine, which are machined accurately and lubricated, we would choose a simple mechanical model in which friction did not appear. Thus, we see that we pick a model to suit the purpose and accuracy required by our task, and we will now investigate how this is done for logic design.

We have already been using an approximate model in the form of Boolean algebra. We saw at the end of the last lecture that there are details that are not represented by Boolean algebra - such as the area of silicon used to implement a circuit - that need to be considered to obtain the best design. We will find a further limitation of Boolean algebra when we come to consider timed circuits. In order to understand those limitations more deeply we now introduce a more detailed model of how digital circuits operate.

The components found in logic gates are of three kinds: resistors, capacitors and transistors. For our simplest models we will ignore the capacitors. The resistors will be modelled by Ohm's law, which states that the electric current flowing in the resistor is related to the voltage difference across the ends by the equation  $V = IR$ . (This of course is only an approximate model but is sufficiently accurate for logic circuit design at any level of detail).

We will start with a very simple, procedural model of the transistor. It has three terminals marked G, S and D, standing for the Gate (G), the Source (S) and the Drain (D), and it obeys the following rules:

1. There is no connection between G and D and G and S
2. If the voltage between G and D ( $V_{gd}$ ) is less than 2 volts the switch is open and there is no connection between D and S.
3. If the voltage  $V_{gd}$  is greater than 2 volts the switch is closed and D is connected directly to S.



Although this is a highly simplified model it contains the sufficient detail to explain how the gates are constructed to compute the correct logic. An inverter is shown in Figure 1, and we shall consider how it works. Suppose that the input voltage (which in this case is the same as the voltage  $V_{gd}$  in the rules above) is set to zero. The switch remains open, and so there is nothing connected to the S terminal. It is therefore not possible for any current to flow through the resistor, and the voltage drop across it is therefore zero by Ohm's law. Thus the output voltage is the same as the battery voltage, that is 5v. Now suppose that a voltage of 5 volts was applied to the input ( $V_{in}=V_{gd}=5v$ ). The switch now closes, and there is a voltage of 5V across the resistor. Current flows, and the voltage at the output becomes zero, since it is connected through the transistor to the 0v line. Hence, if we think of 5v as logic state 1 and 0v as logic state 0, we can see that the circuit is an inverter.

Figure 2 shows how the inverter is simply adapted to make NAND and NOR gates. In the case of the NAND gate, it is clear that both transistor switches must be closed before the output voltage is connected to the 0v

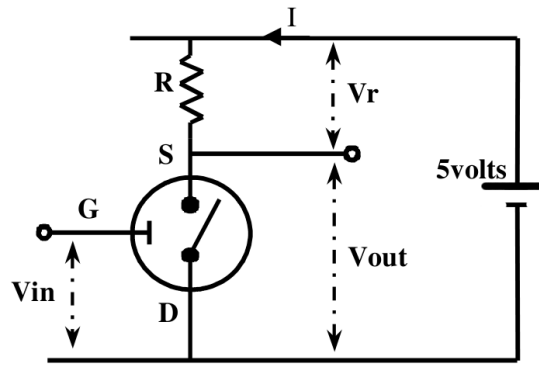


Figure 1: The Inverter gate

battery terminal, and therefore becomes 0. For the NOR gate either transistor can make the connection. Clearly we can construct AND and OR gates by adding an inverter stage after the NAND and NOR gates respectively. This explains why the AND and OR gates take up more silicon area than the NAND and NOR gates. The XOR gate can be constructed from four NAND gates, though in practice there is a simpler implementation which we will not discuss.

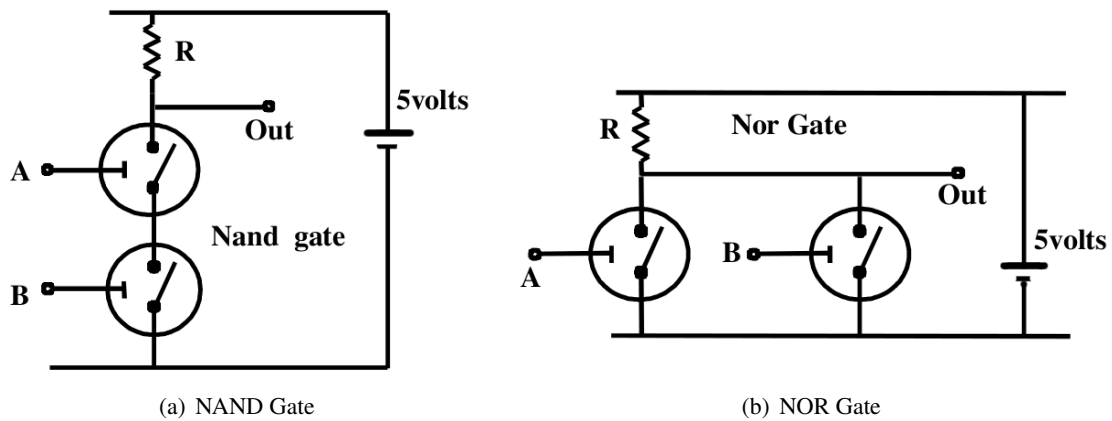
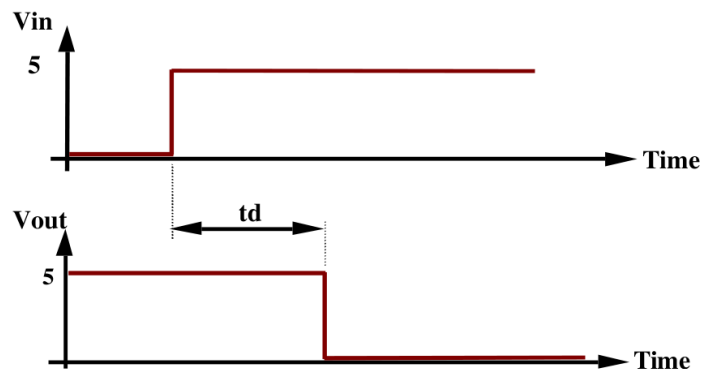


Figure 2: Basic Gate Implementations

Although this simple transistor model captures all the logic behaviour correctly, and tells us something about the size of the final implementation, it does not tell us anything about how fast our circuit will work. As we will see this is a very important question, particularly for modern computer design. Thus the next new property we consider is the transistor time delay which we denote  $t_d$ . We can think of this as the length of time it takes to close or open the switch. This can be expressed best in the form of an idealised timing diagram.



Consideration of the time delay is important since we need to ensure that the correct logic values are present

on the input of any circuit at the correct time. This is the synchronisation problem, and typically occurs in large circuits where the inputs to a final stage of processing are computed using different numbers of gates. An artificial example of this is shown in Figure 3 in which a fragment of a circuit is shown. Suppose that A and B both change state at the same time as shown in the timing diagram. Logically the output should not change, however, because of the transistor delay time it takes longer for the signal from A to reach the output than for the signal from B. Hence, while waiting for the change from A to arrive, the output goes to logic 1 for a period of  $t_d$ . Thus, a false logic level is introduced momentarily into the circuit. These false logic states are referred to as spikes since they are normally very short, and one of the most difficult parts of digital design is to ensure that spikes do not occur, or that if they do they do not disturb the circuits functionality.

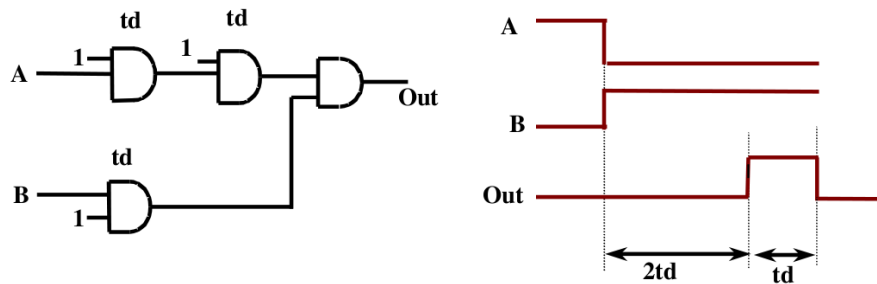


Figure 3: Errors introduced by time delays

The next refinement that we make to our transistor model takes account of the fact that the switch does not change instantaneously. In fact it is more accurately modelled as a variable resistor rather than a switch. The relationship between the gate and the drain and the resistance between the source and drain is called the transistor characteristic, and looks typically like Figure 4(a). It will be seen that the change in resistance is gradual, and there is a band during which the transistor is said to be changing state. This is in contrast to the simpler switch model where the change of state was instantaneous.

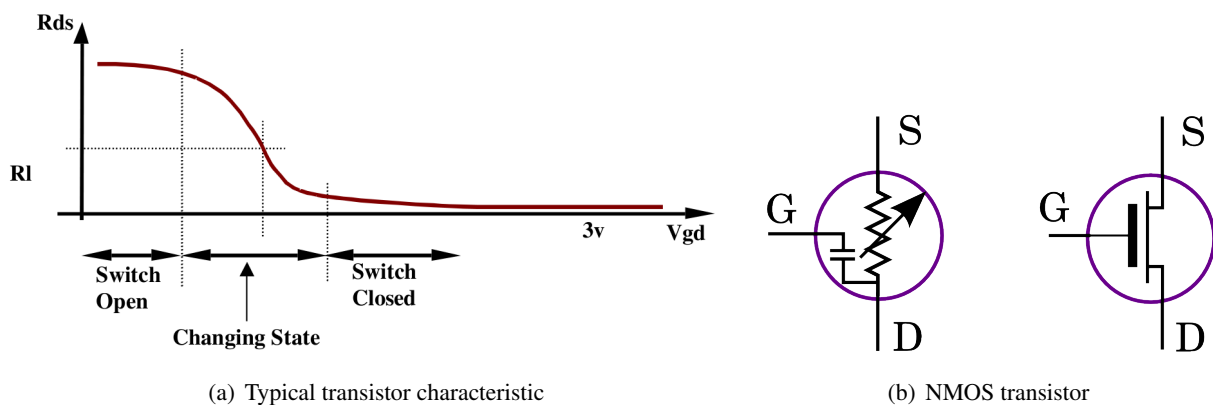


Figure 4: Realistic transistor characteristics

To complete the picture we must make one further refinement which is to add a capacitor between the G terminal and the D terminal. The model of the transistor could now be depicted by the symbol shown in Figure 4(b), but to make drawings simpler, we will adopt the normal symbol also shown in Figure 4(b). We now have a fairly accurate model of the NMOS silicon transistor.

The effect of the capacitor is to prevent the voltage on the gate changing instantaneously. To see this we need to discuss the behaviour of a capacitor. This is governed by the equation:

$$I = C \frac{dV}{dt} \quad (C \text{ is a constant called the capacitance})$$

Now we consider a practical way of feeding an input signal to the transistor, for example from another transistor circuit. Figure 5 shows a transistor connected to a capacitor which we can think of as modelling the input of

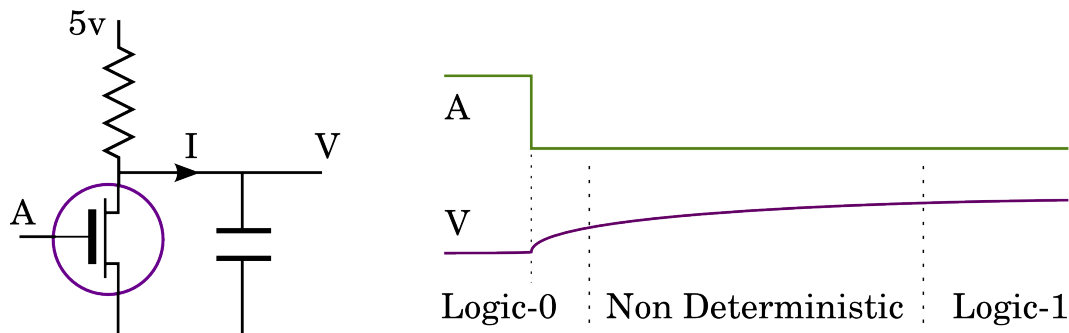


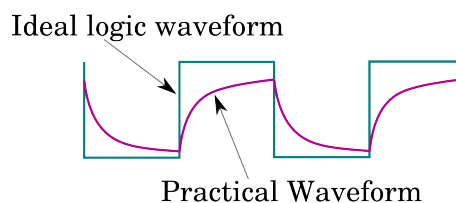
Figure 5: The effect of capacitance

a second transistor. The second transistor isn't shown in the diagram - just its capacitor. To make the analysis simple we will choose the switch model for the first transistor, so while A is at logic 1, the voltage across the capacitor is zero. If we denote it as V we have the following equations:

$$\begin{aligned}
 5 - V &= IR && \text{(Ohm's law modelling the resistors behaviour)} \\
 5 - V &= RC \frac{dV}{dt} && \text{(eliminate I using the capacitor law above)} \\
 (5 - V)dt &= RCdV \\
 \int (1/RC)dt &= \int (1/(5 - V))dV && \text{(remember } \int (1/(aV + b))dV = \frac{1}{a} \ln(aV + b) \text{)} \\
 t/RC + k &= -\ln(5 - V) && \text{(since we take } V = 0 \text{ at } t = 0 \text{ it follows that } k = -\ln(5)) \\
 5 - V &= \exp(-t/RC + \ln(5)) \\
 &= \exp(-t/RC)\exp(\ln(5)) \\
 &= 5\exp(-t/RC) \\
 V &= 5(1 - \exp(-t/RC))
 \end{aligned}$$

This yields the output voltage shown in the timing diagram of Figure 5. Notice that the voltage will never reach 5v, and that the bigger the value of C the longer the time taken for V to rise. The second thing that we now must note is that since no voltage ever changes instantaneously, there will be a period of time when it is not possible to determine whether the transistor has changed or not. In other words the gate will not be at either logic 0 or at logic 1, it will be in a non-deterministic state. It is principally the input capacitance that is responsible for the time delay  $t_d$  that we introduced earlier on.

We can now describe with greater accuracy what happens when an inverter changes state. Since in our more accurate model there is a variable resistor between the source and drain, the circuit can be viewed as a potential divider. In practice the resistor used will be chosen to be in the middle of the transistor range, so that it is small compared with the largest resistance of the transistor, and big by comparison with the smallest resistance. Since the resistance of the transistor is never infinite and never zero, the output voltage is never either zero or 5 volts, but in practice ranges between about 0.2 and about 3.5 volts, with a non deterministic band stretching from roughly 0.5 to 1.5 volts. Thus, in practice our waveforms will look like this.



One last feature should be observed, and this is that there is a second capacitor which goes between the gate and the source. (It is in fact a symmetric device, so we can swap the source and drain connections.) The addition of the second capacitor does not change the behaviour except when the gate is not connected to anything when it will drift up towards 5v. Thus the default input is logic 1, and not logic 0 as we might suppose. This means that all inputs to a circuit that we require to be 0v must be physically connected to the 0v terminal of the power supply.