Lecture 5

Physical Realisation of Logic Gates

Boolean Algebra as a Model of Logic Circuits?

- Boolean Algebra is a good framework for describing the behaviour of logic circuits, but it is an abstraction.

- For a practical machine we need to use real voltages, (e.g. $\sim$3.5v for one and $\sim$0.3v for zero), and we need to consider time delays for the signals to propagate through the circuit.

- Boolean Logic is only an approximation to the way in which a digital circuit operates.

Time in Logic Circuits

- We will see in this lecture that the most important deficiency of Boolean Logic is its inability to describe events happening at different moments in time.
- Failure to synchronise events is a very common cause of errors in hardware design.
- Later in the course we will discuss ways in which we can cope with the problems caused by timing.

Physical Models

What we now need is a more accurate physical model of digital circuits. Note that:

- All physical models are approximate.
- Newtonian mechanics was thought to be exact until about 1900 when more accurate measurements showed that real planetary movements differed from the predicted ones.
- However, Newtonian mechanics is enormously useful - you dont need quantum theory to design a car!

A More Detailed Model of Logic Gates

In order to understand how logic gates work in practice we need to look at their operation in more detail.

To do this we need to study how they are constructed, and for this we need three components.

- The Resistor
- The Capacitor
- The Transistor

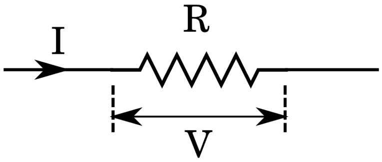
The Resistor

This is a familiar device. It is governed by Ohm's law which states:

$$V = I \times R$$

(V=Voltage, I=Current, R=Resistance)

It is represented by the symbol:



The Transistor

Pure Silicon is a good insulator.

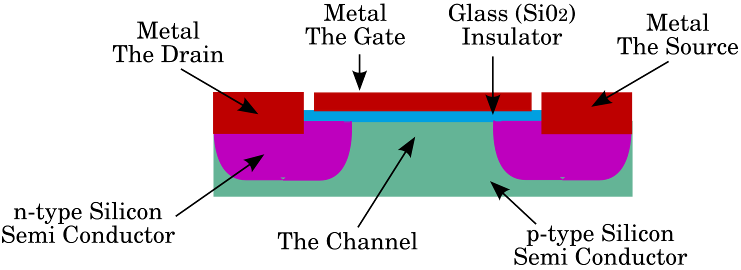
Silicon can be infused with impurities that give it surplus electrons so that it can conduct current like a metal. Since electrons have a negative charge this is called n-type.

Silicon can also be infused with imputities that allow it to conduct with positive charge carriers - really these are missing electrons sometimes called “holes”. This is called p-type silicon.

A transistor is formed by three adjacent pieces of these semi conductors. Either n-p-n or p-n-p.

The Transistor

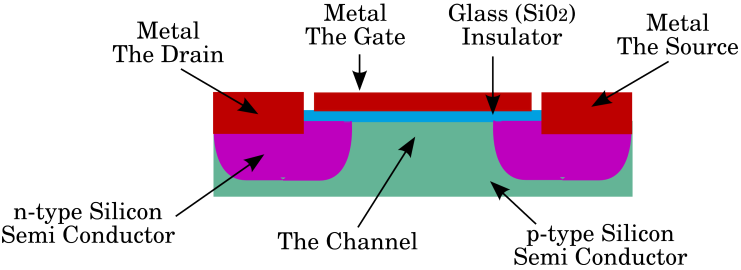
The nMOSFET (negative Metal Oxide Silicon Field Effect Transistor) in cross section:



It is a remarkably simple device and easy to manufacture - but what does it do?

The Transistor

The nMOSFET (negative Metal Oxide Silicon Field Effect Transistor) in cross section:



If a positive voltage is applied between the source and the gate electrons are drawn into the channel and it can conduct.

The Transistor

- Ohms law for the resistor is a simple mathematical model expressed by an equation.
- For more complex devices, such as the transistor, it is possible to derive a set of mathematical equations, but it is much simpler to describe the behaviour of the device as a set of rules.
- Such a description is called a procedural model.

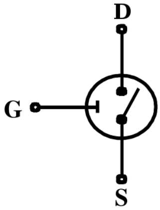
The Transistor

In the simplest procedural model, the transistor may be thought of a a switch with the three terminals labelled:

S : Source

D : Drain

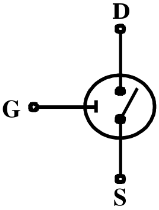
G : Gate



The Transistor

The rules governing the switch are:

1. There is no connection between G and S or G and D
2. If the voltage between G and S (Vgs) is less than or equal to 0.5 volts there is no connection between S and D
3. If the voltage between G and S (Vgs) is greater than 1.7 volts S is connected directly to D



Boolean Truth Values

We will use transistors to create logic gates, but we see from our procedural rules that the transistor switching points are described in voltages not in truth values.

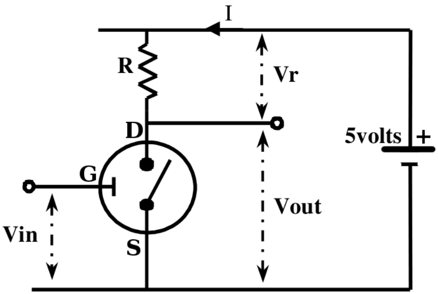
Hence we must now specify what the relationship is between truth values and the voltages that represent them. This is summarised by the following table:

Truth Value	Boolean Algebra	Transistor Circuits
True	1	$>$1.7 Volts
False	0	$<$0.5 Volts

The Inverter Gate

Here is a circuit of an inverter gate. Consider first the case where Vin = 0.2v (Boolean 0):

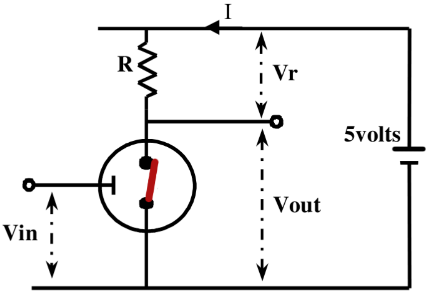
- The procedural rules tell us the switch is open
- I=0
- Ohms law tells us Vr=0
- Vout = 5 Volts (Boolean 1)



The Inverter Gate

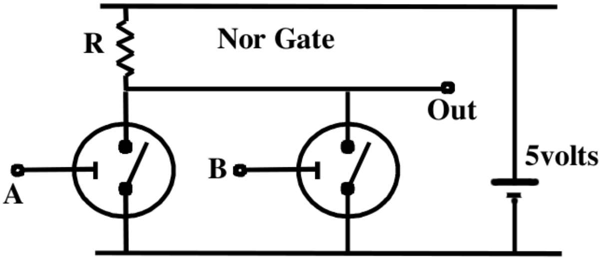
Now consider the case where Vin = 2v (Boolean 1):

- The procedural rules tell us the switch is closed
- Vout = 0 (Boolean 0)
- Vr = 5 Volts
- Ohm's law tells us I=5/R



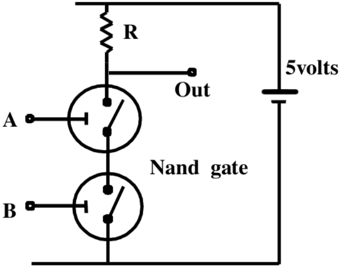
The NOR gate

- If both switches are open (input A and B both Boolean 0), the output is 5v (Boolean 1)
- If either switch is closed (either, or both, ie A and/or B at Boolean 1 value), the output is 0v (Boolean 0)



The NAND Gate

- The output falls to 0v (Boolean 0) only when both switches are closed.
- If either opens it rises to 5v (Boolean 1)

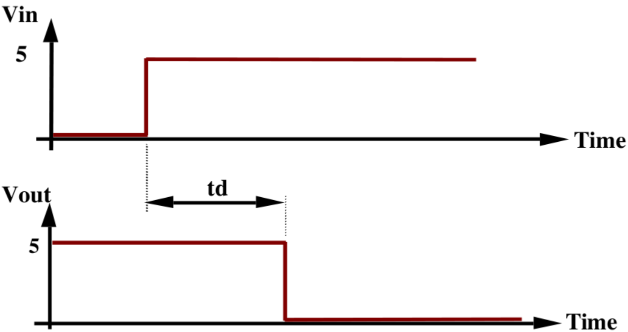


AND and OR gates

- We can construct an AND gate by connecting a NAND gate and an inverter together.
- Similarly we can construct an OR gate by connecting a NOR together with an inverter.
- These models, though simple are surprisingly close to the implementations used in practice.

Time rears its ugly head

Signal Propagation: it takes time for the transistor state to change. This is because electrons move through the transistor.

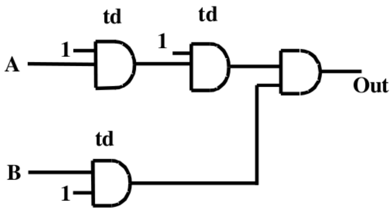


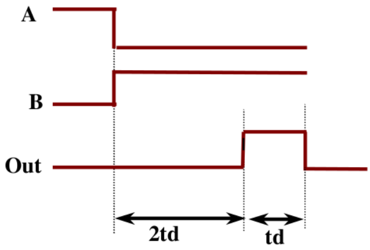
Is it goodbye to Boolean algebra?

- Boolean Algebra does not incorporate a measure of time.
- Although the time delay does not seem very important, in practice it complicates logic circuit design.
- The larger the circuit and greater the difference in the number of gates in different paths from inputs to outputs, and the more difficult it is to reason about time.

The synchronisation problem

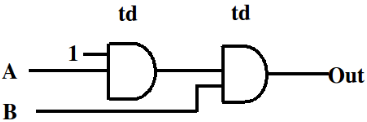
This example is artificial, but illustrates how a false result (sometimes called a spike) can be caused by time delays.

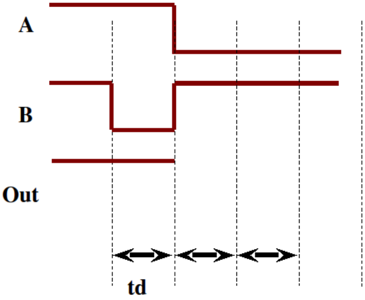




Problem Break

Given that A and B have had their starting values for some time what output would you expect to result from the timing diagram given?





Transistor Characteristics

Unfortunately a transistor is not a perfect switch.

For a perfect switch:

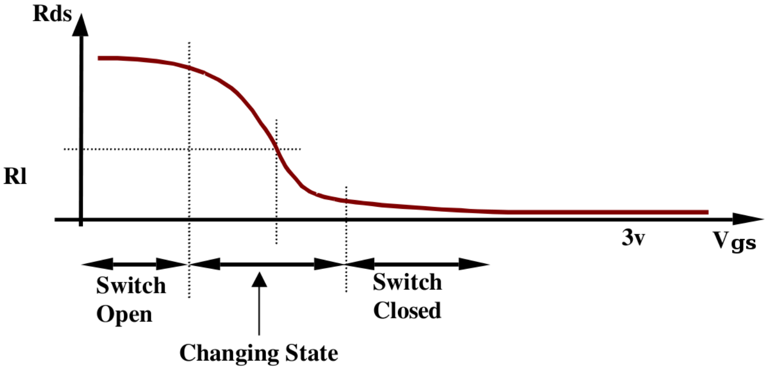
Switch Closed $\rightarrow$ zero resistance

Switch Open $\rightarrow \infty$ resistance

But in practice neither of these extremes are reached.

Transistor Characteristics

In fact a transistor behaves like a variable resistor:

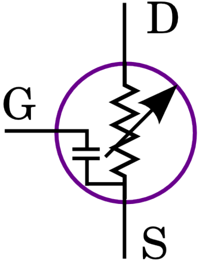


Input capacitance

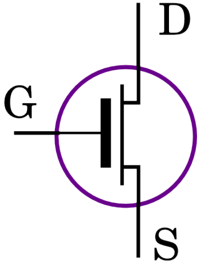
Another feature of the real transistor is that it has a small capacitor connected between the gate and the drain.

We can represent it schematically thus:

Conceptual Model



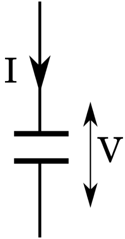
Usual symbol



The effect of the capacitor

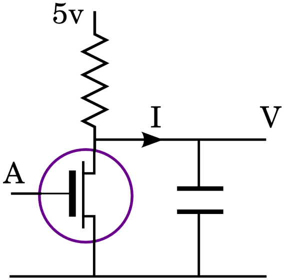
The capacitor has the effect of introducing a time delay. In fact, it is solely responsible for the time delay td that we talked about previously.

To see why this happens we will look at a model of the capacitor:



$$I = C\frac{dV}{dt}$$

The effect of the capacitor



Assume A= 0V

$5 - V = IR$ (Ohm's law)

$V = 5 - IR$

$I = C\frac{dV}{dt}$

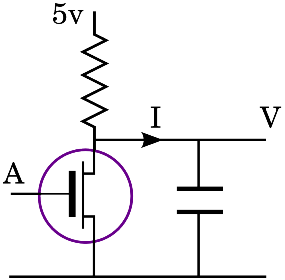
Eliminating *I* we get:

$V = 5 - RC\frac{dV}{dt}$

Rearranging and integrating we get:

$$\int \frac{dV}{5 - V} = \int \frac{dt}{RC}$$

The effect of the capacitor



$$\int \frac{dV}{5-V} = \int \frac{dt}{RC}$$

$$-log(5-V) = \frac{t}{RC} + K$$

If $V = 0$ at $t = 0$ then $K = -log(5)$

$$5 - V = exp(-t/RC + log(5))$$

$$= exp(-t/RC)exp(log(5))$$

$$= 5exp(-t/RC)$$

$$V = 5(1 - exp(-t/RC))$$

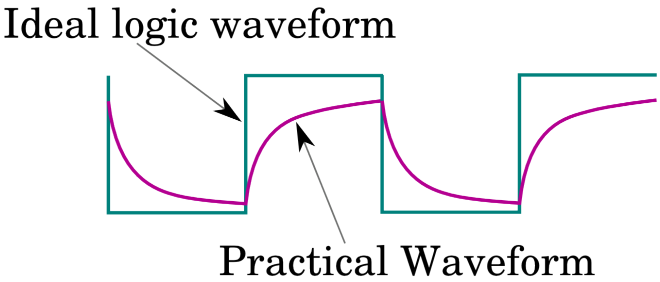
Plotting the voltage rise

From the previous slide we have: $V = 5(1 - exp(-t/RC))$

Note that the voltage will never actually reach 0 or 5V.

Square Waves in Practice

If we try to switch our transistors as fast as possible we get something like this:



So the capacitor limits the speed at which we can compute logic functions.

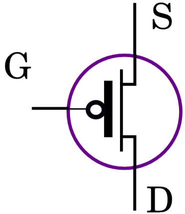
PMOS Transistors

The transistor we have discussed in this lecture is called the NMOS (Negative Metal Oxide Silicon) transistor. There is a dual type of transistor which operates in the opposite manner.

If the voltage between G and D (Vgd) is less than or equal to 0.5 volts there is low resistance between S and D

If the voltage between G and D (Vgd) is greater than 2 volts there is high resistance between S and D.

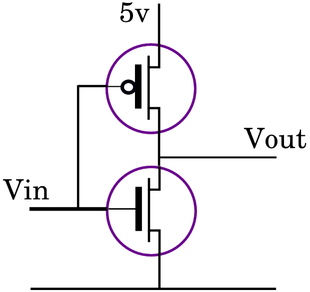
It is called the p-MOS transistor



CMOS Transistors

In practice most integrated circuits do not use resistors but use a combination of NMOS and PMOS transistors. The technology is called CMOS (Complementary MOS).

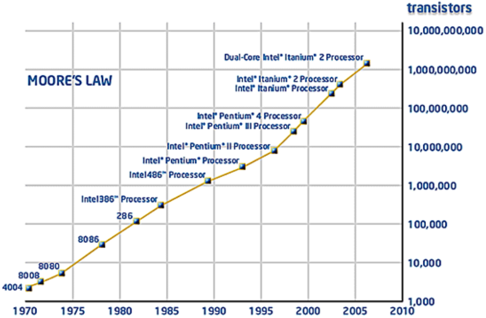
A CMOS inverter circuit has a PMOS transistor in place of the resistor. This means lower power consumption and faster switching.

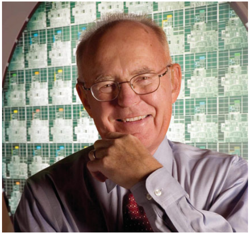


Transistor economics

Moore's Law (1965):

Transistors per square inch doubles every 18 months





Software profligacy

May's Law (1985):

> Software efficiency halves every 18 months,
> compensating Moore's Law.