

Application note

# Vision based navigation system for an endoscope

Gul N. Khan<sup>a</sup>, Duncan F. Gillies<sup>b</sup>

<sup>a</sup>College of Electrical and Mechanical Engineering, National University of Science and Technology, Peshawar Road, Rawalpindi 46000, Pakistan

<sup>b</sup>Department of Computing, Imperial College of Science, Technology & Medicine, 180 Queen's Gate, London SW7 2BZ, UK

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## Abstract

A vision based navigation system to guide an endoscope inside a human colon has been designed and tested. It uses low level vision techniques to extract two types of navigational landmarks, dark regions and curved contours. Dark regions correspond to the distant inner space of the colon, called the lumen. The curved contours represent occlusions due to the inner colon muscles. A hierarchical search space and environment representation, called the QL-tree, was developed to integrate the visual features and implement the navigation system. It uses multiple quadtrees which are linked at all hierarchical levels. A multiprocessor system was employed to achieve real-time performance. The endoscope navigation system has been used successfully in artificial colon models.

*Keywords:* Automatic endoscope; Colonoscope; Contour detection; Environment and search space representation; Region extraction; Vision-based navigation

## 1. Introduction

The vision based navigation system described in this paper has been developed as part of an autonomous guidance system for an endoscope intended to navigate the instrument in the human colon. Although, the work is mainly directed towards colonoscopy, the vision and navigation techniques that have been developed are general enough to be used in other applications of endoscopy including the inspection of pipes and ducts in process plants and engines.

### 1.1. The endoscope

The endoscope is a medical instrument used for observing the inner surfaces of the human body. The endoscopes that are used in investigations of the colon have a flexible shaft, about 1.5 m long which accommodates a CCD video system (see Fig. 1). The shaft contains an air and water supply channel, an illumination channel and an operating channel which allows the passage of flexible instruments such as biopsy forceps. The tip of the shaft can be deflected by two control wheels providing up-down and left-right movements. The shaft transmits rotational movements to the tip.

During conventional colonoscopy, the instrument is inserted into the rectum. It is progressively advanced

by the consultant while he or she manipulates the tip deflection to keep the lumen, or the centre line of the colon, in the centre of the image. This procedure requires simultaneous movement of the two control wheels, in addition to pushing, pulling and rotating the shaft of the endoscope. The colon is a highly flexible tube-like structure with many bends, twists and pockets. When the endoscope bears on its wall it will deform and may produce paradoxical behaviour. The colon can collapse completely due to spasm making the lumen difficult to see. Navigation in the colon is a difficult task, and the goal of this project is to use machine vision to simplify it by guiding the terminal portion of the endoscope allowing the doctor to concentrate on diagnosis [1,2].

### 1.2. Colon image features for guiding an endoscope

The current generation of endoscopes have a single fixed camera. Inner colon surfaces are not Lambertian, and it is difficult to define an accurate reflectance function for them. They also have a texture caused by the veins, which sometimes is not clearly visible due to specular reflections. This means that there are likely to be inaccuracies with the use of photometric stereo methods for reconstruction of the surface depth from the shade information. The colon is illuminated by a point light source located at the endoscope tip very

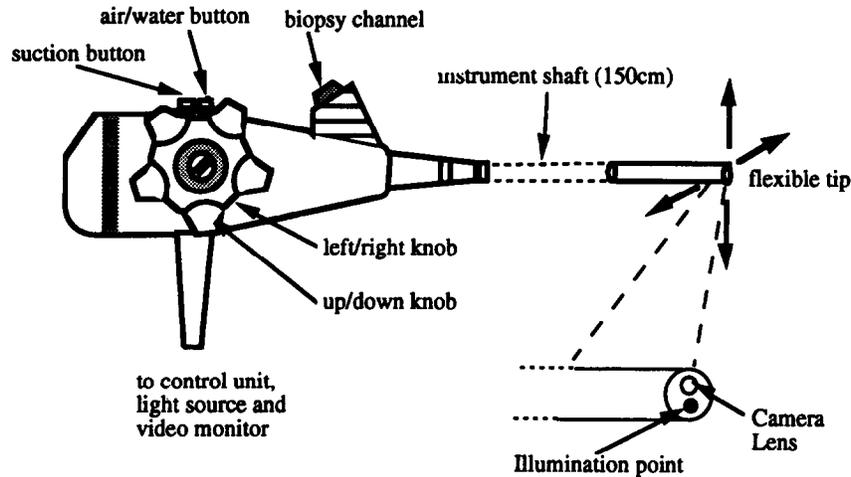


Fig. 1. Diagram of a conventional flexible video endoscope.

close to the camera lens position. Since the light intensity falls off with the square of the distance from the source, the colon surfaces nearer to the camera are more brightly illuminated than the further surfaces. Therefore, the deepest area in the colon with respect to the viewer roughly corresponds to the area in the image where the mean intensity is the lowest. Region based segmentation methods are most appropriate for detecting these regions. A typical colon image is shown in Fig. 7(a). The darkest region is in the lower part of the image and therefore the endoscope tip should be deflected slightly down.

The inner wall of a human colon contains rings of muscle which are clearly seen in Fig. 7(a). These rings form occluding edges with high illumination gradient. When an endoscope is directed along the centre line of a straight section of colon, the muscle rings can appear as closed contours in the image. More commonly, only part of the contours are visible having parts that are hidden either by bends or by other muscles closer to the endoscope tip. The centre of a closed contour indicates the lumen position and therefore the correct insertion direction. For partially visible contours, an estimate of the insertion direction can be obtained from their curvature.

### 1.3. Navigation

From the above discussion we conclude that two types of image features, dark regions and image contours, are important navigational landmarks which do not depend on detailed knowledge of the surface properties. To utilise them effectively, we need to represent them in the computer, in a manner which can be incrementally upgraded, incorporating information from a sequence of images. Normally, a different representation is used for the environment and the search space. This means

that a mapping between the environment representation and search space is required which can be computationally expensive. We therefore propose a single representation for environment and search space in the form of QL-tree. This is based on a linked-list of quadtrees, and has the inherent features of fast access and easy updating which permit efficient search procedures for path planning and navigation.

Endoscope navigation can be compared with that of mobile robots or autonomous vehicles [3]. However, unlike the find-path problem [4-9], endoscope navigation cannot be subjected to a rigorous mathematical treatment because of the inherent complexity of natural systems. Instead our work builds upon many of the ideas put forward for navigating robots in unexplored worlds [10,11].

## 2. Low level vision techniques

### 2.1. Dark region extraction

It was argued in the previous section that one intrinsic characteristic of the lumen is that it is a uniform dark region, and the simplest, and most effective, of the machine vision techniques that we have investigated to identify the colon lumen, is to extract uniform dark regions. An initial algorithm, was devised by Khan and Gillies [12], using a quadtree data structure based on region based segmentation methods which employed pyramidal structures [13-15]. This was improved by using an intensity variance-average pyramid [16].

The intensity of a prominent dark region in a colon image is assumed to correspond to the first peak in its intensity histogram. Thus, for each colon image the histogram is constructed and analysed to find the mean

value corresponding to the region that is being sought. Starting from the pixels, a quadtree based variance-average pyramid is constructed. The quadtree representation of an image is based on a successive subdivision of the image into quadrants and it is represented in memory by a tree of out-degree four. The root node of a pyramid represents the whole image while intermediate nodes represent distinct non-overlapping square image blocks. The nodes maintain the intensity mean and variance of their corresponding image blocks. Intensity mean  $\mu_{k+1}$  and variance  $\nu_{k+1}$  for a node at level  $(k + 1)$  is computed recursively from the mean and variance of its four son nodes at level  $k$ . While the pyramid is constructed it is determined whether the region corresponding to a node is both uniform and close to the estimated dark region intensity. If it is, the node is identified as a possible seed region. If it does not satisfy the dark region criterion, its four sons are examined to identify their largest seed region and a pointer is set to that seed node. Therefore, when the pyramid is completed, we have a pointer to the largest seed of a prominent dark region which is then grown by a merging process.

A parallel implementation of this algorithm extracts dark regions from colon images in about 100 ms on a 21 processor (IMS T800 transputer [17]) pyramidal computer system [18]. The algorithm has also been implemented on a system with five processors, and without merging, can achieve real-time performance.

## 2.2. Contour extraction

The second characteristic feature of colon images which was discussed above is the presence of occluding contours formed by the colon muscles. An algorithm has been devised for extracting these contours using a piecewise linear approximation by short line segments. The method has been presented in detail [19] elsewhere. It involves 'perceptual' filtering of edge points after applying the Hough transform [20] for line segment detection [21]. The line segments obtained are linked into contours using a process which filters out line segments due to noise, again using perceptual criteria. The method proved highly successful in extracting weak but perceptually significant contours [22]. Generally, edge point detection is assumed to be a local and parallel process while grouping has been considered as a global and sequential process. We have found that early and intermediate level grouping techniques, based on the proximity and similarity in orientation and brightness, are implementable in parallel. In other words, the perceptual filters can be devised using purely local relationships [23]. Contour extraction is however an expensive process. Our recent work on this technique has indicated that real-time performance can only be obtained by employing a pyramid of 21 or more processors (IMS T800).

A number of constraints have been put forward for the interpretation of image contours [24]. Some of these order the distance of a contour from other contours or

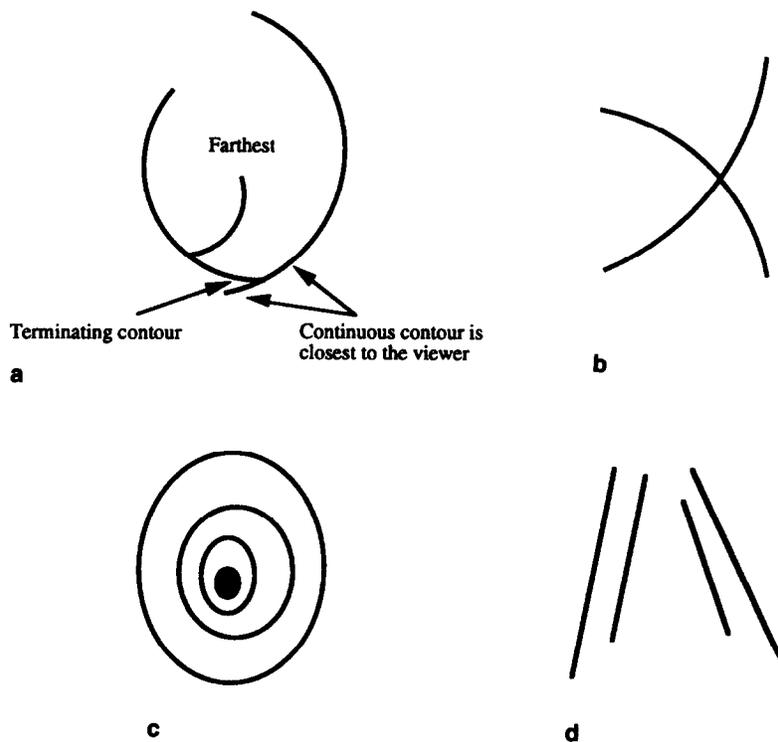


Fig. 2. Shape from contour. (a) Termination at a continuous contour, (b) crossing contours, (c) curved based vanishing point, (d) parallel line vanishing point.

from the viewer and they can therefore determine the inner colon shape. Some selected constraints and their individual interpretations are explained in Fig. 2. The termination at a continuous contour is a powerful constraint indicating that the terminating contour is further from the viewer than the one that occludes it (Fig. 2(a)). The nearest contour in a colon image has to be avoided for navigating the endoscope successfully. When two contours cross each other, as shown in Fig. 2(b), both of them cannot belong to the occluding geometric boundaries. Vanishing points can also be found from the converging image contours (Figs. 2(c) and (d)), and they have been previously employed to solve different navigational problems such as road following [25]. In colon images, circular muscle contours yield a new type of vanishing point in terms of the reduction in radius of curvature as the distance from observer increases (Fig. 2(d)). Thus, when muscle rings are partially visible, the size of their contours also provide a depth estimate.

Other methods have been devised for extracting visual features which could be used for navigation. For example, Rashid and Burger [26] successfully applied a local photometric algorithm to reconstruct the colon surface normals. Further techniques, including the use of range finders and colour, may become available in the future and could be integrated into the present navigation system.

### 3. Endoscope navigation

A typical navigation system consists of three sub-systems: global planner, navigator and pilot. A multi-level production system based on a similar organisation has already been described [27]. In the case of endoscope navigation, our principal focus is on the navigator module. The low level control functions of the pilot module can be implemented using a standard servo mechanism. Global planning, which has been extensively studied by Sucar and Gillies [28], though useful for the purposes of giving advice to the doctor, is not directly concerned with the immediate decisions towards which direction to insert the endoscope.

The selection of a structure for a world or environment representation normally depends on whether the nature of the environment is pre-learned, partially known or completely unknown. When the environment is not known, the construction of its representation for path planning is based only on the information provided by sensors. There is usually some uncertainty in the information provided by a sensing system. For instance, a sensor such as a CCD camera has a field of view, outside which objects become blurred. Thus, a representation of a previously unknown, or partially known, environment must be able to cope with inaccuracy and uncertainty. In particular, it should allow the easy addition or removal

of scene information. Another representation usually employed in navigation systems is the search space for performing search operations to find optimal paths. The generalised cylinder free space [5], the configuration space [6], the Voronoi base space [7] and the medial axis free space [29] have all been used as search spaces. Additional functions are needed to map the environment representation and search space which can limit the on-line reaction capability of a navigation system.

An effective environment representation for path planning in colonoscopy should meet some general objectives. It should provide a simpler means of locating important navigational landmarks such as dark regions, curved contours and vanishing points, it should be able to handle uncertainty, and, for colonoscopy, should provide support for unpredictable movements of the endoscope caused by doctor or patient movements. When a common representation is used for the environmental representation and search space, it should be able to support efficient search procedures.

The construction of an integrated representation of the local environment plays an important role in navigation tasks. An efficient linked-list of quadtrees, the 'QL-tree' structure, has been developed to depict both the environment and search space. Each quadtree in the QL-tree corresponds to a cross-sectional view of the colon and the QL-tree represents a time series of such views. The representation is also compatible with the machine vision techniques described earlier, as the scene primitives supplied by them can be directly integrated.

#### 3.1. The colon model

The main task of building an internal model of colon involves the incorporation of all the available scene information around the endoscope tip as it advances through the colon. This model is used partially to plan the immediate endoscope control actions, but more importantly, the navigator employs it to determine tip-control actions when vision techniques fail to provide any navigational landmarks. A secondary aim is to maintain a global record of the endoscope position in the colon as it is inserted. It is also useful for simply advising doctors during colonoscopy.

In many cases, topological depth organisation of the inner muscles can be deduced following Marr's [30] work on occluding contours. However, in practice, it is quite common for the view of the lumen to be lost during colonoscopy. This is caused by either patient movements, spasms, presence of fluid in the colon, formation of bubbles when air is blown to inflate the colon or the onset of a steep bend. The endoscope insertion process at a colon bend is shown in Fig. 3. In Fig. 3(a), the muscle contours and the colon lumen are clearly visible but the shape of the distant contours and location of the dark

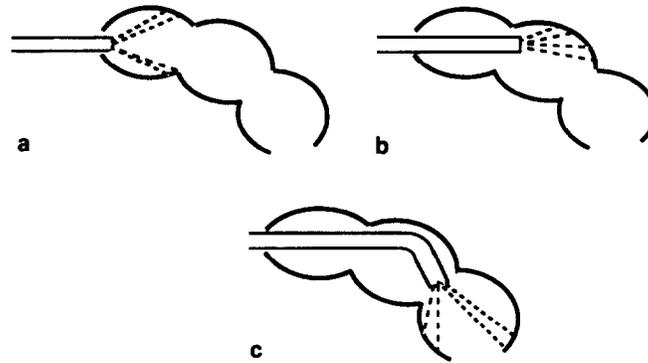


Fig. 3. Endoscope being pushed around a bend. (a) Contours clearly visible, (b) view of the contours blocked, (c) contours are again visible.

region suggest a bend. This information will be incorporated into the colon representation. When the endoscope is advanced further (as shown in Fig. 3(b)), the view of the contours and lumen disappear, and the only visible part is the colon wall. Little reliable information is available from colon wall images, since the light source is so close to the surfaces that reflected light intensity often saturates, exceeding the dynamic range of the camera, and hiding important information. However, the colon representation has recorded the previous navigational landmarks from previous image frames. Thus, it is possible to predict the oncoming colon bend and its type. The endoscope tip can be steered around the bend and a straight view of the colon is restored.

### 3.2. Environment and search space representation

In addition to satisfying the general objectives described earlier, an environment representation should avoid unnecessary excessive details of the colon without affecting the navigational tasks. The colon is represented in the QL-tree as a series of two dimensional planes rather than a true 3D volumetric representation. The QL-tree representation not only models the colon but also represents the search space. The representation is also general enough to be used for representing other 3D cylindrical environments for navigational purposes.

The inside of the colon is represented as a series of cross-sections, allowing for the fact that forward and backward movements of the endoscope at each section are not necessarily orthogonal to the plane of the colon muscle. To cope with this, the coordinate system is shifted from the current plane to the next after each movement. The distance between the object features represented by two consecutive planes (relative depth) has also been incorporated in the representation. This distance can vary for different plane pairs and at different levels in the same plane pair depending on the busyness and resolution of environment. In the present implementation of the navigation system, the reference coordinate system is fixed in the current plane in such a way that its origin coincides with the camera centre and the  $xy$ -plane

coincides with the current image plane. It is further assumed that adjacent planes are almost parallel to each other. The reference coordinate system is moved to the next plane by fixing its origin at the target point and aligning the  $z$ -axis with the line joining the origins of the current and new coordinate system. To move the coordinate system from one frame to the next, requires one translation and two rotations. These operations may be performed by standard matrix methods. This scheme of representing a three-dimensional cylindrical space takes care of bends and twists in the colon.

Quadtrees are best suited for representing cross-sectional planes due to their hierarchical nature. They have been used previously in path planning for representing a 3D environment by three orthogonal two-dimensional projections [31]. Octrees have also been employed for representing three-dimensional space in path planning [29]. The QL-tree based search space and environment representation can be viewed as an intermediate structure between the quadtree and octree. A small section of the representation is shown in Fig. 4. The cross-sectional planes of the colon are represented by separate quadtrees and their nodes have two additional links (previous plane and next plane) to interconnect consecutive planes at all the hierarchical levels of quadtrees. The interconnection of the quadtrees at different hierarchical levels converts multiple 2D representations into a single  $2\frac{1}{2}$  D colon representation. Interconnection links between quadtrees representing objects from consecutive image frames facilitate the incremental construction of representation. Efficient search operations for registering image features in successive image frames are also easily performed in the QL-tree. A depth field is associated with each node at all levels of the three to store relative depth between the object features (contours) stored in successive cross-sectional planes. The QL-tree based search space and environment representation provides a spatially indexed representation of the colon. It can also be viewed at several resolution levels like an octree but it is less complicated in terms of search operations and memory requirements. The representation also allows easy access

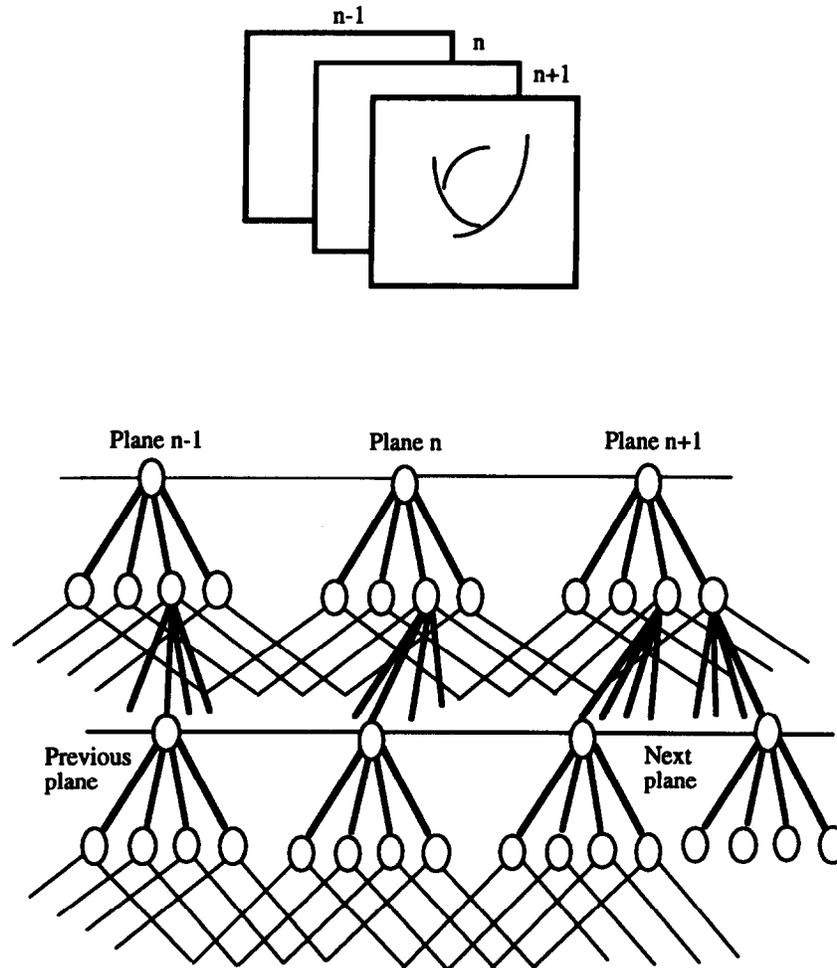


Fig. 4. A section of QL-tree representation.

to each plane represented by a quadtree for which efficient algorithms exist for path planning [32].

The main advantage of QL-tree is the representation of colon as 2D planes in such a way that the current plane contains the most updated information about the colon shape and location of objects around the current position of endoscope tip. An approximation to the shape of the objects further down on the path is also provided by the representation. The accuracy and completeness of the colon shape depends on the camera field of view. The scene information in further planes is refined and corrected incrementally as the machine vision techniques provide more object features. In this way, the QL-tree provides the important capability of updating as new scene information becomes available while forgetting the unimportant. An entire plane is easy to update when vision algorithms provide more data. An entire plane can also be removed from the representation when no longer required. It is interesting to note that in the overall endoscope control and navigation system, the quadtree structure has been used for dark region extraction, contour detection [19], and representing dark regions and colon muscle rings [22].

#### 4. Implementation details and experimental results

To achieve real-time endoscope control performance, the machine vision algorithms and the navigation system have been implemented on a multiprocessor system. The multiprocessor system used is a low cost, coarse-grain and custom built parallel computer system. High performance single processors, such as the Intel i860, DEC alpha and SuperSparc stations, were also considered, but a multiprocessor system was chosen due to its low cost-performance ratio, high flexibility, scalability and the parallel software development expertise available in our group. Parallel processing had the further advantage that the various vision algorithms (dark region extraction, contour extraction and shape from shading) could execute concurrently to achieve real-time performance. Real-time endoscope insertion at normal speed requires the processing and analysis of at least ten image frames per second. The most desirable solution is to use multiple sets of processors and implement individual machine vision algorithms on different sets of processors, and then integrate shape information from these algorithms at the control processor level. The navigator and pilot

have been implemented on the multiprocessor host and control processor respectively. The host computer is a workstation based on the Intel486 processor, though equally a Sparc station could have been used. The basic processing element used in the multiprocessor system is the IMS T800 transputer [17].

Dark regions and contour extraction methods have been implemented separately on two sets of five worker processors each. Both processor sets are organised as interconnected pyramids and the overall ten processor system is illustrated in Fig. 5. The frame grabber processor serves as the root of both pyramids for supplying them the raw image data. It digitises colon images either from a video tape made during colonoscopy (for development work) or from a real endoscope. The navigation system and the building of the QL-tree representation of the colon is executed on the control processor. The control processor receives image analysis results from both sets of worker processors and executes the navigator module. The host processor acts as a pilot by executing the tip control commands it receives from the navigator. This set-up has facilitated the testing and analysis of the vision techniques and the navigation system in a real environment.

The frame grabber digitises the on-line colon images into 256 gray levels, constructs the image histogram and finds the first peak in it to estimate the intensity of dark region. An image quadrant is transferred to each worker processor for dark region extraction. The results are supplied to the control processor for navigation and integration of the QL-tree representation. A typical colon image is shown in Fig. 6 along with its histogram. The

extracted dark region is marked as a set of square areas corresponding to quadtree nodes.

The image quadrants of selected images are also supplied to another set of four worker processors for contour extraction. These processors are connected in a pipeline with the control processor. Each worker processor first detects the edge points and then groups them into short line segments of eight pixels in length using the perceptual Hough transform [21] by dividing the image into 8 by 8 squares with two pixels overlap. These line segments are further grouped hierarchically on the basis of continuity, collinearity, theta-aggregation and curvilinearity to obtain curved contours. The last processor finally gathers the contours of each quadrant and groups them into larger contours by maintaining their hierarchical representation in the form of groups of short line segments. These contours are forwarded to the control processor for integration into the QL-tree representation. An example of the contour extraction process is given in Fig. 7. The output of the line segment extraction process is shown in Fig. 7(b). Short line segments are then perceptually grouped into sets of line segments as depicted in Fig. 7(c). Final image contours are derived from these sets and have been overlaid on the colon image in Fig. 7(d).

In the simplest case, when the lumen is clearly visible, its position in the QL-tree plane, that is the dark region's centre of gravity, is used to navigate the endoscope. The statistics gathered during the test runs of the navigation system on various video tapes of colonoscopy indicate that this happens for about 75% of the image frames in a typical colonoscopy procedure. A precise colon space is

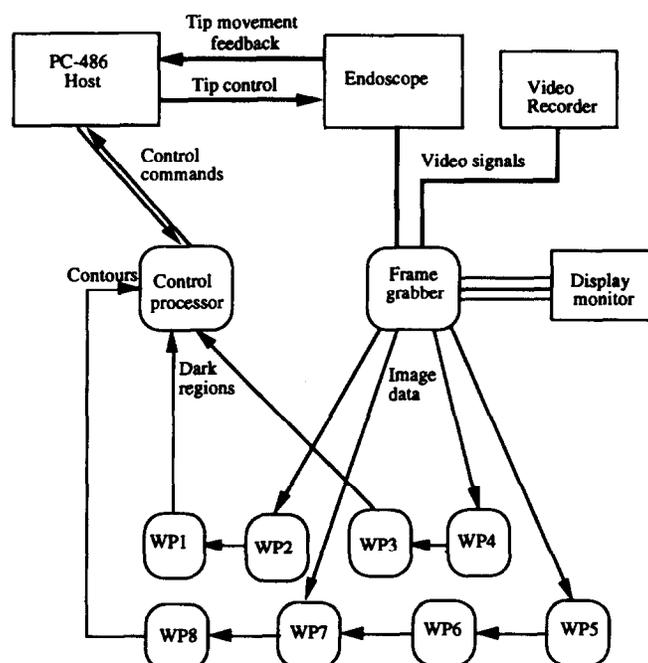


Fig. 5. Multiprocessor system for the automatic endoscope.

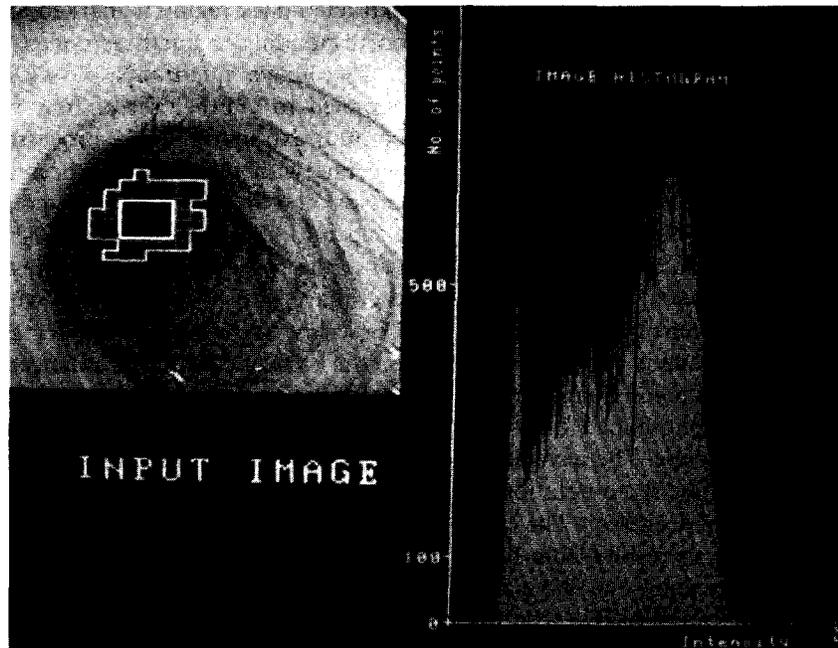


Fig. 6. Dark region extraction from a test image.

constructed by integrating all the dark regions and contours into QL-tree. The individual quadtrees of a QL-tree structure have six levels (in total 62 nodes) for 256 by 256 pixel colon images where the leaves correspond to an  $8 \times 8$  pixel square of the image. The region properties and the contour portions falling in that area are represented at the node. A dark region is stored as a set of non-overlapping square regions in the form of a linked-list structure where the head of the list contains the region perimeter, area and intensity. Similarly a contour is represented by a set of line segments of eight pixels in length. These are arranged as a linked-list of line segments with the head of list containing the contour length, termination line segment and estimates of the curvature. QL-tree nodes are also capable of storing multiple dark regions and the contours present in their corresponding area of the image.

An accurate colon space is maintained of those colon parts through which endoscope has been successfully navigated. This representation is used to confirm the correct endoscope insertion direction when the tip navigation direction provided by a dark region differs significantly from the current tip direction. A complete colon representation is necessary for guiding the endoscope around the bends especially when the view of the lumen is lost and muscle contours are not visible. It is also used to avoid any pockets and unforeseen obstacles when the vision techniques fail to provide any navigational landmarks.

The vision-based navigation system has been tested using material drawn from video tapes. This has enabled the low level algorithms to be thoroughly investigated in

a wide variety of real colonoscopy procedures. In these cases, correct navigation information could be computed. However, the real effectiveness of the navigation system cannot be fully tested in vitro. Thus, an automatic endoscope has been developed in which the tip direction can be set by two computer controlled motors. The tip direction movements can be fed back to the pilot, which in turn can set the tip directions. The automatic endoscope has been navigated successfully in plastic colon models, using some of the methods outlined above. Although the images provided by the plastic models are simpler and more regular than real colon images, they still provide a good illustration of the feasibility of the algorithms.

## 5. Conclusions and future work

Dark regions and occluding contours have been successfully detected in colon images and used to find an obstacle free path to navigate an endoscope. Using this information, an endoscope navigation system, based on a single search space and environment representation, has been successfully constructed. Due to the real-time nature of endoscope control, the machine vision algorithms have been implemented in parallel on two different sets of processors. The QL-tree structure developed for the representation of the colon is an efficient, piecewise,  $2\frac{1}{2}$  D hierarchical search space suitable for representing cylindrical environments for autonomous navigation. An important feature of QL-tree is the

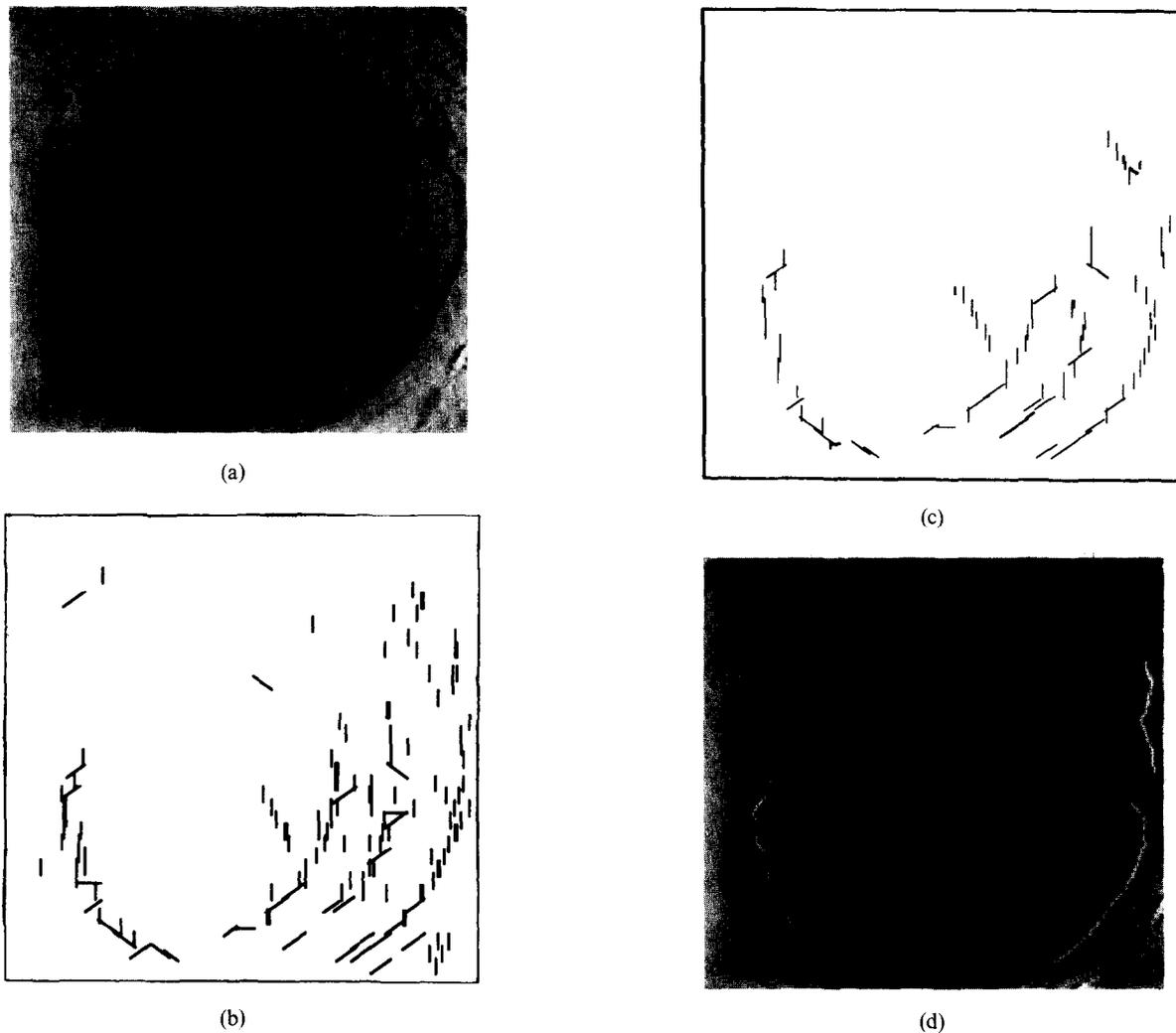


Fig. 7. Contours detection from a test colon image. (a) Colon image, (b) line segments extracted using Perceptual Hough transform, (c) groups of line segments formed, (d) contours overlaid on the image.

facilities it provides for efficient fusion of scene information. The QL-tree based colon model can be utilised to navigate the endoscope around colon bends, twists and pockets where the view is obscured. Navigation results on different videotapes of human colonoscopy procedures are encouraging.

Although a prototype endoscope has been navigated successfully in plastic colon models, with all types of realistic bends and twists, it is difficult to foresee when an automatic endoscope will be safely available for colonoscopy procedures. The present work is laying a foundation which will lead to an important medical application of machine vision in future. Complete automation is a difficult task, however, in the initial phase it will be practical to build an advisory system for the insertion process. In addition to giving global advice, the use of the QL-tree structure be able to suggest tip control actions in cases where the view is obscured. This will be useful for teaching unskilled endoscopists, and will also

familiarise the users with the new automated instrument and enhance their confidence.

Further work is planned in the application of the system to the automation of industrial endoscopes used for inspection of tubes and ducts in process plants and engines. The navigation techniques developed for colonoscopy will, hopefully, prove more successful when used in simple tubes and ducts where the environment is simpler and the navigational landmarks are clearer.

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