In Congestion Games, Taxes Achieve Optimal Approximation

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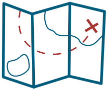
Imperial College London

UNIVERSITY OF LIVERPOOL



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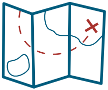

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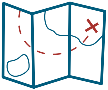

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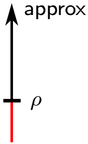

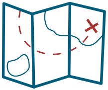

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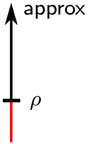

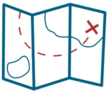

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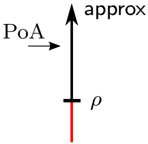

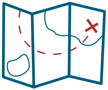

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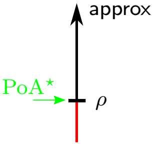

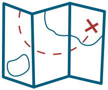

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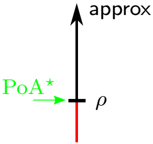

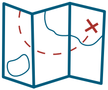

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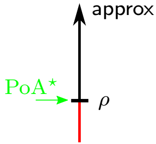

Judiciously designed taxes achieve optimal approx,
and no other tractable intervention can improve

Atomic congestion games

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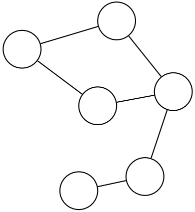
- Set of resources $\mathcal{R}$



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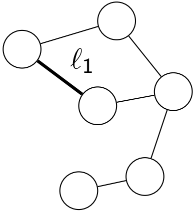

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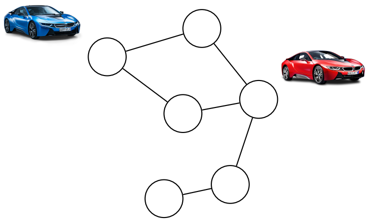

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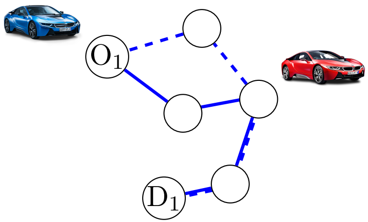

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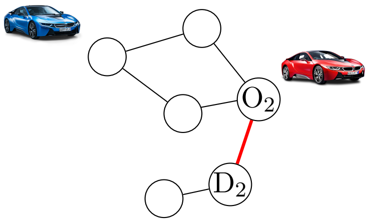

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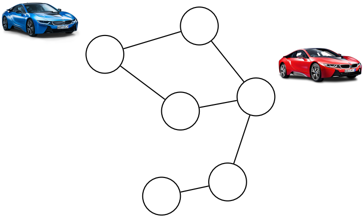

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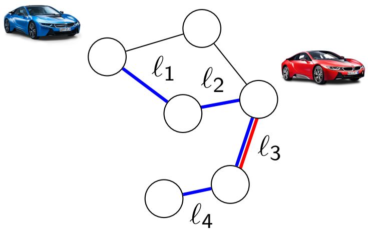

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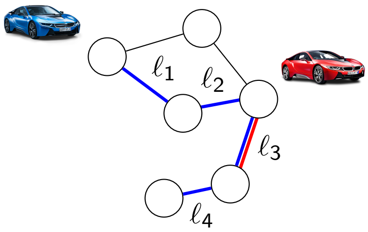

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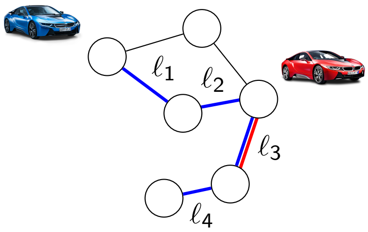

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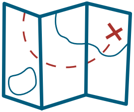
Applications: routing, sensor allocation, scheduling, minimum power, ...



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Take-away: so far no tight computational lower bound

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$\vdots$

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Gap-label-cover [Feige JACM'98; Dudyciz/Manurangsi/Marcinkowski/Sornat IJCAI'20]

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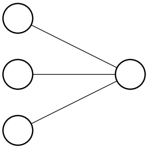

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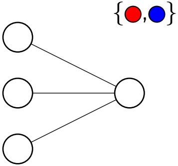

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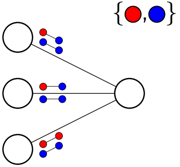

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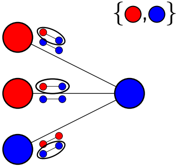

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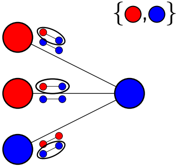

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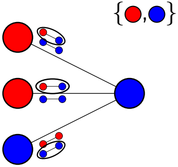

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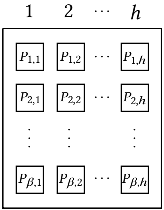

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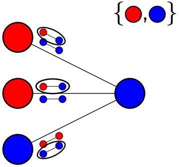

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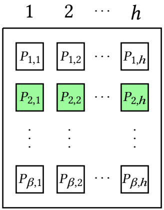

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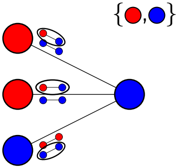

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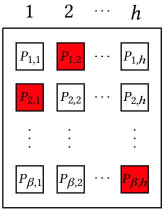

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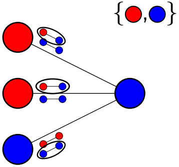

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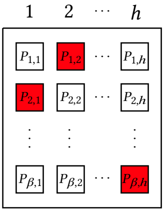
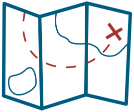
$$\frac{SC(scr)}{SC(row)} \approx \rho_b$$





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Poly-time algorithm based on taxes

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Background:

– price of anarchy measures equilibrium quality, e.g., $SC(a^{\mathrm{NE}})/SC(a^{\mathrm{OPT}})$

[Koutsoupias/Papadimitriou STACS'99; Christodoulou/Koutsoupias STOC'05; Aland/Dumrauf/Gairing/Monien/Schoppmann, STACS'06; Roughgarden JACM'15]

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[Papadimitriou/Roughgarden, JACM'08; Xin Jiang/Leyton-Brown, GEB'15; Hart/Mas-Colell, Econometrica'00; Blum/Hajiaghayi/Ligett/Roth, STOC'08]

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Price of anarchy as approximation ratio

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$\rightsquigarrow$ **Q:** How to improve PoA?

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Take-away: so far no matching approx in general

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Corollary: For any $\varepsilon > 0$, there exists a polynomial time algorithm producing an allocation a^* with cost

$$SC(a^*) \leq (\max_j \rho_{b_j} + \varepsilon) \cdot OPT$$

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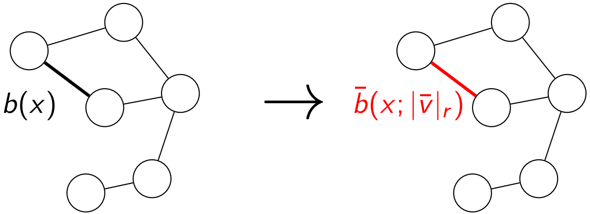

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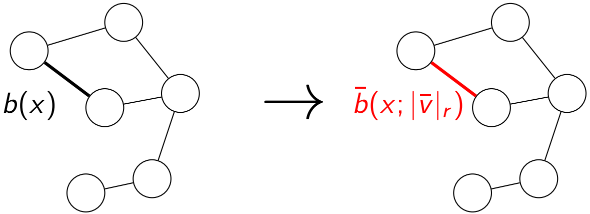

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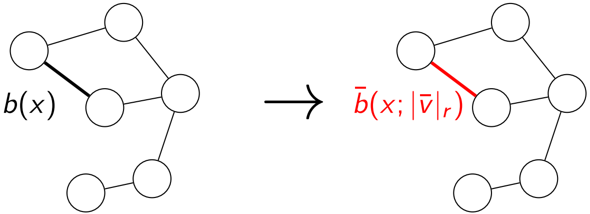

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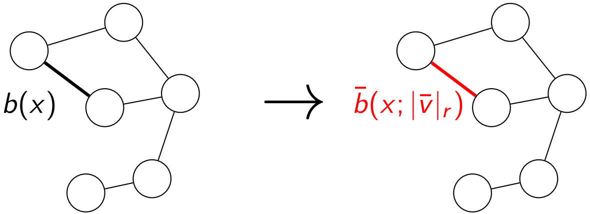

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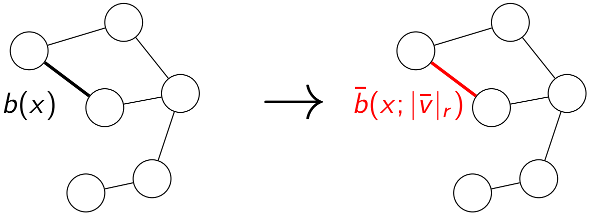

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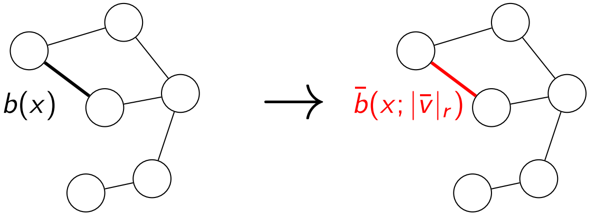

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Main result I: tight NP-hardness of approximation

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* Main result II extends to network CG

“Judiciously designed taxes achieve optimal approximation, and no other tractable intervention can improve upon this result”