

Tutorial 3: Colour

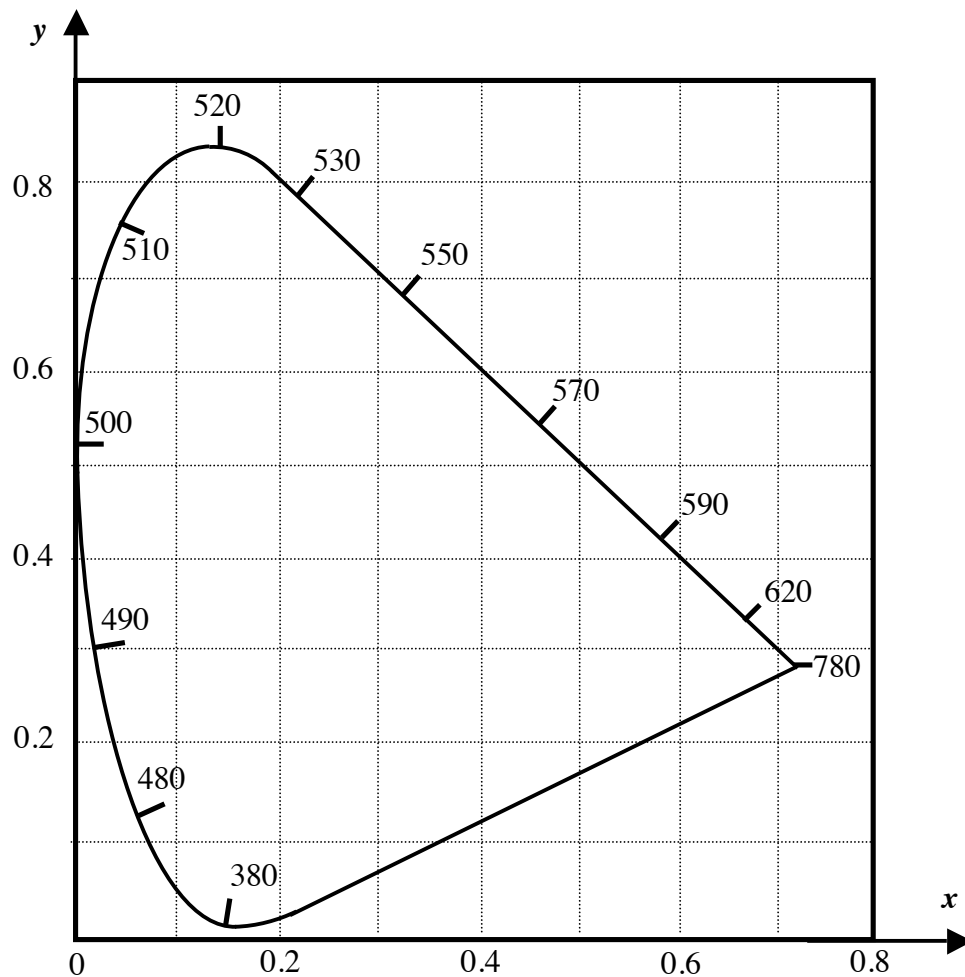
A colour is defined by the following three tri-stimulus intensities obtained during a colour matching experiment:

$$r = 3 \quad g = 3 \quad b = 6$$

1. What is its (x, y) coordinate on the CIE diagram
2. Plot this point on the CIE diagram. Construct a line on the CIE diagram to obtain a measurement of:
 - a. The wavelength of the corresponding pure hue
 - b. The wavelength of the complement of its pure hue
 - c. The (x, y) coordinate of the corresponding pure hue
 - d. The saturation of the colour.
3. A monitor is calibrated and the three phosphors have the following CIE diagram coordinates:

	x	y
Red	0.62	0.34
Green	0.27	0.59
Blue	0.15	0.07

Plot the area of colours that can be displayed by the monitor on your CIE diagram



4. The monitor is driven by a computer system that represents colours by RGB values in the range $[0, 255]$. Find the matrix that will convert (r, g, b) values into (x, y, z) coordinates.

5. The inverse of the matrix of Q4 should be approximately:

$$\begin{pmatrix} 542 & -228 & -84 \\ -316 & 574 & 9 \\ 29 & -91 & 330 \end{pmatrix}$$

You can use this to check your result in Q4.

Compute the RGB values that would be used to represent the original colour defined in Q1.

6. Using the RGB result from part 5 calculate the corresponding HSV values of the colour.

7. A colour given by point $x=0.1, y=0.2$ on the CIE diagram cannot be represented by the light sources of the monitor. How could a light source of this colour be calibrated by subtractive matching, using light sources with the same coordinates as the monitor.

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Solutions

Q1.

$$x = \frac{r}{r+g+b} = 0.25$$

$$y = \frac{g}{r+g+b} = 0.25$$

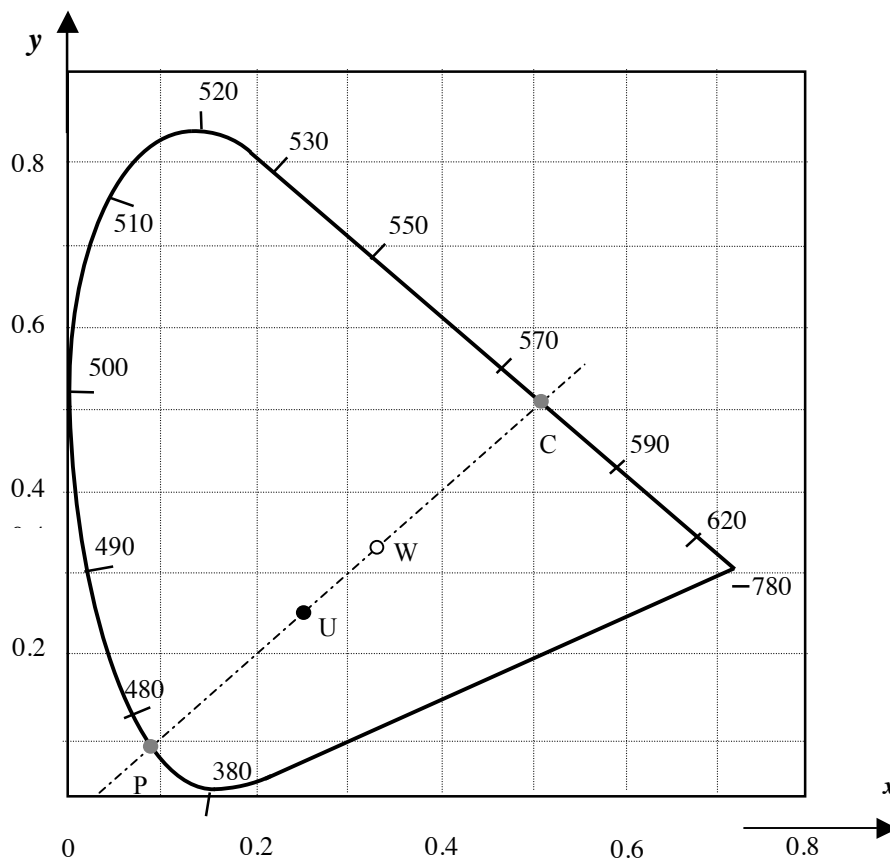
Q2. In the CIE plot below, the point U represents $(x, y) = (0.25, 0.25)$. The white point is W.

a. The point representing the pure hue is P, wavelength 388

b. The complement is point C, wavelength 575

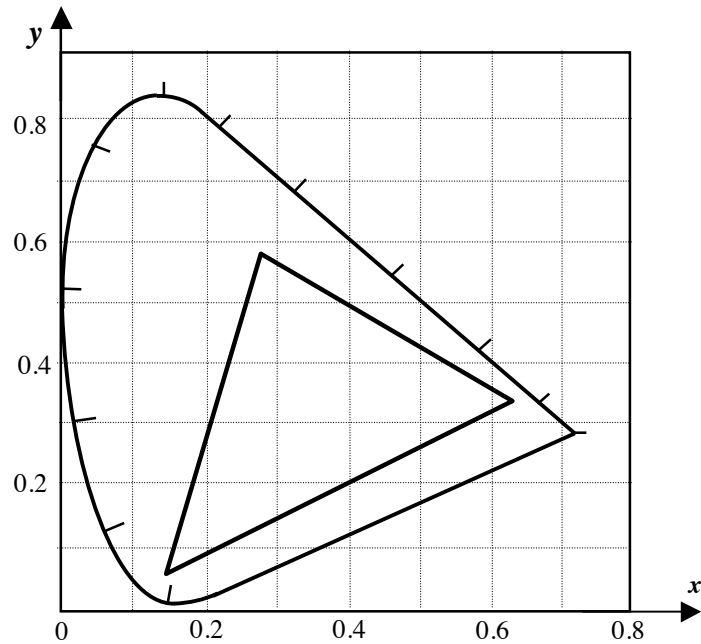
c. (0.08, 0.09)

d. The saturation is the ratio of the distances, $\frac{WU}{WP} \approx 0.335$



Q3. With the values given, the monitor can display colours inside the triangle shown below.

	x	y
Red	0.62	0.34
Green	0.27	0.59
Blue	0.15	0.07



Q4. The R, G, B values need to be normalised in the range $[0, 1]$ and, using the CIE coordinates for the monitor's three phosphors, the transformation from (R, G, B) to (x, y, z) is given as follows

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{255} \begin{pmatrix} 0.62 & 0.27 & 0.15 \\ 0.34 & 0.59 & 0.07 \\ 0.04 & 0.14 & 0.78 \end{pmatrix} \begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

We can apply the division to the matrix to get:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \approx \begin{pmatrix} 0.0024 & 0.0011 & 0.0006 \\ 0.0013 & 0.0023 & 0.0003 \\ 0.0002 & 0.0005 & 0.0031 \end{pmatrix} \begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

Q5. The RGB values are approximately $(37, 69, 150)$:

$$\begin{pmatrix} 542 & -228 & -84 \\ -316 & 574 & 9 \\ 29 & -91 & 330 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.5 \end{pmatrix} \approx \begin{pmatrix} 37 \\ 69 \\ 150 \end{pmatrix}$$

Q6.

$$V = \max(r, g, b) = 150$$

$$S = \frac{\max(r, g, b) - \min(r, g, b)}{\max(r, g, b)} = \frac{150 - 37}{150} \approx 0.75$$

To find H , note that we have $g < r$ and $b < r$ so

$$H = 120 + 120 \times \frac{b - r}{(g - r) + (b - r)}$$

$$H = 120 + 120 \times \frac{150 - 37}{(69 - 37) + (150 - 37)} \approx 215^\circ$$

Q7. First we find out what the corresponding RGB values would be:

$$\begin{pmatrix} 542 & -228 & -84 \\ -316 & 574 & 9 \\ 29 & -91 & 330 \end{pmatrix} \begin{pmatrix} 0.1 \\ 0.2 \\ 0.7 \end{pmatrix} \approx \begin{pmatrix} -50 \\ 90 \\ 216 \end{pmatrix}$$

Since the red component is negative we can mix light from the red source with the given *test* colour to obtain a match. After matching, the result will be a mixture of the green and blue sources.