Tutorial 1: Analysis of three-dimensional (3D) space.

This tutorial is about the use of vector algebra in the analysis of 3D scenes used in computer graphics system. The following notation is used:

- Position vectors are denoted by boldface capital letters: $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{V}$ etc. Position vectors are the same as Cartesian coordinates, and represent position relative to the origin.

- Direction vectors are indicated by boldface lowercase letters $\mathbf{d}$, $\mathbf{n}$ etc. Direction vectors are independent of any origin.

- Scalars are represented by italics: a, b, etc.

A plane is an object that is only defined in Cartesian space, however, each plane has a normal vector, whose size is non zero, and whose direction is at right angles to that plane. We can find a normal vector by taking the cross product of any two direction vectors which are parallel to the plane.

1. Given three points:

$$\mathbf{P}_1 = (10, 20, 5)$$
$$\mathbf{P}_2 = (15, 10, 10)$$
$$\mathbf{P}_3 = (25, 20, 10)$$

find two direction vectors which are parallel to the plane defined by $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$. Hence find a normal vector to the plane.

2. A plane is defined in vector terms by the equation:

$$\mathbf{n} \cdot (\mathbf{P} - \mathbf{P}_1) = 0$$

where $\mathbf{P} = (x, y, z)$ is the locus of a point on the plane, and $\mathbf{P}_1$ is any point known to be in the plane.

For the points given in part 1, expand the vector plane equation to find the Cartesian form of the plane equation, (i.e. $ax + by + cz + d = 0$).

Verify that you get the same result using either $\mathbf{P}_1$ or $\mathbf{P}_2$.

3. Write a procedure, in any programming language you like, which takes as input three points and returns the coefficients of the Cartesian plane equation (a, b, c and d).

4. Starting from any point on a face of a polyhedron, an inner surface normal is a normal vector to the plane of the face whose direction points into the polyhedron.

A tetrahedron is defined by the three points of part 1, and a fourth point $\mathbf{P}_4 = (30, 20, 10)$. Determine whether the normal vector that you calculated in part 1 is an inner surface normal, and if not find the inner surface normal.

5. Two lines intersect at a point $\mathbf{P}_1$, and are in the directions defined by $\mathbf{d}_1$ and $\mathbf{d}_2$. Provided that $\mathbf{d}_1$ and $\mathbf{d}_2$ represent different directions, the two lines define a plane.

Any point on the plane can be reached by travelling from $\mathbf{P}_1$ in direction $\mathbf{d}_1$ by some distance μ and then in direction $\mathbf{d}_2$ by a distance ν.

Using this fact construct the parametric equation of any point on the plane of part 1 in terms of μ,ν, $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$. By taking the dot product with a normal vector to the plane $\mathbf{n}$, show that the parametric plane equation is equivalent to the vector plane equation of part 2.