Tutorial 9:

Animations, Transformations, Projections and Normalisation

This tutorial is a little out of place, but it will serve for revision.

1. *Animating Objects*

A cube in a graphics scene is defined by six square polygons. It is placed in the scene with its centre at (5, 5, 10). In an animation sequence it shrinks by 1/100th of its size in each successive frame until it is too small to be seen. Determine the transformation matrix, which when applied to the co-ordinates of the vertices of the cube will achieve the shrinkage required in each frame.

2. *Viewing transformations*

In a viewer centred animation (*Translator's note*: this means first person shoot 'em up), the viewer is at the point (10, 10, 10). The direction of view is $\mathbf{w} = (0.6, -0.2, 0.77)^T$. The horizontal direction to the right is $\mathbf{u} = (0.79, 0, -0.61)^T$.

Find the third unit vector **v** making up the axis system. Your result for **v** should point mainly upwards so if the y component you calculate is negative you have got the cross product the wrong way round.

Hence write down the viewing transformation matrix. If you've forgotten how to do this look in lecture notes on Scene Transformation and Animation, near the end.

3. *Projection*

The scene from the previous question is to be drawn in perspective projection on the plane $z = 2$. Find the required perspective projection matrix, and combined transformation and projection matrix.

A vertex of the scene has coordinate (10, 10, 20, 1). Where does it project to?

4. *Normalisation*

The world coordinate window in the plane $z = 2$ is between $(x, y) = (-5,-5)$ and $(x, y) = (5,5)$.

The window on the computer screen is of resolution 100 by 100 pixels, and the origin is in the top right hand side (bitmap organisation).

What is the pixel address of the vertex that was projected in the previous question?

Tutorial 9: Solutions

Animations, Transformations, Projections and Normalisation

1. *Animating Objects*

The transformation is in three parts:

> Translate the coordinates so that the centre of the cube is at the origin
> Scale the coordinates by 99/100
> Translate the cube back to its original position

In matrices this becomes

$$\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.99 & 0 & 0 & 0 \\ 0 & 0.99 & 0 & 0 \\ 0 & 0 & 0.99 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

which multiplies out to

$$\begin{pmatrix} 0.99 & 0 & 0 & 0.05 \\ 0 & 0.99 & 0 & 0.05 \\ 0 & 0 & 0.99 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. *Viewing transformations*

The third vertical direction is $\boldsymbol{v} = \boldsymbol{w} \times \boldsymbol{u}$.

This can be evaluated using the cross product determinant which gives

$$\mathbf{v} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0.6 & -0.2 & 0.77 \\ 0.79 & 0 & 0.61 \end{vmatrix} = 0.122\ \hat{\mathbf{i}} + 0.9743\ \hat{\mathbf{j}} + 0.158\ \hat{\mathbf{k}} = \begin{pmatrix} 0.122 \\ 0.9743 \\ 0.158 \end{pmatrix}$$

and is pretty much upwards.

The transformation is:

$$\begin{pmatrix} u_x & u_y & u_z & -\mathbf{C}\cdot u \\ v_x & v_y & v_z & -\mathbf{C}\cdot v \\ w_x & w_y & w_z & -\mathbf{C}\cdot w \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.79 & 0 & -0.61 & -1.8 \\ 0.122 & 0.974 & 0.158 & -12.54 \\ 0.6 & -0.2 & 0.77 & -11.7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. *Projection*

The combined transformation with the projection matrix applied is:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} 0.79 & 0 & -0.61 & -1.8 \\ 0.122 & 0.974 & 0.158 & -12.54 \\ 0.6 & -0.2 & 0.77 & -11.7 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.79 & 0 & -0.61 & -1.8 \\ 0.122 & 0.974 & 0.158 & -12.54 \\ 0.6 & -0.2 & 0.77 & -11.7 \\ 0.3 & -0.1 & 0.385 & -5.85 \end{pmatrix}$$

The coordinate (10, 10, 20, 1) projects as follows:

$$\begin{pmatrix} 10 \\ 10 \\ 20 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0.79 & 0 & -0.61 & -1.8 \\ 0.122 & 0.974 & 0.158 & -12.54 \\ 0.6 & -0.2 & 0.77 & -11.7 \\ 0.3 & -0.1 & 0.385 & -5.85 \end{pmatrix} \begin{pmatrix} 10 \\ 10 \\ 20 \\ 1 \end{pmatrix} = \begin{pmatrix} -6.1 \\ 1.58 \\ 7.7 \\ 3.85 \end{pmatrix}$$

To convert this to a Cartesian coordinate, we divide by the last ordinate to get (-1.58, 0.41, 2).

The value of z is 2, which confirms that the point has projected into the viewing plane.

4. *Normalisation*

This can be done with ratios. In the diagram below, let **P** represent the projected point, i.e. (-1.58, 0.41) in world coordinates, and let **O** represent the origin.

In the world coordinate system the ratio of the distance from x to the right hand side to the total window width is

$$\frac{5-(-1.58)}{10} = 0.658$$

The same ratio must be preserved if we measure in pixels so $x_{pix} = 66$.

Similarly the y ratio is 0.459 so $y_{pix} = 46$.