Tutorial 8: Radiosity

1. *Form factors*:

In a radiosity scene the patches are triangular. Two patches are defined as follows:

Patch	Points		
i	(10, 12, 8)	(10, 13, 8)	(10, 11, 9)
j	(5, 6, 12)	(5, 6, 13)	(8, 6, 12)

Assuming that these two patches are visible from each other calculate the two form factors F_{ij} and F_{ji}. (Use the centroid of each triangle, $\frac{1}{3}(\mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3)$, to estimate the distance)

2. *The Hemicube*:

A hemicube is defined by the top plane $z = 1$ and side planes $x = 1, y = 1, x = -1, y = -1$. Assume that the hemicube pixels all have area ΔA . Derive a formula for the delta form factors of the pixels on the side planes in terms of the distance r of their centre to the origin (Hint: evaluate $\cos\phi$ using a dot product).

3. *The Hemisphere*:

A form factor is to be computed by a ray casting algorithm. Rays are to be cast from the centre of the patch with the aim of finding the nearest patch visible by that ray.

The rays are defined by the spherical polar coordinates (θ, ϕ) and are to be spaced at equal intervals of 1 degree ($\pi/180$ radians) in the range $0 < \theta < 180^\circ$, $0 < \phi < 180^\circ$.

If the rays are thought to pass through a unit hemisphere which is divided into approximately square patches around each ray, derive a formula for the delta form factor for the ray.

4. *r-refinement*:

An r-refinement scheme for a triangular mesh moves each point in the direction of greatest change. Let (**P**, B) represent the pairing of a point **P** with a radiosity value of B. Let its neighbours be represented by the pairs ($\mathbf{P}_1$, B_1), ($\mathbf{P}_2$, B_2), ($\mathbf{P}_3$, B_3) and ($\mathbf{P}_4$, B_4).

One suggestion for refining the mesh is to find the direction of greatest change by adding up the vectors

$$|B_1 - B|(\mathbf{P}_1 - \mathbf{P}) \quad |B_2 - B|(\mathbf{P}_2 - \mathbf{P}) \quad |B_3 - B|(\mathbf{P}_3 - \mathbf{P}) \quad |B_4 - B|(\mathbf{P}_4 - \mathbf{P})$$

Suggest a way in which the distance each point should be moved. Given the following points:

Point	Coordinate	Radiosity
P	(20, 6, 0)	30
$\mathbf{P}_1$	(10, 10, 0)	50
$\mathbf{P}_2$	(10, 30, 0)	20
$\mathbf{P}_3$	(15, 2, 0)	30
$\mathbf{P}_4$	(10, 0, 0)	50

use your method to determine how to move point **P**

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Solutions

1. *Form factors*:

Patch	Points		
i	(10, 12, 8)	(10, 13, 8)	(10, 11, 9)
j	(5, 6, 12)	(5, 6, 13)	(8, 6, 12)

The form factor is given by $F_{ij} = \dfrac{\cos\phi_i \cos\phi_j \left|A_j\right|}{\pi r^2}$.

The centroids are: (10, 12, 8.33) and (6, 6, 12.33).

The vector **r** joining the patches is (4, 6, -4) and $r^2 = 4^2 + 6^2 + (-4)^2 = 68$, i.e. $r = |\mathbf{r}| = \sqrt{68}$.

Normal vectors can be found from the cross product of the edge vectors:

Patch	Edge vectors	Cross product	Unit normal
i	(0, 1, 0) (0, -1, 1)	(1, 0 , 0)	(1, 0, 0)
j	(0, 0, 1) (3, 0, 0)	(0, 3, 0)	(0, 1, 0)

Thus

$$\cos\phi_i = \frac{\mathbf{n}_i \cdot \mathbf{r}}{|\mathbf{r}|} = \frac{4}{r} \text{ and } \cos\phi_j = \frac{\mathbf{n}_j \cdot \mathbf{r}}{|\mathbf{r}|} = \frac{6}{r}$$

The area of a parallelogram spanned by two vectors is given by the magnitude of the cross product vector. The triangle spanned by the vectors is half the parallelogram, so the patch areas are:

$$\left|A_i\right| = \frac{1}{2} \times 1 = \frac{1}{2} \text{ and } \left|A_j\right| = \frac{1}{2} \times 3 = \frac{3}{2}$$

So

$$F_{ij} = \frac{\left(\frac{4}{r}\right)\left(\frac{6}{r}\right)\frac{3}{2}}{\pi\, r^2} = \frac{36}{\pi\, r^4} = \frac{36}{4624\pi} \approx 0.0025 \text{ and } F_{ji} = \frac{12}{4624\pi} \approx 0.0025$$

2. The hemicube:

Consider one face, say $x = -1$. The unit normal vector is $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. The vector from a point on the face to the origin is $\begin{pmatrix} 1 \\ -y \\ -z \end{pmatrix}$. Making this into a unit vector **p** gives

$$\mathbf{p} = \frac{1}{\sqrt{1+y^2+z^2}}\begin{pmatrix} 1 \\ -y \\ -z \end{pmatrix} = \frac{1}{r}\begin{pmatrix} 1 \\ -y \\ -z \end{pmatrix} \quad \text{and} \quad \cos\phi_i = \mathbf{n}\cdot\mathbf{p} = \frac{1}{r}$$

At the origin the unit normal vector is $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

The unit vector from the origin towards the point is $-\mathbf{p}$ so $\cos\phi_j = \frac{z}{r}$. Therefore the form factor is

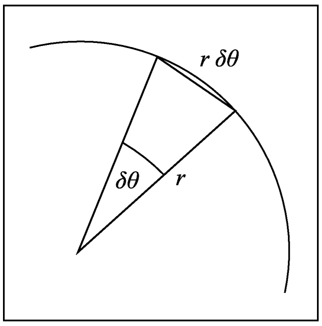
$$\frac{\Delta A \cos\phi_i \cos\phi_j}{\pi r^2} = \frac{\Delta A\, z}{\pi r^4}$$

The form factors for the other side faces are all the same by symmetry.

3. The hemisphere:

The situation is simpler than for the hemicube because $r = 1, \cos\phi_i = 1$, and $\cos\phi_j = z$.

Thus the delta form factor is just $\frac{\Delta A\, z}{\pi}$



Now we need to estimate ΔA. Each ray passes through a patch bounded by four small arcs that can be viewed as approximating a square. The radius equals 1 so the length of each arc subtended by a small angle of 1 degree is $\frac{\pi}{180}$(see right).

Assuming that each arc is a side of the approximated square patch gives an area estimate of $\left(\frac{\pi}{180}\right)^2$

Thus the form factor for each patch is $\left(\frac{\pi}{180}\right)^2 \frac{z}{\pi} = \frac{\pi z}{180^2}$.

4. r-refinement:

The direction of movement can be normalised to the maximum radiosity. One scheme could be to divide the direction vector by the sum of the total radiosity change

$$|B_1 - B| + |B_2 - B| + |B_3 - B| + |B_4 - B| = \sum_{i=1}^{4} |B_i - B|$$

A problem with this would be the case where *all* the radiosity change was to one neighbour. This would move the point all the way to that neighbour. This suggests that it would be better to relax the change by at least a further half.

For the numeric example:

$$|B_1 - B|\,(\mathbf{P}_1 - \mathbf{P}) = 20 \times \begin{pmatrix} -10 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -200 \\ 80 \\ 0 \end{pmatrix} \qquad |B_2 - B|\,(\mathbf{P}_2 - \mathbf{P}) = 10 \times \begin{pmatrix} -10 \\ 24 \\ 0 \end{pmatrix} = \begin{pmatrix} -100 \\ 240 \\ 0 \end{pmatrix}$$

$$|B_3 - B|\,(\mathbf{P}_3 - \mathbf{P}) = 0 \times \begin{pmatrix} -5 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad |B_4 - B|\,(\mathbf{P}_4 - \mathbf{P}) = 20 \times \begin{pmatrix} -10 \\ -6 \\ 0 \end{pmatrix} = \begin{pmatrix} -200 \\ -120 \\ 0 \end{pmatrix}$$

These values give the direction $\begin{pmatrix} -500 \\ 200 \\ 0 \end{pmatrix}$. Normalising by the sum of the radiosity changes gives a direction of:

$$\frac{1}{50}\begin{pmatrix} -500 \\ 200 \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ 4 \\ 0 \end{pmatrix}$$

Relaxing by a factor of 2 gives

$$\begin{pmatrix} -5 \\ 2 \\ 0 \end{pmatrix}$$

So **P** moves from (20, 6, 0) to (15, 8, 0) . This is perhaps rather too large a change for an iterative process. A better scheme might involve using the distances between the points.