Counterpart Semantics for Quantified Modal Logic

Francesco Belardinelli

1 Introduction

In this paper we deal with the semantics for quantified modal logic, QML in short, and their philosophical relevance. In the first part we introduce Kripke semantics for the first-order modal language $\mathcal{L}^{=}$ with identity, then we consider some unsatisfactory features of this account from an actualist point of view. In addition, we show that the calculus $Q^E.K + BF$ on free logic, with the Barcan formula is incomplete for this interpretation. In the second part of the paper we present counterpart semantics, as defined in (Brauner & Ghilardi, 2007; Corsi, 2001). We show that it faithfully formalizes Actualism, encompasses Kripke semantics, while analysing the modal properties of individuals in a more refined way.

Quantified modal logic has always had a strong philosophical appeal, since it first appeared in papers by Barcan Marcus (Barcan, 1946a; 1946b; 1947), Hintikka (Hintikka, 1961; 1969), Prior (Prior, 1956; 1957; 1968) and Kripke (Kripke, 1959; 1963a; 1963b). Besides the topics of propositional modal logic - necessity and possibility, individual knowledge, obligations and permissions, programs and computations - quantified modal logic especially focuses on individuals: we can talk about *actual* and *possible objects*, the *existence* and the *modal properties* of individuals, as well as *counterfactual situations*. In the philosophy of QML we find dramatically relevant issues such as *Actualism/Possibilism*, *realism about possible worlds*, *trans-world identity of individuals*[1]. It is clear that the formal development of quantified modal logic will provide an useful tool to precisely define the concepts above.

2 Kripke Semantics

Kripke semantics is widely used to assign a meaning to modal languages; it stems from Leibniz's intuition of defining necessity as truth in every possible world.

We start with introducing the first-order modal language $\mathcal{L}^{=}$, which contains an infinite set of individual variables $x_1, x_2, \ldots$; an infinite set of n-ary predicative constants $P_1^n, P_2^n, \ldots$, for every $n \in \mathbb{N}$; the propositional connectives $\neg$, $\rightarrow$; the universal quantifier $\forall$; the modal operator $\Box$ and the identity symbol $=$. The first-order modal formulas $\phi, \phi', \ldots$ in $\mathcal{L}^{=}$ are defined as follows:

$$\phi \quad ::= \quad P^n(y_1, \ldots, y_n) \mid y = y' \mid \neg\phi \mid \phi \rightarrow \phi' \mid \Box\phi \mid \forall y \phi$$

[1] See (Chihara, 1998; Loux, 1979; Menzel Summer 2005) for surveys of these subjects.

The logical constants $\bot$, $\wedge$, $\vee$, $\leftrightarrow$, $\exists$ and $\diamond$ are defined by means of those above in the standard way. By $\phi[y_1, \ldots, y_n]$ we mean that the free variables in ϕ are among $y_1, \ldots, y_n$; while $\phi[y/y']$ denotes the formula obtained by substituting some, possibly all, free occurrences of y in ϕ with y', renaming bounded variables if necessary.

Note that no symbol for constants or functors appears in $\mathcal{L}^=$, therefore the only terms in our language are individual variables.

In order to assign a meaning to the formulas in $\mathcal{L}^=$ we extend the Kripke structures for propositional modal logic to the first-order.

Definition 2.1 (Kripke Frame) *A Kripke frame $\mathcal{F}$ - K-frame in short - is a 4-tuple $\langle W, R, D, d\rangle$ s.t.*

- *W is a non-empty set;*
- *R is a relation on W;*
- *for $w, w' \in W$, $D(w)$ is a non-empty set s.t. wRw' implies $D(w) \subseteq D(w')$;*
- *for $w \in W$, $d(w)$ is a possibly empty subset of $D(w)$.*

Intuitively, W is the set of possible worlds and R is the accessibility relation between worlds. Each outer domain $D(w)$ contains the individuals which it makes sense to talk about in w, while each inner domain $d(w)$ is the set of inviduals actually existing in w.

We say that a K-frame $\mathcal{F}$ has *constant* (resp. *increasing*, *decreasing*) inner domains iff wRw' implies $d(w) = d(w')$ (resp. $d(w) \subseteq d(w')$, $d(w) \supseteq d(w')$).

Definition 2.2 (Kripke Model) *A Kripke model $\mathcal{M}$ - K-model in short - is a couple $\langle \mathcal{F}, I\rangle$ where $\mathcal{F}$ is a K-frame and the interpretation I is a function s.t.*

- *for every n-ary predicative constant P^n and $w \in W$, $I(P^n, w)$ is an n-ary relation on $D(w)$;*
- *$I(=, w)$ is the equality relation on $D(w)$.*

Finally, we consider the truth conditions for a formula $\phi \in \mathcal{L}^=$ at a world w w.r.t. a w-assignment σ from the variables to the elements in $D(w)$:

$$
\begin{array}{rll}
(\mathcal{M}^\sigma, w) \models P^n(y_1, \ldots, y_n) & \text{iff} & \langle \sigma(y_1), \ldots, \sigma(y_n)\rangle \in I(P^n, w) \\
(\mathcal{M}^\sigma, w) \models y = y' & \text{iff} & \sigma(y) = \sigma(y') \\
(\mathcal{M}^\sigma, w) \models \neg\psi & \text{iff} & (\mathcal{M}^\sigma, w) \not\models \psi \\
(\mathcal{M}^\sigma, w) \models \phi \to \psi & \text{iff} & (\mathcal{M}^\sigma, w) \not\models \phi \text{ or } (\mathcal{M}^\sigma, w) \models \psi \\
(\mathcal{M}^\sigma, w) \models \Box\phi & \text{iff} & \text{for every } w' \in W,\ wRw' \text{ implies } (\mathcal{M}^\sigma, w') \models \phi \\
(\mathcal{M}^\sigma, w) \models \forall y\psi & \text{iff} & \text{for every } a \in d(w), (\mathcal{M}^{\sigma\binom{y}{a}}, w) \models \psi
\end{array}
$$

where $\sigma\binom{y}{a}$ is the w-assignment that differs from σ at most on y and assigns element a to y. Note that the clause for $\Box$-formulas is well-defined, as by the increasing outer domain condition σ is a w'-assignment whenever it is a w-assignment.

The truth conditions for the formulas containing the logical constants $\wedge$, $\vee$, $\leftrightarrow$, $\exists$ and $\diamond$ are defined from those above. Furthermore, a formula $\phi \in \mathcal{L}^=$ is said to be

true at a world w	iff it is satisfied at w by every w-assignment σ
valid on a model $\mathcal{M}$	iff it is true at every world in $\mathcal{M}$
valid on a frame $\mathcal{F}$	iff it is valid on every model based on $\mathcal{F}$
valid on a class $\mathcal{C}$ *of frames*	iff it is valid on every frame in $\mathcal{C}$

While a w-assignment σ has outer domain $D(w)$ as codomain, the quantifiers range over the inner domain $d(w)$. This means that the classic theory of quantification is not valid on the class of all Kripke frames.

In the next paragraph we highlight the unsatisfactory features of Kripke semantics from an actualist point of view.

3 Actualism

Kripke semantics assumes the increasing outer domain condition: for all $w, w' \in W$, if wRw' then $D(w) \subseteq D(w')$. This constraint is required for evaluating $\Box$-formulas - otherwise a variable y s.t. $\sigma(y) \in D(w)$ might have no denotation in $D(w')$ - but is it philosophically motivated? In this section we negatively answer this question, on the grounds of problems related to the existence and trans-identity of individuals. Thus, we lay the foundations of a counterpart-theoretic approach to quantified modal logic.

3.1 Increasing outer domains

In par. 2 we presented Kripke semantics for the first-order modal language $\mathcal{L}^=$. We remind the evaluation clause for $\Box$-formulas:

$$(\mathcal{M}^\sigma, w) \models \Box\phi \quad \text{iff} \quad \text{for every } w', wRw' \text{ implies } (\mathcal{M}^\sigma, w') \models \phi$$

The same assignment σ to the variables in $\mathcal{L}^=$ appears in evaluating both $\Box\phi$ and ϕ. This means that $\Box\phi$ is true at a world w for the individuals $a_1, \ldots, a_n$ in the outer domain of w, iff in all the worlds accessible from w formula ϕ is true for the same $a_1, \ldots, a_n$. This definition lays down a problem of trans-world existence: in order to evaluate $\Box$-formulas in a K-model, we have to assume that the individuals $a_1, \ldots a_n$ in a world w exist in all the worlds accessible from w. Kripke semantics requires the increasing outer domain condition, which was assumed in def. 2.1.

Nonetheless, there is a number of contexts in which this constraint is not intuitive at all, just consider temporal logics: things now existing probably will not exist in some future time[2]. Even in epistemic and modal logic, we may be willing to think of

[2]As a roman epigraph states: *Fui non sum, es non estis, nemo immortalis.* This ontological account is known as *presentism*, for a survey of the eternalism/presentism issue see (Loux, 1998; Lowe, 1998).

epistemic states and possible worlds containing fewer individuals than the present one. After all, actualists deny the existence of all the possible individuals but the actual ones:

> Actualism is the philosophical position that everything there is - everything that can be said to exists in any sense - is *actual*. Put another way, actualism denies that there is any kind of being beyond actuality; to be is to be actual.[3]

If we accept the actualist account of existence, then we are eventually forced to dropping the increasing outer domain condition.

3.2 Varying domain K-models

In Kripke semantics we have a way to reconcile increasing outer domains and Actualism. It consists in distinguishing for each possible world w an outer domain $D(w)$ of objects, to which it makes sense to ascribe properties and relationships, from an inner domain $d(w)$ of existing individuals, over which the quantifiers range. In this way we obtain the varying domain K-models in par. 2, that first appeared in (Kripke, 1963b) as a formal representation of Actualism in the author's intent. This approach has some point, as the varying domain K-models formalize the idea of diverse individuals existing in different instants. Moreover, possibilist principles such as the Barcan formula $\forall x \Box \phi \to \Box \forall x \phi$, its converse $\Box \forall x \phi \to \forall x \Box \phi$ and the necessity of existence $\forall x \Box E(x)$ - which are all rejected by actualists - are no longer valid. In conclusion, can actualists be content with the varying domain settings in Kripke semantics?

In (Menzel Summer 2005) Menzel lists two actualist issues, which are not completely satisfied by this solution:

1. In the object-language the quantifiers range only over the individuals in the inner domain, as it is expressed by the evaluation clause for $\forall$-formulas:

   $$(\mathcal{M}^{\sigma}, w) \models \forall y \phi \quad \text{iff} \quad \text{for every } a \in d(w), \ (\mathcal{M}^{\sigma\binom{y}{a}}, w) \models \phi$$

   but in the meta-language of K-frames we deal with two distinct types of sets, i.e. $D(w)$ and $d(w)$, for each $w \in W$. Thus, the *possibilia* swept out by the door, come back through the window. Furthermore, since the classic theory of quantification is no longer valid, we are eventually forced to the existence predicate E and free logic to recover a sound first-order calculus. This is a quite ironic consequence for a philosophical account which does not want to discriminate between actual and possible existence.

2. In varying domain K-models it can be the case that some individual a belongs to $D(w)$ but not to $d(w)$, for some $w \in W$, nonetheless properties and relationships are usually ascribed to a even in w. From a certain perspective this

[3](Menzel Summer 2005), p. 1. See also (Linsky & Zalta, 1994), p. 436.

is quite intuitive: think about Plato who is considered, at the present time, a great philosopher even if he died in 347 BC. But this characteristic of Kripke semantics conflicts with the fundamental thesis of *Strong Actualism*[4]: if an object a does not exist in a world w, then nothing can be said about a in w. If we accept Strong Actualism, then we must admit truth-value gaps in Kripke semantics, even for modal formulas evaluated on existing objects $a_1, \ldots, a_n$, whenever any a_i does not appear in some accessible world.

We conclude that Kripke models with varying inner domains are not a satisfactory proposal for reconciling increasing outer domains and the actualist account, in particular w.r.t. Strong Actualism. These last remarks seem to deny the very possibility of a formal representation for Actualism in Kripke semantics.

3.3 Trans-world identity

There is a further question, concerning the increasing outer domain condition, which deserves more insight. The definition of satisfaction for $\Box$-formulas is an *a priori* construction, the well-definiteness of which is guaranteed by the recursive process. When *a posteriori* we want to check whether a modal statement $\Box\phi$ is true for an individual a, we need a method to recognize the same a across possible worlds. This tantamounts to the well-known problem of *trans-world identity*, the bibliography of which has been enlarging during the last half-century[5]. This issue is not our concern for the moment, we consider only the (negative) solution to the problem given by Lewis in (Lewis, 1979). But before, we list two other unsatisfactory aspects of Kripke semantics.

The necessity of identity $x = y \to \Box(x = y)$ and the necessity of difference $x \neq y \to \Box(x \neq y)$ hold in every K-model, as consequences of the unrestricted validity of Leibniz's Law $x = y \to (\phi \to \phi[x/y])$. But in temporal logics, for instance, we may wish to talk about fusion and fission of individuals.

The calculi $Q^E.K + BF$ (resp. $Q^E.K + CBF + BF$) on free logic, with the Barcan formula (resp. BF and CBF) are incomplete for Kripke semantics, that is, they both validate the necessity of fictionality $\neg E(x) \to \Box\neg E(x)$, but none of them prove this formula. See (Belardinelli, 2006) for a formal proof of this fact. These incompleteness results extends to modalities stronger than K. Furthermore, in (Ghilardi, 1991) Ghilardi proved that Kripke semantics is incomplete for a wide range of QML calculi.

We conclude that Kripke semantics is far from being completely satisfactory from an actualist point of view, and it cannot handle fusion and fission of individuals. Moreover, the incompleteness results reveal confusion in the meaning of formulas. QML demands a more perspicuous semantics.

[4]See (Prior, 1968) for a brief presentation of Strong Actualism.

[5]We refer to (Loux, 1979), which contains relevant papers on this subject.

4 Counterpart Semantics

In the second part of this paper we introduce the counterpart semantics for QML, which is based on Lewis' intuition in (Lewis, 1979) that it is not possible to identify individuals across possible worlds. He even denies that an individual may exist in different worlds. Lewis substitutes the notion of trans-world identity with a not further explained counterpart relation C, that - he claims - need to be neither transitive, nor symmetric, nor functional, nor injective, nor surjective, nor everywhere defined, but is only reflexive. Now a formula $\Box\phi$ is true at a world w for the individuals $a_1, \ldots, a_n$ iff in every world w' accessible from w, ϕ is true not for the same $a_1, \ldots, a_n$, but for their counterparts $b_1, \ldots, b_n$ in w'.

In (Brauner & Ghilardi, 2007) Ghilardi, Corsi in (Corsi, 2001) and Kracht Kutz in (Kracht & Kutz, 2001; 2002) present various semantics for quantified modal logic based on counterparts. We start with the definition of counterpart frame.

Definition 4.1 (Counterpart Frame) *A counterpart frame $\mathcal{F}$ - c-frame in short - is a 5-tupla $\langle W, R, D, d, C\rangle$ s.t.*

- *W, R, D, d are defined as for K-frames, but D need not to satisfy the increasing outer domain condition;*
- *C is a function assigning a subset of $D(w) \times D(w')$ to every couple $\langle w, w'\rangle \in R$.*

Note that we relax Lewis's Counterpart Theory and allow individuals to exist in more than one world. Interpretations and models are defined as in Kripke semantic, but now we run into problems if the truth conditions of formulas are given by means of infinitary assignments. Consider the following clause which appears in (Fitting, 2004):

$$(\mathcal{M}^\sigma, w) \models \Box\phi[y_1, \ldots, y_n] \text{ iff for every } w' \in W, \text{ for every } \tau, \\ wRw' \text{ and } C_{w,w'}(\sigma(y_i), \tau(y_i)) \text{ imply } (\mathcal{M}^\tau, w') \models \phi[y_1, \ldots, y_n]$$

By this definition Aristotle's Law $\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$ is no longer valid, see (Corsi, 2001) for a counterexample. For recovering this principle either we have to assume Kracht and Kutz's Counterpart-Existence Property: for $w, w' \in W$, for every $a \in D(w)$ there exists $b \in D(w')$ s.t. $C_{w,w'}(a, b)$; or we adopt *finitary assignments* and *typed languages* as Corsi and Ghilardi do. Kracht and Kutz's condition is rather strong and has no deep philosophical motivation, so we choose the second approach.

First of all, we say that each variable x_i is an n-term, for $n \geq i$. The typed language $\mathcal{L}_T^=$ is the set of first-order modal formulas inductively defined as follows:

- if P^k is a k-ary predicative constant and $\vec{t}$ is a k-tuple of n-terms, then $P^k(t_1, \ldots, t_k)$ is an n-formula;
- if ϕ, ϕ' are n-formulas, then $\neg\phi$ and $\phi \to \phi'$ are n-formulas;
- if ϕ is an $n+1$-formula, then $\forall x_{n+1}\phi$ is an n-formula;

- if ϕ is a k-formula and $\vec{t}$ is a k-tuple of n-terms, then $(\Box\phi)(t_1, \ldots, t_m)$ is an n-formula.

We write $\Box(\psi[t_1, \ldots, t_k])$ as a shorthand for $(\Box(\psi[t_1, \ldots, t_k]))(x_1, \ldots, x_n)$.

Now, for $w \in W$, let a finitary n-assignment $\vec{a}$ be an n-tuple of elements in $D(w)$. The valuation $\vec{a}(x_i)$ of an n-term x_i is tantamount to a_i. Finally, the truth conditions for an n-formula ϕ at a world w w.r.t. a finitary n-assignment $\vec{a}$ are inductively defined as follows:

$$
\begin{array}{rcl}
(\mathcal{M}^{\vec{a}}, w) \models P^k(t_1, \ldots, t_k) & \text{iff} & \langle \vec{a}(t_1), \ldots, \vec{a}(t_k) \rangle \in I(P^k, w) \\
(\mathcal{M}^{\vec{a}}, w) \models t = t' & \text{iff} & \vec{a}(t) = \vec{a}(t') \\
(\mathcal{M}^{\vec{a}}, w) \models \neg\psi & \text{iff} & (\mathcal{M}^{\vec{a}}, w) \not\models \psi \\
(\mathcal{M}^{\vec{a}}, w) \models \psi \to \psi' & \text{iff} & (\mathcal{M}^{\vec{a}}, w) \not\models \psi \text{ or } (\mathcal{M}^{\vec{a}}, w) \models \psi' \\
(\mathcal{M}^{\vec{a}}, w) \models (\Box\psi)(t_1, \ldots, t_k) & \text{iff} & \text{for every } w' \in W, \text{for every } b_1, \ldots, b_k \in D(w'), \\
& & wRw', C_{w,w'}(\vec{a}(t_i), b_i) \text{ imply } (\mathcal{M}^{\vec{b}}, w') \models \psi \\
(\mathcal{M}^{\vec{a}}, w) \models \forall x_{n+1}\psi & \text{iff} & \text{for every } a^* \in d(w), (\mathcal{M}^{\vec{a}\cdot a^*}, w) \models \psi
\end{array}
$$

where $\vec{a} \cdot a^*$ is the $n+1$-assignment $\langle a_1, \ldots, a_n, a^* \rangle$.

The truth conditions for the formulas containing the logical constant $\wedge$, $\vee$, $\leftrightarrow$, $\exists$ and $\diamond$ are standardly defined from the ones above. The definitions of truth and validity go as in Kripke semantics.

Note that in counterpart semantics the n-formulas $(\Box\psi)(t_1, \ldots, t_k)$ and $\Box(\psi[t_1, \ldots, t_k])$ are not equivalent: the former has a *de re* reading, while the latter is *de dicto*. Only the implication from the first to the second one holds, while the implication $\Box(\psi[x_1, \ldots, x_n]) \to (\Box\psi)(x_1, \ldots, x_n)$ holds iff the counterpart relations is everywhere defined. Thus, substitution commutes with the modal operators only in particular cases.

In the next paragraph we consider the advantages of counterpart semantics.

5 Counterparts and Actualism

In par. 3.2 we focused on three features of varying domain K-models, which are not completely satisfactory from an actualist point of view:

1. the presence of *possibilia* at least in the meta-language of Kripke semantics;
2. the recourse to the existence predicate E and free logic to recover quantification;
3. the violation of the principle of Strong Actualism, according to which something not existing in a world w cannot have properties in w.

We show that counterpart semantics can deal with all these problems and solve them, thus giving Actualism the first adequate formal representation probably. As regards the presence of *possibilia* in the meta-language of semantics, we assume that for every $w \in W$, $D(w) = d(w)$, i.e. the individuals, which it makes sense to talk about in w, are all and only the objects existing in w. By this choice the classic theory of quantification holds, therefore neither the existence predicate E nor free logic are needed.

Pay attention to the different consequences of assuming $D(w) = d(w)$ in Kripke and counterpart semantics. In the former this constraint validates some principles the kripkean reading of which is rejected by actualists, i.e. the converse of BF. Hence, Kripke semantics seems to force actualists towards varying domain K-models and free logic. In counterpart semantics we have none of this, we can set $D(w) = d(w)$ for every $w \in W$ and reject Possibilism and free logic at once. Clearly CBF is still valid in this framework, but its counterpart-theoretic interpretation no longer clashes with the actualist account, as it only corresponds to the following condition:

$$\text{for } w, w' \in W, \text{ for } a \in d(w) = D(w), \; C_{w,w'}(a, b) \text{ implies } b \in d(w') = D(w')$$

This constraint is actualistically acceptable, as it just says that every counterpart in w' of an existing object exists in w'.

As to the third point, if an individual a does not belong to $D(w')$, we need not to ascribe properties or relationships to a in w' in order to avoid truth-value gaps. In evaluating modal formulas w.r.t. the individual a, we consider the features of a only in the actual world, and of its counterpart(s) in the other accessible worlds. Thus, counterpart semantics nicely formalizes Actualism, as it is free from all the three faults listed above.

Furthermore, counterpart semantics can discriminate formulas which are equivalent in Kripke semantics. In K-frames both BF and the necessity of fictionality $\neg E(x) \rightarrow \Box\neg E(x)$ correspond to decreasing inner domains: wRw' implies $d(w') \subseteq d(w)$. On the other hand, in c-frames BF tantamounts to the surjectivity of the counterpart relation:

$$\text{for } w, w' \in W, \text{ for every } b \in d(w') \text{ there exists } a \in d(w) \text{ s.t. } C_{w,w'}(a, b)$$

while $\neg E(x) \rightarrow \Box\neg E(x)$ holds iff

$$\text{for } w, w' \in W, \text{ for every } b \in d(w'), \; C_{w,w'}(a, b) \text{ implies } a \in d(w)$$

These are quite different constraints, which collapse into decreasing inner domains only in virtue of the strong assumptions on individuals underlying Kripke semantics. In fact, K-frames can be seen as a limit case of c-frames, where the counterpart relation is everywhere defined and it is identity. In this case both surjectivity and fictional faithfulness reduce to decreasing inner domains. We refer to (Belardinelli, 2006) for a formal proof of this fact.

Finally, in counterpart semantics the necessity of identity and the necessity of difference are not unrestrictedly valid, contrarily to what happens in Kripke semantics, but correspond to precise constraints on the counterpart relation:

a c-frame $\mathcal{F}$ is

$$\begin{array}{rll} \text{functional} & \text{iff} & wRw', C_{w,w'}(a,b) \text{ and } C_{w,w'}(a,b') \text{ imply } b = b' \\ & \text{iff} & \mathcal{F} \models (x = y) \rightarrow \Box(x = y) \\ \text{injective} & \text{iff} & wRw', C_{w,w'}(a,b) \text{ and } C_{w,w'}(a',b) \text{ imply } a = a' \\ & \text{iff} & \mathcal{F} \models (x \neq y) \rightarrow \Box(x \neq y) \end{array}$$

Nonetheless, Leibniz's Law unrestrictedly holds, without implying either the necessity of identity or the necessity of difference.

6 Conclusions

We conclude that counterpart semantics is a major improvement in comparison to the kripkean framework. The former encompasses the latter, in addition it adequately formalizes the actualist account of existence. In c-frames we do discriminate formulas deemed equivalent in Kripke semantics and make further subtle distinctions. Counterpart semantics is philosophically and logically motivated, thus deserves a thorough analysis. We briefly outline some possible developments: (a) There is no standard formalism for typed modal languages, the one used here has to be improved and made more natural. (b) Counterpart semantics is context-sensitive; contexts are represented by the types of formulas, that make explicit the (finite string of) individuals w.r.t. which formulas are meaningful. This feature is relevant in applications to linguistics, in order to explicitly state the background in which a statement is meaningful. (c) In typed languages we syntactically discriminate between the *de re* and *de dicto* reading of formulas; this characteristic is useful for epistemic logic.

References

Barcan, R. C. (1946a). The deduction theorem in a functional calculus of first order based on strict implication. *Journal of Symbolic Logic* 11:115–118.

Barcan, R. C. (1946b). A functional calculus of first order based on strict implication. *Journal of Symbolic Logic* 11:1–16.

Barcan, R. C. (1947). The identity of individuals in a strict functional calculus of second order. *Journal of Symbolic Logic* 12:12–15.

Belardinelli, F. (2006). *Quantified Modal Logic and the Ontology of Physical Objects*. Ph.D. Dissertation, Scuola Normale Superiore, Pisa.

Brauner, T., and Ghilardi, S. (2007). First-order modal logic. In Blackburn, P.; van Benthem, J.; and Wolter, F., eds., *Handbook of Modal Logic*. Elsevier. 549–620.

Chihara, C. S. (1998). *The Worlds of Possibility: modal realism and the semantics of modal logic*. Oxford: Clarendon Press.

Corsi, G. (2001). *Counterparts and possible worlds. A study on quantified modal logic*, volume 21. Bologna: CLUEB.

Fitting, M. (2004). First order intensional logic. *Annals of pure and applied logic* 127:173–191.

Ghilardi, S. (1991). Incompleteness results in kripke semantics. *Journal of Symbolic Logic* 56:516–538.

Hintikka, J. (1961). Modality and quantification. *Theoria* 27:119–128.

Hintikka, J. (1969). *Models for Modalities*. Dordrecht: Reidel Publishing Company.

Kracht, M., and Kutz, O. (2001). The semantics of modal predicate logic ii. modal individuals revisited. In Kahle, R., ed., *Intensionality, Lecture Notes in Logic*, volume 22. Los Angeles: ASL.

Kracht, M., and Kutz, O. (2002). The semantics of modal predicate logic i. counterpart frames. In Wolter, F.; Wansing, H.; de Rijke, M.; and Zakharyaschev, M., eds., *Advances in Modal Logic*, volume 3. World Scientific Publishing.

Kripke, S. (1959). A completeness theorem in modal logic. *Journal of Symbolic Logic* 24:1–14.

Kripke, S. (1963a). Semantical analysis of modal logic. *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 9:67–96.

Kripke, S. (1963b). Semantical considerations on modal logic. *Acta Philosophica Fennica* 16:83–94.

Lewis, D. (1979). Counterpart theory and quantified modal logic. In Loux, M., ed., *The Possible and the Actual*. Ithaca, NY: Cornell University Press. 110–128.

Linsky, B., and Zalta, E. N. (1994). In defense of the simplest quantified modal logic. In *Philosophical Perspectives, 8, Logic and Language*. Cornell University Press. 431–457.

Loux, M. J. (1979). *The Possible and the Actual*. Ithaca, NY: Cornell University Press.

Loux, M. J. (1998). *Metaphysics: A contemporary introduction*. London: Routledge.

Lowe, E. J. (1998). *The Possibility of Metaphysics*. Oxford: Clarendon Press.

Menzel, C. (Summer 2005). Actualism. In Zalta, E. N., ed., *The Stanford Encyclopedia of Philosophy*.

Prior, A. N. (1956). Modality and quantification in $s5$. *Journal of Symbolic Logic* 21:60–62.

Prior, A. N. (1957). *Time and Modality*. Oxford: Oxford University Press.

Prior, A. N. (1968). Tense logic for non-permanent existents. In *Paper on Time and Tense*. Oxford: Clarendon Press.

Francesco Belardinelli

Department of Computing
Imperial College
180 Queen's Gate
London SW7 2AZ
F.Belardinelli@imperial.ac.uk
http://homepage.sns.it/belardinelli