

First-order Modal Logic and the Ontology of Physical Objects

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The interaction between Logic and Ontology

First-order temporal logic and the ontology of physical objects share:

1. a common domain of entities (i.e. the physical objects);
2. the problems related to persistence and change in time.

Despite being closely related, the research on these topics has been developed independently:

- philosophers rarely use formal methods;
- logicians analyse \Box but not ∇ .

Overview

1. Three ontologies of physical objects and three semantics for first-order modal logic.
2. **Correspondence Thesis**: each semantics soundly formalises an ontological account.
3. Technical results about first-order modal logic [5, 11, 12] can be used to formally compare the ontological theories.

Three Theories on Persistence

1. **Endurantism**: physical objects persist in time by remaining *wholly present* in every moment which they exist. (Aristotle, Rea [18], Wiggins [24])
2. **Perdurantism**: physical objects are extended in time as well as in space, with *temporal parts* formally analogous to spatial parts. (Hawley [7], Heller [8], Sider [19])
3. **Sequentialism**: we cannot speak of strict identity in time, the temporal stages are connected by a *counterpart relation* into a unique *ens successivum*. (Chisholm [3], Lewis [14], Varzi [22])

Problem: what is the relationship among these different accounts?

In [9, 16, 17] the equivalence of endurantism and perdurantism is conjectured.

The pros of formalisation

1. It clarifies the conceptual content of the various accounts by formulating them in the same (formal) language.
2. It makes possible their comparison, thus rigorously establishing any reduction or independence result.

First-order modal logic will be our formal framework.

First-order Modal Languages

The first-order modal language \mathcal{L} contains:

- individual variables x_1, x_2, \dots
- n -ary predicative constants P_1^n, P_2^n, \dots , for $n \in \mathbb{N}$
- propositional connectives \neg and \rightarrow
- identity symbol $=$
- universal quantifier \forall
- modal operator \Box

First-order modal formulas in \mathcal{L} are defined as follows:

$$\phi ::= P^n(x_1, \dots, x_n) \mid x = y \mid \neg\phi \mid \phi \rightarrow \phi' \mid \Box\phi \mid \forall x\phi$$

Three Semantics for QML

1. The Objectual Interpretation [6, 13]
2. The Intensional Interpretation [6, 10]
3. The Semantics of Counterparts [1, 4, 14]

A specific ontology underlies each of these semantics.

On the other hand, each semantics soundly formalises one of the ontologies.

The Objectual Interpretation

A Kripke model \mathcal{M}_K is a 4-tuple $\langle W, R, D, I \rangle$ s.t.

- $W \neq \emptyset$ and $R \subseteq W^2$
- for $t, t' \in W$, $D(t) \neq \emptyset$ and tRt' implies $D(t) \subseteq D(t')$
- for $t \in W$, $I(P^n, t) \subseteq D(t)^n$

Let $\sigma : Var(\mathcal{L}) \rightarrow D(t)$. The relation of satisfaction is defined as follows:

$$(\mathcal{M}_K^\sigma, t) \models P^n(x_1, \dots, x_n) \quad \text{iff} \quad \langle \sigma(x_1), \dots, \sigma(x_n) \rangle \in I(P^n, t)$$

$$(\mathcal{M}_K^\sigma, t) \models x = y \quad \text{iff} \quad \sigma(x) = \sigma(y)$$

$$(\mathcal{M}_K^\sigma, t) \models \neg\psi \quad \text{iff} \quad (\mathcal{M}_K^\sigma, t) \not\models \psi$$

$$(\mathcal{M}_K^\sigma, t) \models \phi \rightarrow \psi \quad \text{iff} \quad (\mathcal{M}_K^\sigma, t) \not\models \phi \text{ or } (\mathcal{M}_K^\sigma, t) \models \psi$$

$$(\mathcal{M}_K^\sigma, t) \models \Box\phi \quad \text{iff} \quad tRt' \text{ implies } (\mathcal{M}_K^\sigma, t') \models \phi$$

$$(\mathcal{M}_K^\sigma, t) \models \forall x\phi \quad \text{iff} \quad \text{for every } a \in D(t), (\mathcal{M}_K^{\sigma(x/a)}, t) \models \phi$$

Endurantist features of the Objectual Interpretation

- In K -models the individual variables in \mathcal{L} are interpreted on the set $D(t)$, which contains the objects existing at moment $t \in W$.
- For evaluating at moment t a formula $\Box\phi[x_1, \dots, x_n]$, where x_1, \dots, x_n are free, in every moment t' accessible from t we refer to the same individuals $\sigma(x_1), \dots, \sigma(x_n)$ in $D(t) \subseteq D(t')$.
- Reidentification through time is a primitive notion in K -models, as it is for endurantism.

Objectually valid principles accepted by endurantists

- Leibniz's Law: $x = y \rightarrow (\phi \rightarrow \phi[x/y])$

How, if a is b , could there be something true of the object a which was untrue of the object b ? *They are the same object.* People sometimes speak of counterexamples to Leibniz's Law. But these are scarcely more impressive than the counter-examples to the Law of Non-Contradiction. ([24], p. 27)

- the necessity of identity: $x = y \rightarrow \Box(x = y)$
- the necessity of difference: $x \neq y \rightarrow \Box(x \neq y)$
- For both the Objectual Interpretation and Endurantism individuals can neither merge nor separate.

Qualitative change

- A table is clean in the morning and dirty at the evening [22].
By the principle of substitution of identicals *salva veritate*:
(SI) if the individuals a and b are one and the same object, then every statement true of a is also true of b ,
the persistence of the table in change is denied.
- The endurantist solution of indexing properties to time [15, 21, 24] can be formalised within the Objectual Interpretation:
 - $\mathbf{table} \in I(\text{Clean}, \text{morning})$
 - $\mathbf{table} \notin I(\text{Clean}, \text{evening})$
- Once solved the logical problem, there remains the ontological one [22].

Mereological change

- The table in the evening is a proper part of the table in the morning [22].
- If we accept the hypotheses:
 1. $\diamond(tab- = tab_e)$
 2. $\diamond(tab_e = tab_m)$
 3. $tab- < tab_m$
 4. $tab_m \not\prec tab_m$

we can prove a contradiction in the Objectual Interpretation by means of endurantistically valid principles.

Mereological change

1)	$\diamond(tab- = tab_e)$	(1)
2)	$\diamond(tab- = tab_e) \rightarrow (tab- = tab_e)$	nec. of dif.
3)	$(tab- = tab_e)$	1, 2 by MP
4)	$\diamond(tab_e = tab_m)$	(2)
5)	$\diamond(tab_e = tab_m) \rightarrow (tab_e = tab_m)$	nec. of dif.
6)	$(tab_e = tab_m)$	4, 5 by MP
7)	$(tab- = tab_m)$	3, 6 by trans.
8)	$(tab- = tab_m) \rightarrow ((tab- < tab_m) \rightarrow (tab_m < tab_m))$	Leibniz's Law
9)	$(tab- < tab_m) \rightarrow (tab_m < tab_m)$	7, 8 by MP
10)	$(tab- < tab_m)$	(3)
11)	$(tab_m < tab_m)$	9, 10 by MP
12)	$(tab_m \not< tab_m)$	(4)
13)	\perp	11, 12

- Chisholm [2], Van Inwagen [20] and Wiggins [24] deny (2), (3) and (1) respectively and maintain the validity of Endurantism.

Coincident but distinct objects

- A lump of clay is modelled into a statue. The lump persists after the statue has been reshaped, but not the statue [8].
- If we accept the hypotheses:
 1. $s = l$
 2. $\neg \diamond Dif(s)$
 3. $\diamond Dif(l)$

also in this case we can prove a contradiction in the Objectual Interpretation by endurantistically valid principles.

Coincident but distinct objects

- | | | |
|-----|---|-------------------------------|
| 1) | $(s = l) \rightarrow (\neg Dif(s) \rightarrow \neg Dif(l))$ | Leibniz's Law |
| 2) | $\Box(s = l) \rightarrow (\Box\neg Dif(s) \rightarrow \Box\neg Dif(l))$ | 1 by distributivity of \Box |
| 3) | $(s = l) \rightarrow \Box(s = l)$ | necessity of identity |
| 4) | $(s = l) \rightarrow (\Box\neg Dif(s) \rightarrow \Box\neg Dif(l))$ | 2, 3 by transitivity |
| 5) | $s = l$ | (1) |
| 6) | $\Box\neg Dif(s) \rightarrow \Box\neg Dif(l)$ | 4, 5 by MP |
| 7) | $\Box\neg Dif(s)$ | (2) |
| 8) | $\Box\neg Dif(l)$ | 6, 7 by MP |
| 9) | $\neg\Box\neg Dif(l)$ | (3) |
| 10) | \perp | 8, 9 |

- Wiggins' theory of sortals denies (1) and keeps Endurantism valid: the statue s and the lump l belong to two different sortals, therefore they are different objects filling the same space.

The Objectual Interpretation and Endurantism: conclusions

- In K -models each individual domain $D(t)$ contains objects wholly present at moment t .
- For evaluating the modal formulas in \mathcal{L} we refer to the same objects at different moments in time.
- Leibniz's Law, the necessity of identity and the necessity of difference are valid principles.
- The endurantistic solution to the puzzle of qualitative change can be formalised within the K -models.
- The inconsistencies of mereological change and of coincident but distinct objects can be deduced in the Objectual Interpretation.

The Intensional Interpretation

An intensional model \mathcal{M}_i is a 5-tuple $\langle W, R, D, F, I \rangle$ s.t.

- $W \neq \emptyset$ and $R \subseteq W^2$ and $D(t) \neq \emptyset$
- F is non-empty set of functions f on W such that $f(t) \in D(t)$
- for $t \in W$, $I(P^n, t) \subseteq D(t)^n$

Let $\sigma : Var(\mathcal{L}) \rightarrow F$. The relation of satisfaction is defined as follows:

$$(\mathcal{M}_i^\sigma, t) \models P^n(x_1, \dots, x_n) \quad \text{iff} \quad \langle \sigma(x_1)(t), \dots, \sigma(x_n)(t) \rangle \in I(P^n, t)$$

$$(\mathcal{M}_i^\sigma, t) \models x = y \quad \text{iff} \quad \sigma(x)(t) = \sigma(y)(t)$$

$$(\mathcal{M}_i^\sigma, t) \models \Box\phi \quad \text{iff} \quad tRt' \text{ implies } (\mathcal{M}_i^\sigma, t') \models \phi$$

$$(\mathcal{M}_i^\sigma, t) \models \forall x\phi \quad \text{iff} \quad \text{for every } f \in F, (\mathcal{M}_i^{\sigma_f}, t) \models \phi$$

Perdurantist features of the Intensional Interpretation

- In i -models the variables in \mathcal{L} are interpreted on the set F of functions from the moments in W to the temporal stages in each $D(t)$.

[t]hings are world-lines, or functions from times into time slices, and so they correspond to term intensions or individual concepts. Since our ontology takes thing, not their slices as ontologically basic, it is natural to quantify over term intensions in temporal logic. ([6], p. 281)

- For evaluating at moment t a formula $\Box\phi[x_1, \dots, x_n]$, where x_1, \dots, x_n are free, in every moment t' accessible from t we refer to the temporal parts $\sigma(x_1)(t'), \dots, \sigma(x_n)(t') \in D(t')$ of the individuals $\sigma(x_1), \dots, \sigma(x_n)$ in F .

Intensionally valid principles accepted by perdurantists

- Leibniz's Law is not unrestrictedly valid.
In the same way, for a perdurantist two coinciding objects may have different temporal properties.
- The necessity of identity and the necessity of difference do not hold.
- For both the Intensional Interpretation and Perdurantism individuals can merge and separate.

Qualitative change

- A perdurantist accepts the consequences of (SI) and distinguishes the temporal parts of the table.
- The perdurantist solution can be formalised within the Intensional Interpretation: $\mathbf{table} \in F$
 - $\mathbf{table}(\mathit{morning}) \in I(\mathit{Clean}, \mathit{morning})$
 - $\mathbf{table}(\mathit{evening}) \notin I(\mathit{Clean}, \mathit{evening})$
- The logical problem is solved and the ontological opacity is avoided (provided that we accept temporal parts).

Mereological change and coincident but distinct objects

- A Perdurantist avoids the contradiction by rejecting the necessity of identity and the necessity of difference.
- The Intensional Interpretation provides models of the perdurantist solutions to these puzzles:
- Mereological change: $\mathbf{table}, \mathbf{table-} \in F$
 - $\mathbf{table-}(\mathit{morning}) < \mathbf{table}(\mathit{morning})$
 - $\mathbf{table-}(\mathit{evening}) = \mathbf{table}(\mathit{evening})$
- Coincident but distinct objects: $\mathbf{statue}, \mathbf{lump} \in F$
 - $\mathbf{statue}(t) = \mathbf{lump}(t)$
 - $\mathbf{lump}(t') \in I(Dif, t')$
 - $\mathbf{statue}(t') \notin I(Dif, t')$

The Intensional Interpretation and Perdurantism: conclusions

- In i -models the domain F contains individuals extending across points in W . We can think of $f(t) \in D(t)$ as the temporal part of the four-dimensional object f at moment t .
- For evaluating the modal formulas we refer to the temporal parts of a unique object at different moments.
- Leibniz's Law, the necessity of identity and the necessity of difference do not hold.
- The Intensional Interpretation soundly formalises the perdurantist solutions to the puzzles of change.

The Semantics of Counterparts

A counterpart model \mathcal{M}_c is a 5-tuple $\langle W, R, D, C, I \rangle$ s.t.

- $W \neq \emptyset$ and $R \subseteq W^2$ and $D(t) \neq \emptyset$
- for $t, t' \in R$, $C_{t,t'} \subseteq D(t) \times D(t')$
- for $t \in W$, $I(P^n, t) \subseteq D(t)^n$

Let $\vec{a} \in D(t)^m$. The satisfaction relation for m -formulas is defined as follows:

$$(\mathcal{M}_c^{\vec{a}}, t) \models P^n(x_{i_1}, \dots, x_{i_n}) \quad \text{iff} \quad \langle a_{i_1}, \dots, a_{i_n} \rangle \in I(P^n, w)$$

$$(\mathcal{M}_c^{\vec{a}}, t) \models x_{i_1} = x_{i_2} \quad \text{iff} \quad a_{i_1} = a_{i_2}$$

$$(\mathcal{M}_c^{\vec{a}}, t) \models \Box\phi \quad \text{iff} \quad tRt' \text{ and } C_{t,t'}(\vec{a}, \vec{b}) \text{ implies } (\mathcal{M}_c^{\vec{b}}, t') \models \phi$$

$$(\mathcal{M}_c^{\vec{a}}, t) \models \forall x_{n+1}\phi \quad \text{iff} \quad \text{for every } a^* \in D, (\mathcal{M}_c^{\vec{a} \cdot a^*}, w) \models \phi$$

Sequentialist features of Counterpart Semantics

- Lewis' Counterpart Theory extends to alethic modalities the sequentialist account of trans-identity:

[w]e might say, speaking causally, that your counterparts are you in other worlds, that they and you are the same; but this sameness is no more literal identity than the sameness between you today and you tomorrow. ([14], p. 112)

- For evaluating at moment t a formula $\Box\phi[x_1, \dots, x_n]$, where x_1, \dots, x_n are free, in every moment t' accessible from t we do not necessarily refer to the same individuals $a_1, \dots, a_n \in D(t)$, but to their counterparts $b_1, \dots, b_n \in D(t')$.
- Variables are interpreted on world-bound individuals, but reidentification is not taken for granted.

Principles deemed valid by counterpart-theoreticians and sequentialists

- Leibniz's Law is unrestrictedly valid, but...
- ...the necessity of identity and the necessity of difference hold iff the counterpart relation is functional and injective.
 - Functionality: $C_{t,t'}(a, b)$ and $C_{t,t'}(a, b')$ imply $b = b'$
 - Injectivity: $C_{t,t'}(a, b)$ and $C_{t,t'}(a', b)$ imply $a = a'$
- For both Counterpart Semantics and Sequentialism individuals can have more than a counterpart and be counterpart to more than one individual.

Qualitative change

- A sequentialist accepts (SI) and distinguishes between the table in the morning and its counterpart at the evening [19, 23].
- The sequentialist solution can be formalised within Counterpart Semantic:
 - $\mathbf{table}_{morning} \in I(Clean, morning)$
 - $\mathbf{table}_{evening} \notin I(Clean, evening)$
 - $C_{morning, evening}(\mathbf{table}_{morning}, \mathbf{table}_{evening})$
- Also in this case, the logical problem is solved and the ontological criticisms are avoided (provided that we accept counterparts).

Mereological change

- A Perdurantist avoids the contradiction by rejecting the necessity of difference.
- Counterpart Semantics provides a model for the sequentialist solution of the puzzle:
 - $\mathbf{table-} < \mathbf{table}_{morning}$
 - $C_{morning,evening}(\mathbf{table-}, \mathbf{table}_{evening})$
 - $C_{morning,evening}(\mathbf{table}_{morning}, \mathbf{table}_{evening})$

Coincident but distinct objects

- Leibniz's Law is valid for *de re* modalities, but not *de dicto*.
- If the hypotheses are *de re*: $\neg(\diamond Dif(x_1))(s)$, $(\diamond Dif(x_1))(l)$, then we have a contradiction.
- This is not the case if the hypotheses are *de dicto*: $\neg\diamond Dif(s)$, $\diamond Dif(l)$.
- In Counterpart Semantics we can distinguish the two readings of modal formulas.

Counterpart Semantics and Sequentialism: conclusions

- Lewis' Counterpart Theory applies the sequentialist ontology to alethic modalities: the notion of reidentification is substituted by counterparts.
- For evaluating modal formulas we refer to the counterparts of an object at different moments.
- The necessity of identity and the necessity of difference hold iff the counterpart relation is functional and injective.
- Counterpart Semantics soundly formalises the sequentialist solutions to the puzzles of qualitative and mereological change.
- Coincident but distinct objects demands further investigation.

Conclusions

We compared three semantics for *QML* and three theories on the persistence conditions for physical objects by considering the following points:

- the ontological assumptions underlying the logical accounts;
- the agreement on principles for identity;
- the representability of the ontological solutions to the puzzles of change within our formal frameworks.

After this discussion, we uphold the correspondence thesis between semantics for quantified modal logic and ontologies for physical objects.

Applications to the comparison of ontologies

- If the counterpart relation is everywhere defined and it is identity, then a formula ϕ is valid in Counterpart Semantics iff it is valid in the Objectual Interpretation.
- In [5, 11, 12] we have constraints on counterpart models such that a formula ϕ is valid in Counterpart Semantics iff it is valid in the Intensional Interpretation.
- For comparing Kripke and counterpart models we do not have to assume identity as the counterpart relation, weaker constraints are sufficient.
- By the Correspondence Thesis, both Endurantism and Perdurantism can be reduced to Sequentialism.

References

- [1] T. Brauner and S. Ghilardi. First-order modal logic. In P. Blackburn, J. van Benthem, and F. Wolter, editors, *Handbook of Modal Logic*. Elsevier, in preparation.
- [2] R. M. Chisholm. Parts as essential to their wholes. *Review of Metaphysics*, 26:581–603, 1973.
- [3] R. M. Chisholm. *Person and Object*. Open Court, La Salle, IL, 1976.
- [4] G. Corsi. *Counterparts and possible worlds. A study on quantified modal logic*, volume 21. CLUEB, Bologna, 2001.
- [5] M. Fitting. First order intensional logic. *Annals of pure and applied logic*, 127:173–191, 2004.
- [6] J. W. Garson. Quantification in modal logic. In D. Gabbay and F. Günthner, editors, *Handbook of Philosophical Logic*, pages 267–323. Kluwer Academic Publishers, Dordrecht, 2001.
- [7] K. Hawley. *How things persist*. Clarendon Press, Oxford, 2001.
- [8] M. Heller. *The ontology of physical objects: Four-dimensional hunks of matter*. Cambridge University Press, Cambridge, 1990.
- [9] E. Hirsch. *The Concept of Identity*. Oxford University Press, Oxford, 1982.

- [10] G. E. Hughes and M. J. Cresswell. *A New Introduction to Modal Logic*. Routledge, London, 1996.
- [11] M. Kracht and O. Kutz. The semantics of modal predicate logic ii. modal individuals revisited. In R. Kahle, editor, *Intensionality, Lecture Notes in Logic*, volume 22. ASL, Los Angeles, 2001.
- [12] M. Kracht and O. Kutz. The semantics of modal predicate logic i. counterpart frames. In F. Wolter, H. Wansing, M. de Rijke, and M. Zakharyashev, editors, *Advances in Modal Logic*, volume 3. World Scientific Publishing, 2002.
- [13] S. Kripke. Semantical considerations on modal logic. *Acta Philosophica Fennica*, 16:83–94, 1963.
- [14] D. Lewis. Counterpart theory and quantified modal logic. In M. Loux, editor, *The Possible and the Actual*, pages 110–128. Cornell University Press, Ithaca, NY, 1979.
- [15] E. J. Lowe. Lewis on perdurance versus endurance. *Analysis*, 47:152–154, 1987.
- [16] S. McCall. *A Model of the Universe*. Clarendon Press, Oxford, 1994.
- [17] K. Miller. The metaphysical equivalence of three and four dimensionalism. *Erkenntnis*, 62:91–117, 2004.
- [18] M. C. Rea. Temporal parts unmotivated. *Philosophical Review*, 107:225–260, 1998.
- [19] T. Sider. *Four dimensionalism: An ontology of Persistence and Time*. Oxford University Press, Oxford, 2001.
- [20] P. van Inwagen. The doctrine of arbitrary undetached parts. *Pacific Philosophical Quarterly*, 62:123–137, 1981.

- [21] P. van Inwagen. Four-dimensional objects. *Nous*, 24:245–255, 1990.
- [22] A. Varzi. Entia successiva. *Rivista di estetica*, 43(1):139–158, 2003.
- [23] A. Varzi. Riferimento, predicazione e cambiamento. In C. Bianchi and A. Bottani, editors, *Significato e Ontologia*, pages 221–249. Franco Angeli, Milano, 2003.
- [24] D. Wiggins. *Sameness and substance renewed*. Cambridge University Press, Cambridge, 2001.