A Logic for Global and Local Announcements

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Outline

1. Background: logics for (public, semi-private, private) announcements [vDHvdHK15]
   In PAL announcements are
   - **public**: all agents listen to (and are aware of!) the announcement
   - **global**: how the new information is processed depends on the model (i.e., public announcements are model transformers)
2. Goal: to generalise PAL by weaking **publicity** and **globality**
   - **privacy**: announcements to any subset $A \subseteq Ag$ of agents
   - **locality**: announcements are **pointed model** transformers
3. Dynamic Epistemic Logic: action models allow private announcements, but
   - updated indistinguishability relations are not necessarily equivalences
   - updated models might be strictly larger . . .
   - . . . several problems are undecidable
4. GLAL: an extension of PAL supporting both **private** and **local** announcements
   - updated indistinguishability relations **are** equivalences
   - updated models are normally "smaller" . . .
   - . . . the model checking and satisfaction problems are decidable

Let Ag be a set of agents and AP a set of propositional atoms.

Definition (GLAL)

Formulas ϕ in $\mathcal{L}_{glal}$ are defined by the following BNF:

$$\psi \quad ::= \quad p \mid \neg\psi \mid \psi \wedge \psi \mid K_a\psi \mid C_A\psi \mid [\psi]_A^+\psi \mid [\psi]_A^-\psi$$

- $[\psi]_A^+\phi$ ::= after **globally** announcing ψ to the agents in A, ϕ is true
- $[\psi]_A^-\phi$ ::= after **locally** announcing ψ to the agents in A, ϕ is true

$$\mathcal{L}_{pl} \subseteq \mathcal{L}_{el} \subseteq \mathcal{L}_{pal^+} \subseteq \mathcal{L}_{glal}$$

Formulas in GLAL are interpreted on (multi-modal) Kripke models.

Definition (Frame)

A **frame** is a tuple $\mathcal{F} = \langle W, \{R_a\}_{a \in Ag} \rangle$ where

- W is a set of **possible worlds**
- for every agent $a \in Ag$, $R_a \subseteq 2^{W \times W}$ is an **equivalence relation** on W.

A **model** is a pair $\mathcal{M} = \langle \mathcal{F}, V \rangle$ where $V : AP \to 2^W$ is an assignment to atoms.

- $R_A^C = (\bigcup_{a \in A} R_a)^*$ is the reflexive and transitive closure of $\bigcup_{a \in A} R_a$
- $R(w) = \{w' \in W \mid R(w, w')\}$ is the R-equivalence class of $w \in W$

Satisfaction & Refinements

The **satisfaction set** $[\![\varphi]\!]_{\mathcal{M}} \subseteq W$ is defined as

$$
\begin{array}{lcl}
[\![p]\!]_{\mathcal{M}} & = & V(p) \\
[\![\neg\psi]\!]_{\mathcal{M}} & = & W \smallsetminus [\![\psi]\!]_{\mathcal{M}} \\
[\![\psi \wedge \psi']\!]_{\mathcal{M}} & = & [\![\psi]\!]_{\mathcal{M}} \cap [\![\psi']\!]_{\mathcal{M}} \\
[\![C_A\psi]\!]_{\mathcal{M}} & = & \{w \in W \mid \text{for all } w' \in R^C_A(w), w' \in [\![\psi]\!]_{\mathcal{M}}\} \\
[\![[\psi]^-_A\psi']\!]_{\mathcal{M}} & = & \{w \in W \mid \text{if } w \in [\![\psi]\!]_{\mathcal{M}} \text{ then } w \in [\![\psi']\!]_{\mathcal{M}^-_{(w,\psi,A)}}\} \\
[\![[\psi]^+_A\psi']\!]_{\mathcal{M}} & = & \{w \in W \mid \text{if } w \in [\![\psi]\!]_{\mathcal{M}} \text{ then } w \in [\![\psi']\!]_{\mathcal{M}^+_{(w,\psi,A)}}\}
\end{array}
$$

where **refinements** $\mathcal{M}^-_{(w,\psi,A)} = \langle W^-, \{R^-_a\}_{a\in Ag}, V^-\rangle$ and $\mathcal{M}^+_{(w,\psi,A)} = \langle W^+, \{R^+_a\}_{a\in Ag}, V^+\rangle$ have

- $W^- = W^+ = W$ and $V^- = V^+ = V$
- for every agent $b \notin A$, $R^-_b = R^+_b = R_b$; while for $a \in A$,

$$
R^-_a(v) = \begin{cases} R_a(v) \cap [\![\psi]\!]_{\mathcal{M}} & \text{if } v \in R_a(w) \cap [\![\psi]\!]_{\mathcal{M}} \\ R_a(v) \cap [\![\neg\psi]\!]_{\mathcal{M}} & \text{if } v \in R_a(w) \cap [\![\neg\psi]\!]_{\mathcal{M}} \\ R_a(v) & \text{otherwise} \end{cases}
$$

$$
R^+_a(v) = \begin{cases} R_a(v) \cap [\![\psi]\!]_{\mathcal{M}} & \text{if } v \in R^C_A(w) \cap [\![\psi]\!]_{\mathcal{M}} \\ R_a(v) \cap [\![\neg\psi]\!]_{\mathcal{M}} & \text{if } v \in R^C_A(w) \cap [\![\neg\psi]\!]_{\mathcal{M}} \\ R_a(v) & \text{otherwise} \end{cases}
$$

Remark

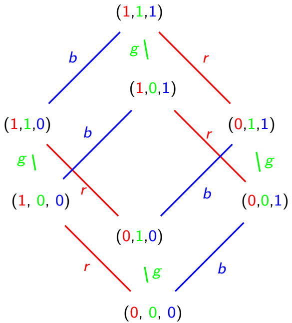
- for every agent $a \in Ag$, R^-_a and R^+_a are equivalence relations
- $[\psi]^+_A$ and $[\psi]^-_A$ are interpreted as local (pointed model) transformers
- the difference between global and local announcements collapse whenever A is a singleton

Example:

Example: the Muddy Children Puzzle

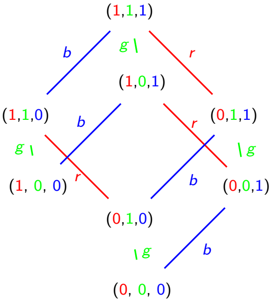
Example: the Muddy Children Puzzle

The model $\mathcal{M}$ for 3 children (red, blue, and green), where no child knows whether she is muddy, can be represented as follows:



Example: the Muddy Children Puzzle

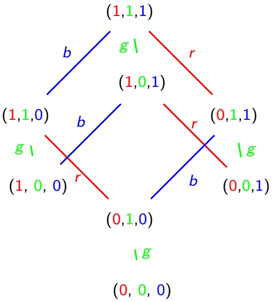
- Suppose that only red is muddy, i.e., the actual world is $(1, 0, 0)$
- then, the father **locally** announces to red and blue that at least one child is muddy: $\alpha := m_r \vee m_b \vee m_g$
- the updated model $\mathcal{M}^-_{(100,\alpha,rb)}$ is as follows:



- only the indistinguishability relation for red is updated
- now red and blue both know that at least one child is muddy: $(\mathcal{M}, 100) \vDash [\alpha]^-_{rb} E_{rb}\alpha$
- the father's announcement does not make α common knowledge: $(\mathcal{M}, 100) \nvDash [\alpha]^-_{rb} C_{rb}\alpha$
- In general, for every world $w \neq 000$, $(\mathcal{M}, w) \nvDash [\alpha]^-_{rb} C_{rb}\alpha$

Example: the Muddy Children Puzzle

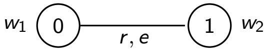
- Suppose that the father **globally** announces to red and blue that at least one child is muddy
- the updated model $\mathcal{M}^+_{(100,\alpha,rb)}$ is as follows:



- now the indistinguishability relations for both red and blue are updated and ...
  ... they acquire common knowledge that at least one child is muddy: $(\mathcal{M}, 100) \vDash [\alpha]^+_{rb} C_{rb}\alpha$
- but the father's announcement is not enough to make α common knowledge *amongst all children*: $(\mathcal{M}, 100) \nvDash [\alpha]^+_{rb} C_{rgb}\alpha$

Example: Communication Scenario

Consider communication between sender s and receiver r over a reliable channel that is listened to by eavesdropper e:



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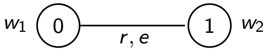
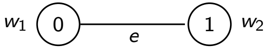


After s has communicated to r the value of the bit, we obtain the updated model $\mathcal{N}_{(w_1, bit=0, r)}$:

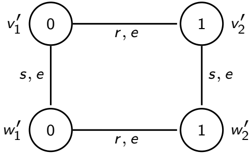


Hence, receiver r learns the value of the bit: $(\mathcal{N}, w_1) \models [bit = 0]_r K_r(bit = 0)$

On the other hand, eavesdropper e learns that r knows it: $(\mathcal{N}, w_1) \models [bit = 0]_r K_e Kw_r(bit = 0)$

Example: Communication Scenario

Compare model $\mathcal{N}$ above with the following **bisimilar** model $\mathcal{N}'$,



Example: Communication Scenario

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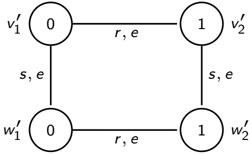
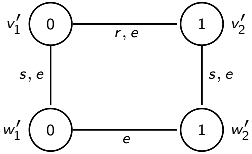


However, after communicating to r the value of the bit, the updated model $\mathcal{N}'_{(w_1', bit=0, r)}$ is not bisimilar to $\mathcal{N}_{(w_1, bit=0, r)}$:



In particular, in w_1' eavesdropper e does not learn that r knows the value of the bit: $(\mathcal{N}', w_1') \not\models [bit = 0]_r K_e Kw_r(bit = 0)$.

$\Rightarrow$ GLAL is not preserved under standard modal bisimulations.

Comparison with PAL

GLAL is at least as expressive as PAL:

Proposition

For all formulas ϕ, ψ in PAL, $(\mathcal{M}, w) \vDash [\phi]\psi$ iff $(\mathcal{M}, w) \vDash [\phi]^{+}_{Ag}\psi$.

By this result we can define a truth-preserving embedding τ from PAL to GLAL.

Proposition

For all formulas ϕ in PAL, $(\mathcal{M}, w) \vDash \phi$ iff $(\mathcal{M}, w) \vDash \tau(\phi)$.

Actually, by the example above,

Theorem

GLAL is strictly more expressive than PAL, and therefore than epistemic logic.

Comparison with Attentive Announcements

- **Attention-based Announcements** [BDH^+16]: agents process the new information only if they are paying attention.
- whether they pay attention is handled by a designated set of atoms.
- close relationship with GLAL: in $(\mathcal{N}', w_1')$ although r processes the new information, agent s is uncertain about this fact.
- consider adding an 'attention atom' h_r for receiver r such that h_r is true in w_1' and w_2' but false in v_1' and v_2'.
- then, announcing $bit = 0$ to r in $(\mathcal{N}', w_1')$ corresponds to the attention-based announcement wherein sender s is uncertain as to whether r is paying attention.

Differences:

- [BDH^+16] models truly private announcements [GG97] (equivalence relations **are not** preserved), whereas our proposal considers semi-private announcements that **do** preserve equivalence relations.
- Our announcements are not necessarily public.

Comparison with Semi-Private Announcements

- **Semi-Private Announcements** [GG97, vD00, vdHP06, BvDM08]: after announcing semi-privately ϕ to coalition A, all agents in A know ϕ, and the agents in $Ag \setminus A$ know that all agents in A know whether ϕ.

- In GLAL agents in $Ag \setminus A$ do not necessarily know that all agents in A know whether ϕ.

- Semi-private announcements can be modeled by refinement $\mathcal{M}^{sp}_{(w,\psi,A)}$ according to which $W^{sp} = W$, $V^{sp} = V$, and for $a \in A$,

$$R^{sp}_a(v) = \begin{cases} R_a(v) \cap [\![\psi]\!]_{\mathcal{M}} & \text{if } v \in R^C_{Ag}(w) \cap [\![\psi]\!]_{\mathcal{M}} \\ R_a(v) \cap [\![\neg\psi]\!]_{\mathcal{M}} & \text{if } v \in R^C_{Ag}(w) \cap [\![\neg\psi]\!]_{\mathcal{M}} \\ R_a(v) & \text{otherwise} \end{cases}$$

- The two frameworks are not directly comparable.

Validities

No complete axiomatisation, but some interesting validities.

- Truthfully announcing a propositional formula $\phi \in \mathcal{L}_{pl}$ entails the knowledge thereof:

$$\vDash [\phi]^-_A E_A \phi$$
$$\vDash [\phi]^+_A C_A \phi$$

- Differently from PAL, announcements in GLAL cannot be rewritten as simpler formulas. Nonetheless, the following are validities in GLAL:

$$\begin{array}{rcl} [\phi]^-_A p & \leftrightarrow & \phi \rightarrow p \\ [\phi]^-_A \neg\psi & \leftrightarrow & \phi \rightarrow \neg[\phi]^-_A \psi \\ [\phi]^-_A (\psi \wedge \psi') & \leftrightarrow & [\phi]^-_A \psi \wedge [\phi]^-_A \psi' \end{array}$$

- Further, epistemic operators and nested announcements commute with announcement operators if they refer to the same coalition (but not in general):

$$\begin{array}{rcl} [\phi]^-_A E_A \psi & \leftrightarrow & \phi \rightarrow E_A [\phi]^-_A \psi \\ [\phi]^-_A [\phi']^-_A \psi & \leftrightarrow & [\phi \wedge [\phi]^-_A \phi']^-_A \psi \\ [\phi]^+_A [\phi']^+_A \psi & \leftrightarrow & [\phi \wedge [\phi]^+_A \phi']^+_A \psi \end{array}$$

- Operators $[\phi]^+_A$ and $[\phi]^-_A$ are "normal" modalities.
  None of schemes T, S4 and B hold.

A New Notion of Bisimulation

We remarked that GLAL is not preserved under modal bisimulation.

- define $R_A(w, v)$ as: $R_a(w, v)$ iff $a \in A$.

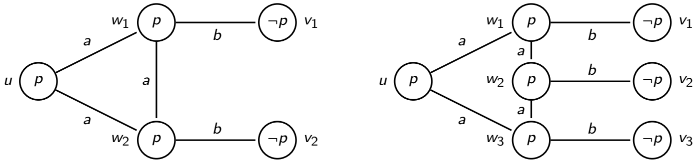
Definition (±-Simulation)

Given models $\mathcal{M}$ and $\mathcal{M}'$, a ±-**simulation** is a relation $\mathbf{S} \subseteq W \times W'$ such that $\mathbf{S}(w, w')$ implies

Atoms $w \in V(p)$ iff $w' \in V'(p)$, for every $p \in AP$

Forth for every $A \subseteq Ag$ and $v \in W$, if $R_A(w, v)$ then for some $v' \in W'$, $R'_A(w', v')$ and $\mathbf{S}(v, v')$

Reach for every $v, v' \in W$, $a \in Ag$, if $\mathbf{S}(v, v')$ then $R_a(w, v)$ iff $R'_a(w', v')$



Theorem

If states s and s' are bisimilar, then for every formula ψ in GLAL, $(\mathcal{M}, s) \models \psi$ iff $(\mathcal{M}', s') \models \psi$.

Model Checking and Satisfiability

Definition (Model Checking and Satisfiability)

- **Model Checking Problem**: given a finite pointed model $(\mathcal{M}, w)$, and formula ϕ in GLAL, determine whether $(\mathcal{M}, w) \models \phi$.
- **Satisfiability Problem**: given a formula ϕ in GLAL, determine whether $(\mathcal{M}, w) \models \phi$ for some pointed model $(\mathcal{M}, w)$.

Theorem

The model checking problem for GLAL is PTIME-complete.

Model refinements can be computed in polynomial time.

Theorem

The satisfiability problem for GLAL is decidable.

Decision procedure inspired by tableaux for epistemic logic.

Conclusions

Contributions:

- GLAL: a logic for global and local announcements
- strictly more expressive than PAL
- alternative to action models to represent private announcements
- however, not preserved under standard modal bisimulation
- but we have a novel, truth-preserving notion of bisimulation
- the model checking problem is no harder than for epistemic logic
- the satisfiability problem is decidable.

Future Work:

- axiomatisation
- closer comparison with DEL
- more elaborate form of communication (asynchronous, FIFO, LIFO, etc.)
- real-life scenarios and applications

Advertising Space: EUMAS 2017

15th European Conference on Multi-agent Systems (EUMAS 2017):

- to be held in Evry (UEVE), December 14-15
- co-located with Agreement Technologies (AT)
- Winter School on AT, December 12-13
- papers published in other conferences are also accepted!
- `https://eumas2017.ibisc.univ-evry.fr/`

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