Tutorial 2: Solutions

Q1. First move the scene to the standard configuration, that is with the viewpoint at the origin. This translates all vertices by 2 units in the z direction. Thus the points are:

[10,0,12]	[10,10,8]	[0,-10,12]

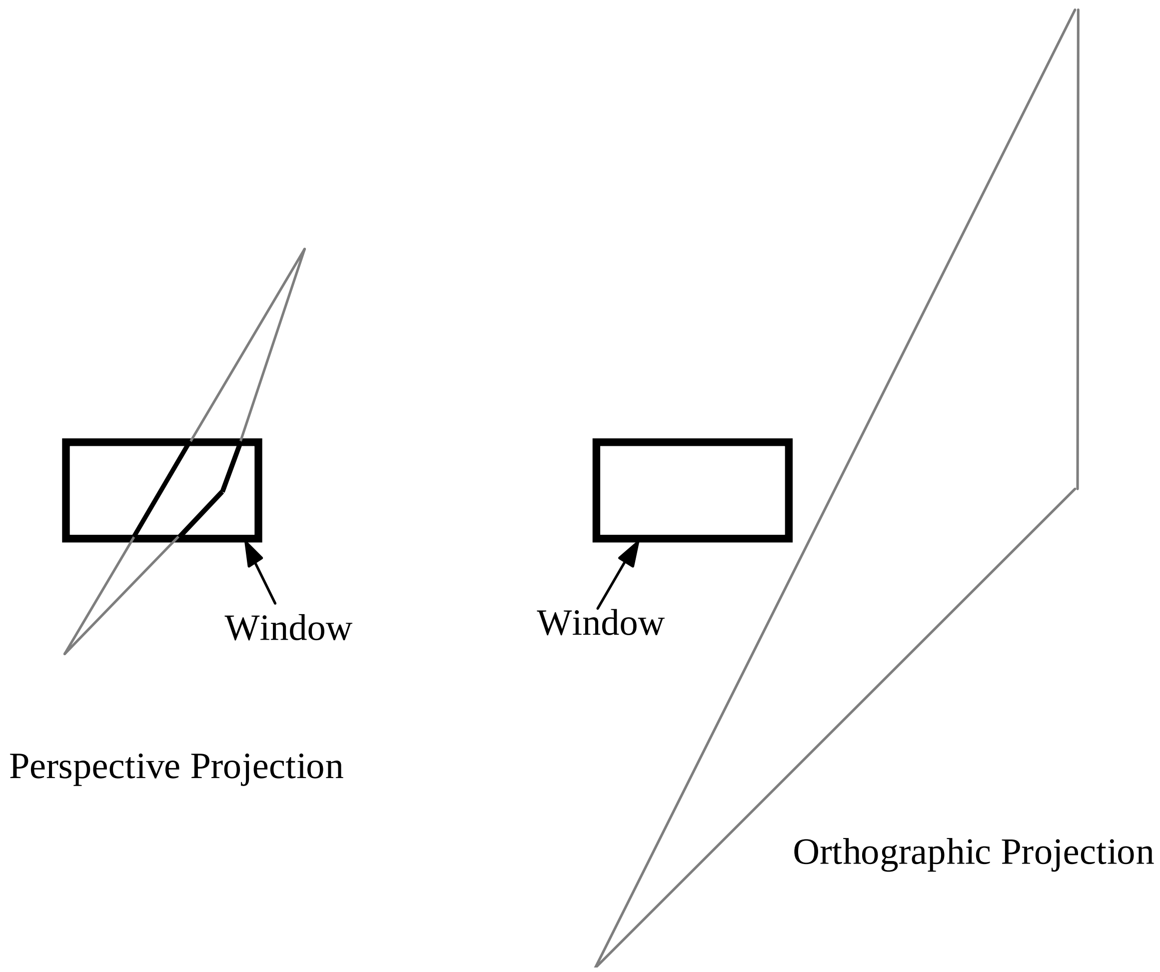
And the plane of projection is z=4

Projecting the points gives:

[10*4/12,0]	[10*4/8,10*4/8]	[0,-10*4/12]
[3.33,0]	[5,5]	[0,-3.33]

The orthographic projections are simply [10,0] [10,10] and [0,-10]

Thus the final screen will show:



Q2. The order of the transformations is Scale Rotate Translate. This may be verified by multiplying out.

b) The rotation is from +ve y to +ve x, ie clockwise. This could be deduced from the derivation, or by learning the rule, or the matrices.

c) The inverse transformation is $T^{-1} R^{-1} S^{-1}$. The inverses of the individual components are

$$T^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -t_x & -t_y & 1 \end{pmatrix} \quad R^{-1} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad S^{-1} = \begin{pmatrix} 1/S_x & 0 & 0 \\ 0 & 1/S_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

which yields inverse:

$$\begin{pmatrix} \cos(\theta)/S_x & \sin(\theta)/S_y & 0 \\ -\sin(\theta)/S_x & \cos(\theta)/S_y & 0 \\ -t_x\mathrm{Cos}(\theta)/S_x+t_y\mathrm{Sin}(\theta)/S_x & -t_x\mathrm{Sin}(\theta)/S_y-t_y\mathrm{Cos}(\theta)/S_y & 1 \end{pmatrix}$$

Q3. The easiest way is to multiply the two matrices out for the two possibilities.

$$\begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} \mathrm{Cos}(t) & -\mathrm{Sin}(t) & 0 \\ \mathrm{Sin}(t) & \mathrm{Cos}(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} Sx\mathrm{Cos}(t) & -Sx\ \mathrm{Sin}(t) & 0 \\ Sy\mathrm{Sin}(t) & Sy\mathrm{Cos}(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{Cos}(t) & -\mathrm{Sin}(t) & 0 \\ \mathrm{Sin}(t) & \mathrm{Cos}(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} Sx\mathrm{Cos}(t) & -Sy\mathrm{Sin}(t) & 0 \\ Sx\mathrm{Sin}(t) & Sy(\mathrm{Cos}(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It can then be seen clearly that the only case for which S*R=R*S is when Sx=Sy.

Q4a.
It is necessary to translate the object to the origin, scale it then translate it back. The combined matrix is:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -10 & -10 & 1 \end{pmatrix} \begin{pmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 10 & 10 & 1 \end{pmatrix}$$

multiplying out we get

$$\begin{pmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

Q4b.

The required rotation is about the z axis, but clockwise.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -10 & -10 & 1 \end{pmatrix} \begin{pmatrix} .98 & -.17 & 0 & 0 \\ .17 & .98 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 10 & 10 & 1 \end{pmatrix}$$

multiplying out we get

$$\begin{pmatrix} .98 & -.17 & 0 & 0 \\ .17 & .98 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1.7 & -9.8 & -10 & 1 \end{pmatrix} \begin{pmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 10 & 10 & 1 \end{pmatrix} = \begin{pmatrix} .98Sx & -.17Sy & 0 & 0 \\ .17Sx & .98Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ -1.7Sx & -9.8Sy+10 & -10Sz+10 & 1 \end{pmatrix}$$