Tutorial 3: Solutions

Q1. Let **P** = (1,2,3) and use

Px′ = (**P**-**C**).***u*** = **P**.***u*** – **C**.***u***
Py′ = (**P**-**C**).***v*** = **P**.***v*** – **C**.***v***
Pz′ = (**P**-**C**).***w*** = **P**.***w*** – **C**.***w***

then:

Px' = (1-C)ux + (2-C)uy + (3-C)uz
Py' = (1-C)vx + (2-C)vy + (3-C)vz
Pz' = (1-C)wx + (2-C)wy + (3-C)wv

Q2a.
Use the cross product to get ***u*** = ***v*** x ***w***.
(Note that, for our normal axis system it is ***v*** x ***w*** and not ***w*** x ***v***. Get this wrong and you swap left and right!)

The cross product is evaluated by the determinant method

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1 & 0 \\ 0.6 & 0 & -0.8 \end{vmatrix}$$

u = [-0.8, 0, -0.6]

Q2b. The translation parts of the matrix are:
-C.u = 11
-C.v = -5
-C.w = -2

And the whole transformation matrix is:

$$\begin{matrix} -0.8 & 0 & 0.6 & 0 \\ 0 & 1 & 0 & 0 \\ -0.6 & 0 & -0.8 & 0 \\ 11 & -5 & -2 & 1 \end{matrix}$$

Q2c.
The projection matrix is:

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 \end{matrix}$$

To get the overall transformation we just multiply the matrices together to get:

$$\begin{matrix} -0.8 & 0 & 0.6 & 0.3 \\ 0 & 1 & 0 & 0 \\ -0.6 & 0 & -0.8 & -0.4 \\ 11 & -5 & -2 & -1 \end{matrix}$$

Q2d.
Multiplying homogenous coordinate [10,1,0,1] by the matrix we get [3,-4,4,2] and normalising this to Cartesian form we get [1.5,-2,2,1]. Thus the two dimensional coordinate is [1.5,-2].

Q3. Let $\mathbf{C} = [10,15,5]$. We then obtain unit vectors $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$ collinear to the given directions by diving each component by the magnitude of the corresponding vector.:

$$\boldsymbol{w} = \left(-\frac{6}{10}, 0, \frac{8}{10}\right) = \left(-\frac{3}{5}, 0, \frac{4}{5}\right)$$

$$\boldsymbol{u} = \left(\frac{p_x}{|\boldsymbol{u}|}, 0, \frac{p_z}{|\boldsymbol{u}|}\right)$$

$$\boldsymbol{v} = \left(\frac{q_x}{|\boldsymbol{v}|}, \frac{1}{|\boldsymbol{v}|}, \frac{q_z}{|\boldsymbol{v}|}\right)$$

Substituting the above values into the general transformation matrix we obtain:

$$\begin{bmatrix} p_x/|\boldsymbol{u}| & q_x/|\boldsymbol{v}| & -3/5 & 0 \\ 0 & 1/|\boldsymbol{v}| & 0 & 0 \\ p_z/|\boldsymbol{u}| & q_z/|\boldsymbol{v}| & 4/5 & 0 \\ -10 * p_x/|\boldsymbol{u}| - 5 * p_z/|\boldsymbol{u}| & -10 * q_x/|\boldsymbol{v}| - 15/|\boldsymbol{v}| - 5 * q_z/|\boldsymbol{v}| & 2 & 1 \end{bmatrix}$$