Tutorial 4: Solutions

1a.
We could take (**P2**-**P1**) = [5,-10,5], or simply [1,-2,1] and (**P3**-**P1**) = [15,0,5] or just [3,0,1], and take the cross product:

i	**j**	**k**		
1	-2	1	=	[-2,2,6]
3	0	1		

So a simple normal vector is [-1,1,3].

1b.
Take the vector from say **P1** to **P4**, ie **P4**-**P1** = [20,0,5], and take the dot product of this with the normal [-1,1,3], which gives -5. Since the result is negative the angle between these two vectors is bigger than 90, and so the normal vector must be the outward surface normal. The inner surface normal is therefore [1,-1,-3]

2a.
First we project the points onto the plane of projection. Since this is at z=1 the projected points are simply [x/z,y/z] that is [2,4] [1,2] [2,2] and [1,1]. As all the vertices are inside the window, the projected tetrahedron must lie completely within the window.

2b.
For each face parallel to the cartesian axes, we can find the inner surface normal by inspection.

[10,20,5] [5,10,5] [10,10,5]	(z = 5) inner surface normal is [0,0,1]
[10,20,5] [10,10,5] [10,10,10]	(x = 10) inner surface normal is [-1,0,0]
[5,10,5] [10,10,5] [10,10,10]	(y = 10) inner surface normal is [0,1,0]

for the fourth face we need to evaluate the cross product of two edge vectors.
[10,20,5] [5,10,5] [10,10,10]
two edge vectors are [5,10,0] and [5,0,5] (NB any of the three edge vectors in either sense could be used)

the cross product is

i	**j**	**k**
5	10	0
5	0	5

that is [50,-25,-50] which simplifies to [2,-1,-2]
by inspection (consider **n** = [10,-5,-10]) we see that this is the inner surface normal (alternatively we could test it using the fourth vertex).

For the point under test , we see by inspection that it is on the inside of the three faces parallel to the axes, hence we only need test the fourth normal.

P-**V** = [8,15,8] - [5,10,5]= [3,5,3]   (any of the vertices could be used here)
(**P**-**V**).**n** = [3,5,3].[2,-1,-2] = 6 - 5 - 6 = -5
Since the dot product is negative, the point is outside the volume.

3a. Obtain two vectors on the plane:

(1,0,-1) – (0,-1,1) = (1,-1,-2)
(1,0,-1) – (1,1,0) = (0,-1,-1)

Then find the normal vector by taking the cross product:

i	**j**	**k**		
1	-1	-2	=	[-3,1,-1]
0	-1	-1		

Thus the equation is $-3x + y - z + d = 0$

Substitute any vertex to obtain d = 2. The final equation of the plane is $f(x,y,z) = -3x + y - z + 2$

3b.
At the viewpoint sign(f(0,0,0)) > 0 and at the internal point sign(f(5,-1,5)) < 0. This means that the face is between the viewpoint and the internal point and is thus VISIBLE!

4.
First find the 2D projections of the points:

(5,5,5) projects to (2,2)
(20,10,10) projects to (4,2)

The mid point of the projected line is (3,2)

Find the corresponding point in 3D by back projection.

The projector in 3D that goes through the mid point and the origin is given by $P = \nu*(3,2,2)$
The line in space is given by $P = \mu*(5,5,5) + (1 - \mu)*(20,10,10)$

Find the intersection by letting $\nu*(3,2,2) = \mu*(5,5,5) + (1 - \mu)*(20,10,10)$ and solving for μ and ν to obtain $\mu = 2/3$, $\nu = 10/3$.

Obtain the point in 3D using the val μ or ν in either the projector or line equations.

P = (10, 20/3, 20/3)