Decision Analysis

Tutorial Problem 3

An “ice cream man” finds that the demand for his ice cream depends on the weather, which is very variable, but can be either “moderate” or “warm”, with equal frequency.

On warm days the total demand is for 2, 3 or 4 “freezersful”, with corresponding probabilities 0.2, 0.4 and 0.4, respectively.

On moderate days the total demand is for 2, 3 or 4 freezersful, with corresponding probabilities 0.4, 0.4 and 0.2, respectively.

Each freezerful sold yields a profit of 2. Unsold freezersful of ice cream must be dumped, and involve a loss 1.

(i) Based on the above information, how many freezersful of ice cream should the man prepare, for sale each day?

The local radio station broadcasts a weather forecast, which has a record of predicting moderate or warm weather with 80% reliability.

(ii) Should the ice cream man take note of the forecast, and, if so, what should be his strategy?

Decision Analysis

Solution to Tutorial Problem 3

(i) If the ice cream man produces at or below the level of demand then he will sell all that he produces. There will be no wastage and, of course, no loss incurred in dumping excess product. In general, the net profit (taking into account loss on dumping excess ice cream) is given by:

net profit = 2*production level if production level ≤ demand

2*demand - 1*(production level - demand) if production level > demand

Consequently, the net profit matrix is:

Net Profit	demand 2	demand 3	demand 4
Produce 2	4	4	4
Produce 3	3	6	6
Produce 4	2	5	8

Taking into account the probability distributions of demand for warm and moderate weather, we can calculate the expected net profit for different actions and different weather conditions, as well as the overall expected net profit for different actions, given that the two weather conditions are equally likely:

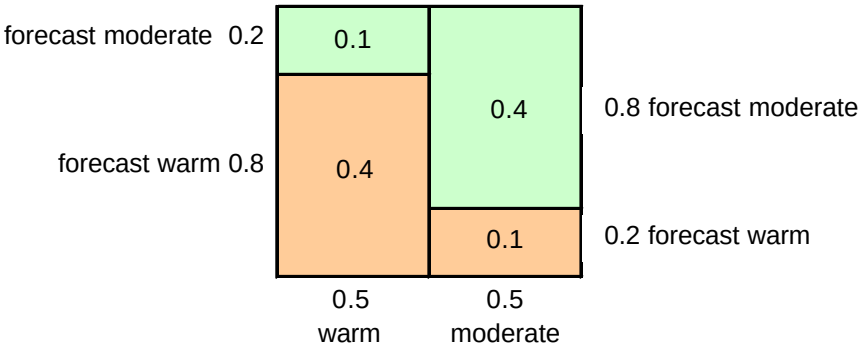
Expected Net Profit	Warm Day	Moderate Day	Overall
Produce 2	0.2*4 + 0.4*4 + 0.4*4 **4.0**	0.4*4 + 0.4*4 + 0.2*4 **4.0**	0.5*4.0 + 0.5*4.0 **4.0**
Produce 3	0.2*3 + 0.4*6 + 0.4*6 **5.4**	0.4*3 + 0.4*6 + 0.2*6 **4.8**	0.5*5.4 + 0.5*4.8 **5.1**
Produce 4	0.2*2 + 0.4*5 + 0.4*8 **5.6**	0.4*2 + 0.4*5 + 0.2*8 **4.4**	0.5*5.6 + 0.5*4.4 **5.0**

It is clear that producing three freezersfull leads to the greatest expected profits.

(ii) 80% reliable information means that there is an 80% chance that the forecast will be correct, given the underlying situation. (In the language of medical testing, both the sensitivity and specificity are 80%).

So: Pr(forecast warm | warm) = 0.8 and Pr(forecast moderate | moderate) = 0.8 .

Using the graphical form of Bayes' Theorem:

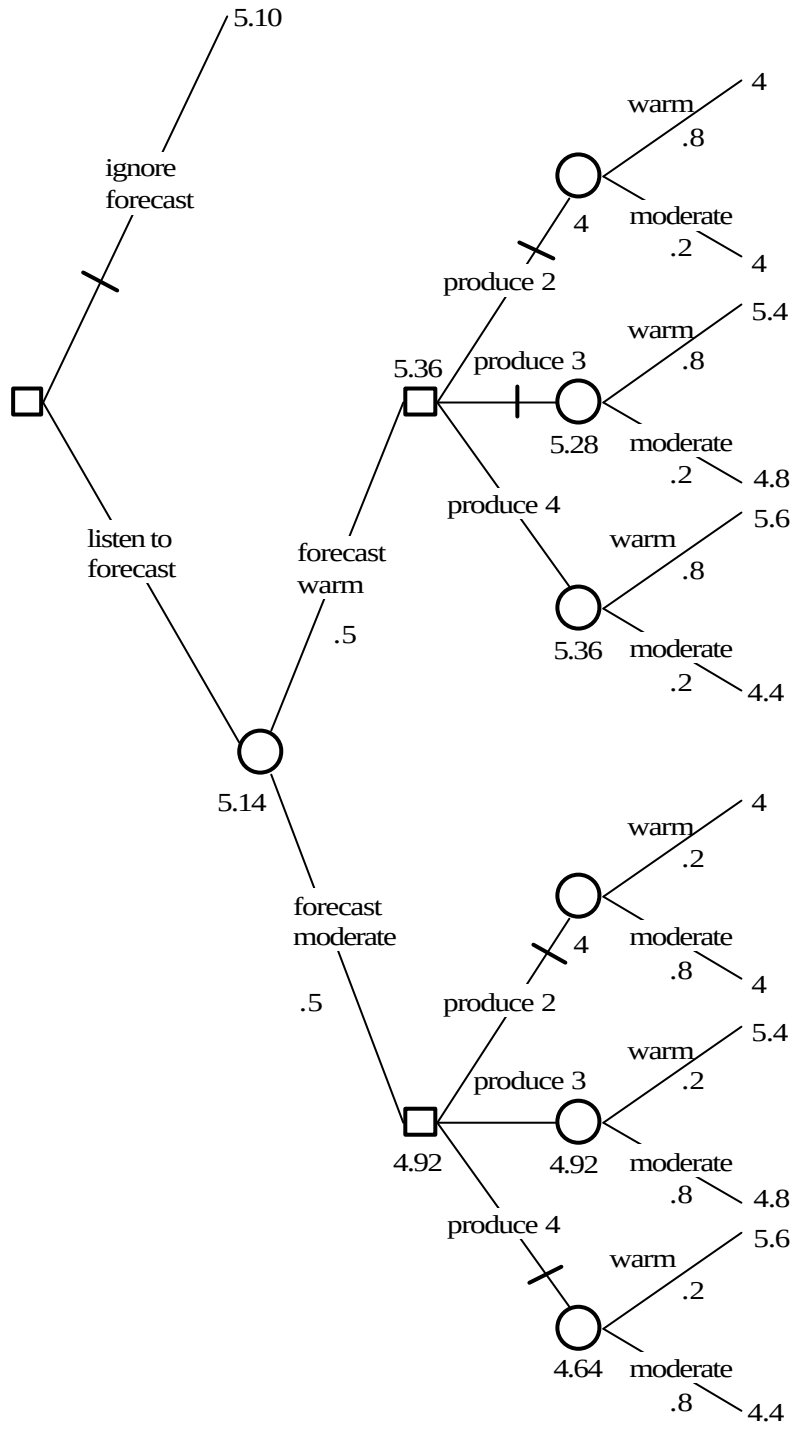


we can deduce that: Pr(forecast warm) = 0.4 + 0.1 = 0.5 and Pr(forecast moderate) = 0.5

Pr(warm | forecast warm) = 0.4 / 0.5 = 0.8, Pr(moderate | forecast warm) = 0.1/0.5 = 0.2

Pr(warm | forecast moderate) = 0.1 / 0.5 = 0.2, Pr(moderate | forecast moderate) = 0.4/0.5 = 0.8

Using these figures and the conclusion of part (i), we can produce the following decision tree:



The expected payoff from listening to the forecast is 5.14, which is marginally better than the best (5.10) that we can achieve otherwise. The decision tree and EMV tells us to listen to the forecast, and to produce three freezersfull if the forecast is moderate but four if the forecast is warm.

Perhaps since the EMVs of the three strategies:

- produce three freezersfull (5.1)
- produce four freezersfull (5.0)
- listen to forecast and produce three freezersfull if the forecast is moderate but four if the forecast is warm (5.14)

are so similar we should compare risk profiles before coming to any firm conclusion.