Decision Analysis

Tutorial Problem 8

A railway company is planning to purchase a new locomotive for one of its busiest lines, and one of two types of vehicle, the Arrow or the Bullet, is to be purchased. The prices of the two locomotives are very similar, so the choice is to be based on two factors: running costs and reliability. It is agreed that these two factors can be represented by the variables: *average weekly operating costs* and *number of breakdowns in the first three month of operation*. The company's operations manager estimates that the following probability distributions apply to the two locomotives. It can be assumed that the probability distributions for operating costs and number of breakdowns are independent

Arrow

Average weekly operating costs (£)	Prob.
20,000	0.6
30,000	0.4

No. of breakdowns	Prob.
0	0.15
1	0.85

Bullet

Average weekly operating costs (£)	Prob.
15,000	0.5
35,000	0.5

No. of breakdowns	Prob.
0	0.2
1	0.7
2	0.1

Details of the manager's utility functions for operating costs and numbers of breakdowns are shown below:

Average weekly operating costs (£)	Utility
15,000	1.0
20,000	0.8
30,000	0.3
35,000	0

No. of breakdowns	Utility
0	1.0
1	0.9
2	0

a) The operations manager's responses to questions reveal that, for him, the two attributes are mutually utility independent. Explain what this means.

b) The production manager also indicates that for him $k_1 = 0.7$ (where attribute 1 = operating costs) and $k_2 = 0.5$. Discuss how these values could have been determined.

c) Which locomotive has the highest expected utility for the operations manager?

Decision Analysis

Solution to Tutorial Problem 7

Parts a and b are bookwork, dealt with in detail in the course notes entitled "Risk and Multi-Attribute Utilities".

c. It follows from the information provided in parts a and b that the following multiplicative model can be used to combine utility values for the two individual attributes:

$U_{joint}(c, b)$

$= 0.7*U_{cost}(c) + 0.5*U_{breakdowns}(b) + (1 – 0.7 - 0.5)* U_{cost}(c)*U_{breakdowns}(b)$

$= 0.7*U_{cost}(c) + 0.5*U_{breakdowns}(b) – 0.2 * U_{cost}(c)*U_{breakdowns}(b)$

For example:

$U_{joint}(20K\ cost, 0\ breakdowns)$

$= 0.7* U_{cost}(20K) + 0.5*U_{breakdowns}(0) – 0.2* U_{cost}(20K)*U_{breakdowns}(0)$

$= 0.7 * 0.8 + 0.5 * 1.0 – 0.2 * 0.8 * 1.0$

$= 0.56 + 0.5 – 0.16$

$= 0.9$

Since the distributions of cost and breakdowns are independent we can lay out a decision tree for the problem as follows, and can see that Arrow has the higher expected joint utility of 0.7677: