Decisions involving Multiple Objectives

In many decision problems each alternative has several attributes associated with it.

Even when risk and uncertainty are not significant, it may not be a trivial task to choose the “best” alternative.

The performance of each alternative has to be measured on each attribute, and then the attributes themselves have to be weighed against each other before a decision can be made.

The central idea is that, by splitting the problem into small parts and focusing on each part separately, the decision maker is likely to achieve a better understanding of his problem than he would by taking a holistic view.

An ***attribute*** is used to measure the performance of courses of action in relation to the objectives of the decision maker.

For example, if the objective was “maximise the exposure of a television advertisement”, the attribute “number of people surveyed who recall seeing the advertisement” could be used to measure the degree to which the objective was achieved.

Sometimes a “***proxy attribute***”, not directly related to the objective, is used.

For example, a company may use the proxy attribute “staff turnover” to measure how well they are achieving their objective of maximising job satisfaction for their staff.

Edwards' Simple Multi-Attribute Rating Technique (SMART)

(for problems that do NOT involve risk)

1. Identify the decision maker(s).
2. Identify the alternative courses of action.
3. Identify the attributes relevant to the decision problem.
4. For each attribute, assign values to measure the performance of the alternatives on that attribute.
5. Determine a weight for each attribute.
6. For each alternative, take the weighted average of the values assigned to that alternative.
7. Make a provisional decision.
8. Perform sensitivity analysis to see how robust the decision is to changes in the figures supplied by the decision maker.

This technique requires only simple responses from the decision maker.

The analysis of these responses is also simple.

The analysis is transparent, so the method is likely to yield an enhanced understanding of the problem.

The method can be applied pretty speedily.

However, the method may not always capture the full detail and complexities of the real problem.

Example: Office Location Problem.

A small printing and photocopying business must move from its existing office because the site has been acquired for redevelopment. The owner of the business is considering seven possible new offices, all of which would be rented:

Location	Annual Rent
Addison Square (A)	£30,000
Bilton Village (B)	£15,000
Carlisle Walk (C)	£5,000
Denver Street (D)	£12,000
Elton Street (E)	£30,000
Filton Village (F)	£15,000
Gorton Square (G)	£10,000

While the owner would like to keep costs as low as possible, he would also like to take other factors into account.

SMART stage 1: The decision maker is the business owner.

SMART stage 2: The alternatives are the different offices the owner can choose.

SMART stage 3: The next step is to identify the **attributes** which the decision maker considers relevant to his problem.

Constructing a Value Tree:

We need to arrive at a set of attributes which can be assessed on a numeric scale.

The initial attributes may be vague and may need to be broken down into more specific attributes before measurement can take place.

A value tree can be useful here.

We start constructing a tree by addressing the attributes which represent the general concerns of the decision maker.

Initially the owner identifies two main attributes which he calls “costs” and “benefits”.

Note that there is no restriction on the number of initially specified attributes.

They might have been “short-term costs”, “long-term costs”, “convenience of the move” and “ benefits”.

Neither need they be categorised as costs and benefits.

In some applications “risk of option” may be an attribute.

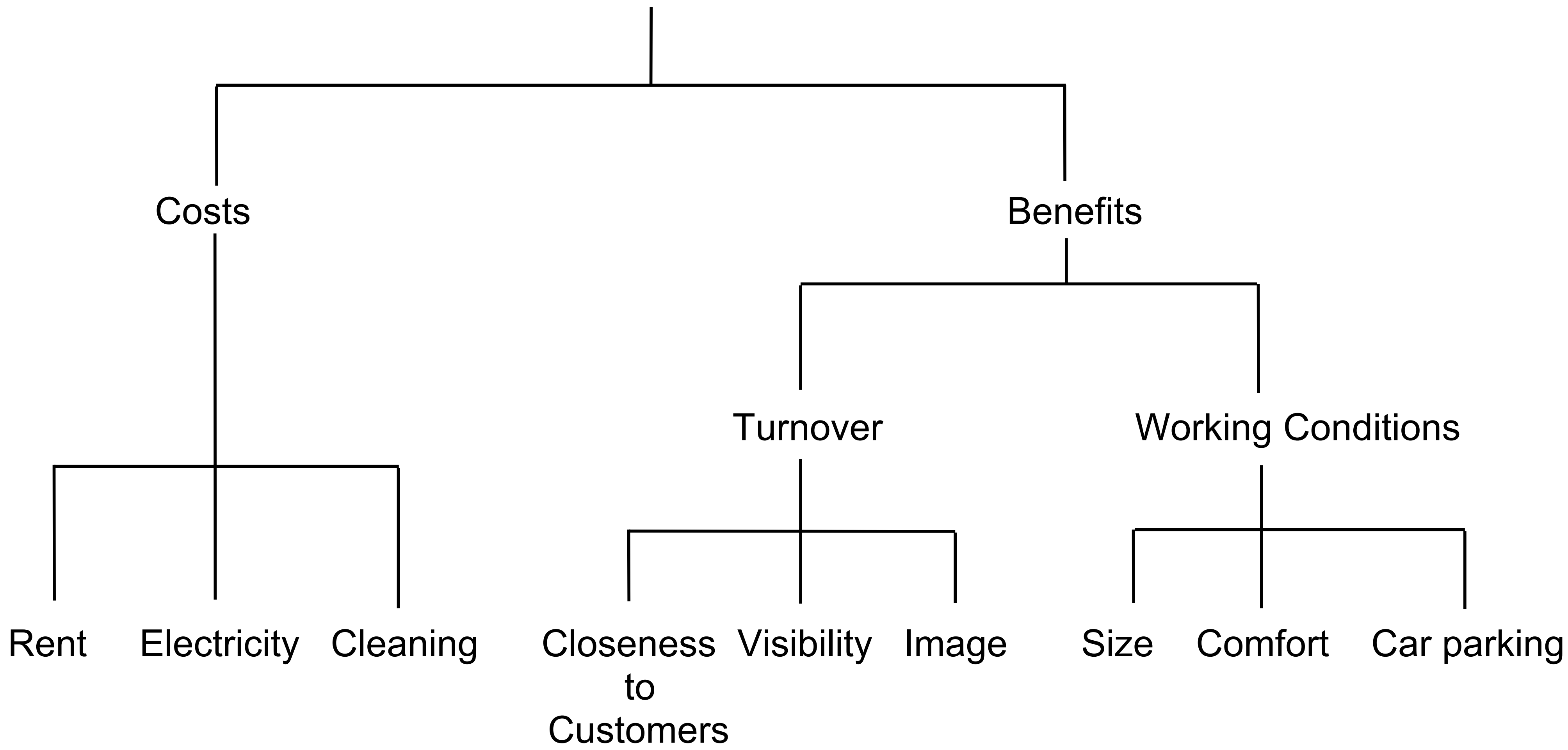
Having established the main attributes, they need to be decomposed to a level where they can be assessed.

The owner identifies three main costs that are of concern to him: “annual rent”, “cost of electricity” and “cost of regular cleaning”.

Similarly, he decides that benefits can be subdivided into “potential for improved turnover” and “staff working conditions”.

To assess potential for improving turnover he will have to consider “the closeness of the office to potential customers”, “the visibility of the site” (business may be generated from people who see the office while passing by) and “the image of the location” (a decaying building in a back street may convey a poor image and lead to a loss of image).

The owner feels that he will be better able to compare the working conditions of the offices if he decomposes this attribute into “size”, “comfort” and “car parking facilities”:



It is sometimes necessary to go through several iterations before we arrive at a tree that is an accurate and useful representation of the decision maker's concerns.

A tree should be judged by:

Completeness

Have all the attributes of concern to the decision maker been included?

Operationality

Are all the lowest-level attributes specific enough for the decision maker to evaluate and compare them for different options?

Decomposability

Can the performance of an option on one attribute be judged independently of it performance on other attributes?

Absence of Redundancy

Do two attributes represent the same thing?

Minimum Size

Don't decompose too far; eliminate attributes that don't distinguish between options.

SMART stage 4: The next step is to find how well the different office locations perform on each of the lowest-level attributes in the value tree.

Measuring how well the Options Perform on each Attribute: Determining the annual costs of operating the offices is relatively straightforward:

Office Location	Rent (p.a.)	Cleaning (p.a.)	Electricity (p.a.)	TotalCosts (p.a.)
Addison Square (A)	£30,000	£3,000	£2,000	£35,000
Bilton Village (B)	£15,000	£2,000	£800	£17,800
Carlisle Walk (C)	£5,000	£1,000	£700	£6,700
Denver Street (D)	£12,000	£1,000	£1,100	£14,100
Elton Street (E)	£30,000	£2,500	£2,300	£34,800
Filton Village (F)	£15,000	£1,000	£2,600	£18,600
Gorton Square (G)	£10,000	£1,100	£900	£12,000

In measuring the benefit attributes, the task will be made easier if we can identify variables to represent the attributes.

For example, the “size” of an office can be represented by its floor area in square feet.

Similarly, the distance of the office from the town centre may provide a suitable approximation for “distance from potential customers”.

However, it will be more difficult to find variables for “image” and “comfort”.

For attributes difficult to quantify we use ***direct rating***. For attributes that can be represented by easily quantifiable variables we use ***value functions***.

Direct Rating: Consider “image”.

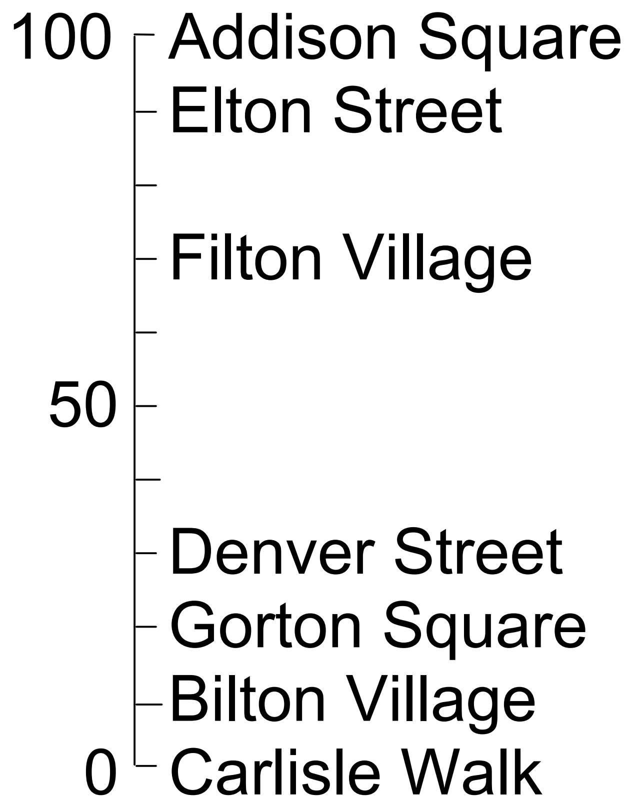
The owner is asked to rank the locations in terms of their image, from the most preferred to the least preferred.

His rankings are:

1. Addison Square
2. Elton Street
3. Filton Village
4. Denver Street
5. Gorton Square
6. Bilton Village
7. Carlisle Walk

We assign Addison Square a value 100 for image, and Carlisle Walk a value 0. (We could have used values 1 and 0 instead).

The owner is now asked to rate the other locations for image, in such a way that the space between the values he gives to the offices represents his strength of preference for one office over another, in terms of image.



The improvement in image from Carlisle Walk to Gorton Square is perceived by the owner to be twice as preferable as the improvement in image from Carlisle Walk to Bilton Village.

Note that it is the interval (or improvement) between the points in the scale which is being compared.

We can’t say that Gorton Square’s image is twice as good as that of Bilton Village. This interval scale is similar to a temperature scale - water at 80^{o} C (175^{o} F) is not twice as hot as water at 40^{o} C (104^{o} F).

Although it may be difficult for the decision maker to position the various options on the scale precisely, a high level of precision is not needed as the ultimate action is generally fairly robust , and it usually requires substantial changes in figures supplied before a different action is preferred.

The direct rating method can be repeated for the other less easily quantified attributes “comfort”, “visibility” and “car parking facilities”.

Value Functions: We first measure the owner's relative strength of preference for offices of different sizes. The floor areas are:

Location	Floor Area (ft^2)
Addison Square (A)	1,000
Bilton Village (B)	550
Carlisle Walk (C)	400
Denver Street (D)	800
Elton Street (E)	1,500
Filton Village (F)	400
Gorton Square (G)	700

An increase in area from 500 ft^2 to 1,000 ft^2 is very attractive as this would considerably improve working conditions.

However, the improvements to be gained through an increase from 1,000 ft^2 to 1,500 ft^2 might be marginal.

Because of this, we need to translate the floor areas into values.

The owner judges that the larger the office, the more attractive it is.

(Note that in the case of some attributes the "ideal point" may be intermediate between the lowest and highest measurements, e.g. age of applicants for a job).

The largest office, Elton Street, has an area of 1,500 ft^2, so we give 1,500 ft^2 a value of 100.

Similarly, the smallest offices, Carlisle Walk and Filton Village, both have an area of 400 ft^2, to which we attach a value of 0.

We could ask the owner to rate the areas of the other offices directly, but because some of the areas involve rather awkward numbers, we derive a value function instead.

This will enable us to estimate the value of any office area between the least and most preferred.

We shall employ the ***bisection method*** to elicit a value function for area.

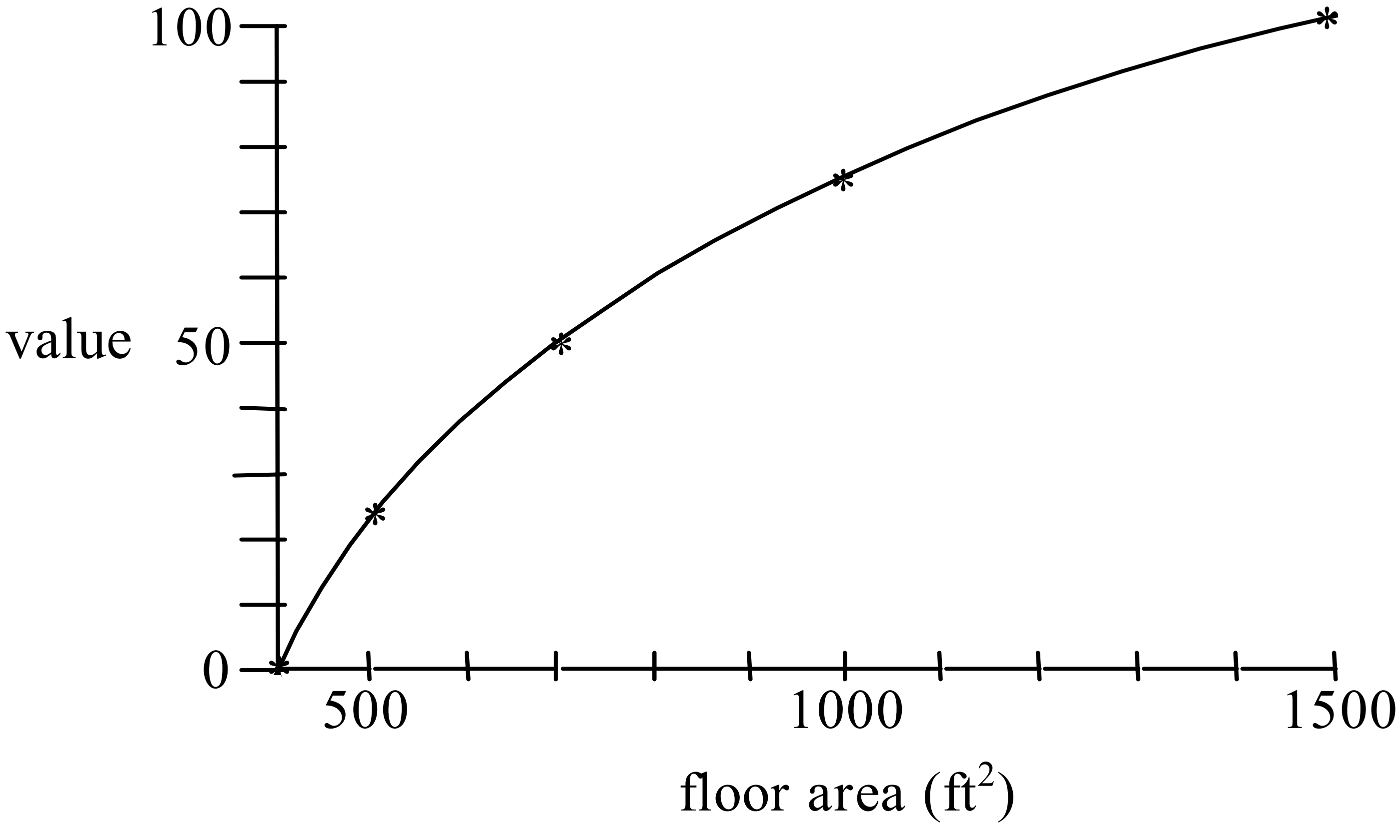
The owner is asked to identify an office area that is halfway between the least-preferred area (400 ft^2) and the most-preferred area (1,500 ft^2). The area does not have to correspond to that of one of the offices under consideration.

After much thought, the owner agrees that an office area of 700 ft^2 would have the midpoint value 50.

The decision maker is now asked to identify the “quarter points”.

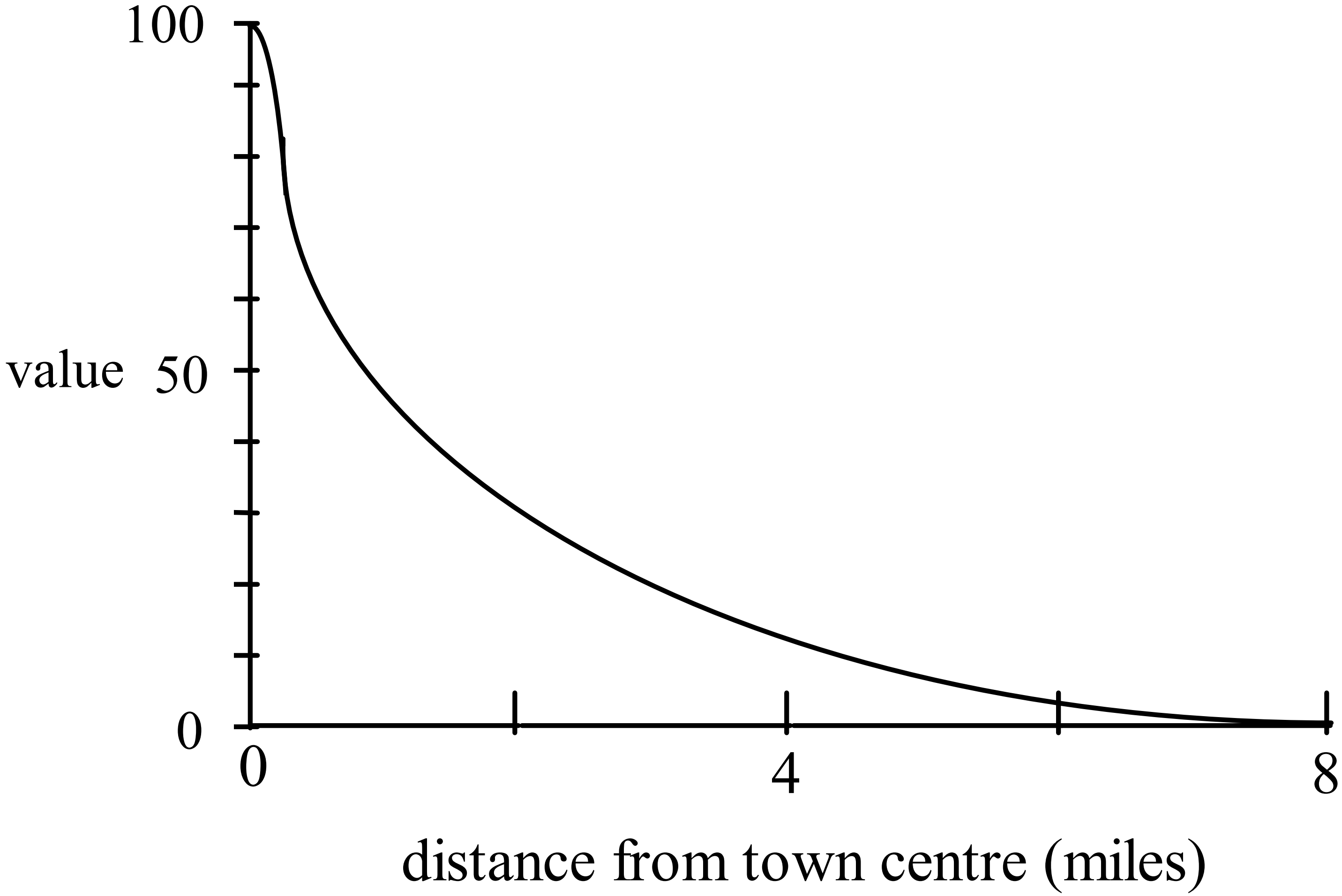
He decides that an office area of (500 ft^2) would be halfway between the least-preferred area (400 ft^2) and the midpoint area (700 ft^2), and so has a value 25. Similarly, he thinks an area of (1,000 ft^2) comes midway in preference between the midpoint area (700 ft^2) and the best area (1,500 ft^2), thus earning a value of 75.

With five points on the value function for area, we can sketch the curve.



From this curve, we can estimate the value of the area of the Bilton Village office (550 ft^2) to be about 30, for example.

A similar method can be applied to the attribute “closeness to customers”, represented by the variable “distance from town centre”.



SMART stage 5: **Determining the Weights of the Attributes.**

In order to make a decision, the owner has to combine the value for the different attributes, so as to gain a view of the overall benefits each office location has to offer.

Note that, at first sight, all that would be needed would be a weight for each attribute that reflected the ***importance*** the decision maker gives to that attribute.

However, that would ignore how wide the range was between the most-preferred option and least-preferred option, for each attribute.

Suppose that, in general, the further out a location was - the greater its floor area.

The owner believed that the locations varied in distance between 2 and 8 miles from their nearest town centre and gave the attribute “closeness from customers” three times the weight of “size”.

Now suppose that he discovers that, in fact, they all lie between 3.2 and 3.8 miles from their nearest town centre.

Almost certainly, the owner should change his weights, for otherwise this would imply that a mere reduction of 0.6 miles in the distance from their town centre would still be three times more important in the choice of office location than an increase in floor area from 400 ft^2 to 1,500 ft^2.

If importance weights are used, they should be adjusted so that the smaller the range over which the attribute is assessed, the smaller the importance weight that should be used. However, we avoid the difficulties with importance weights by using swing weights.

Swing weights are derived by asking the decision maker to compare a change (or swing) from the least-preferred to the most-preferred value on one attribute to a similar change in another attribute.

Consider the lowest-level attributes on the “benefits” branch of the “value tree”.

The owner is asked to imagine a hypothetical office with all these attributes at their least preferred levels.

In other words, an office at the greatest distance (8 miles) from the town centre, with the worst position for visibility, the worst image, smallest size, etc.

He is then asked, if just one of these attributes could be moved to its best level, which would he choose?

The owner selects “closeness to customers”.

After this change is made, he is asked which attribute he would next choose to move to its best level, and so on until all the attributes have been ranked:

1. Closeness to Customers
2. Visibility
3. Image
4. Size
5. Comfort
6. Car-Parking facilities

We can now give “closeness to customers” a weight of 100.

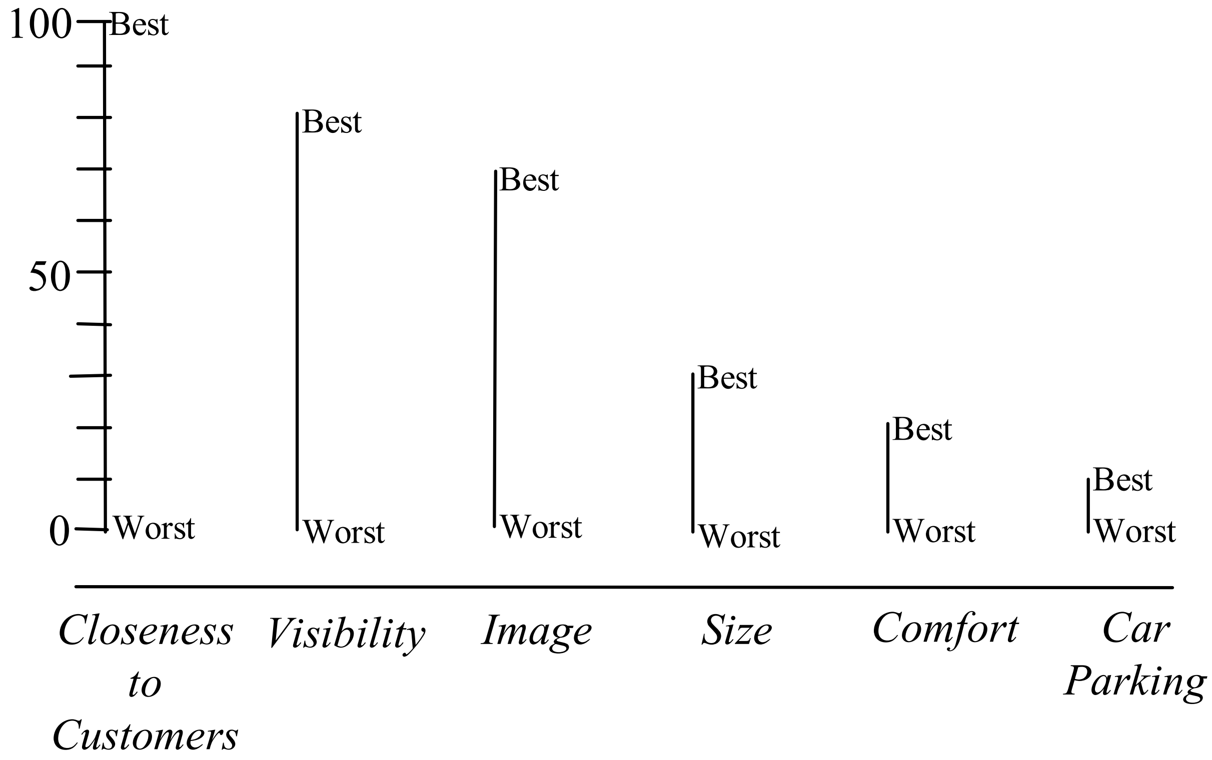
The owner is then asked to compare a swing from the least visible location to the most visible, with a swing from the most distant location to the closest one.

After some thought, he decides that a swing in "visibility" is 80% as important as the swing in "closeness to customers".

So "visibility" is given a weight of 80.

Similarly, a swing from the worst "image" to the best is considered 70% as important as a swing from the worst to best location for "closeness to customers", and so image is assigned a weight of 70.

The process is repeated for the other low-level attributes.



The six weights obtained sum to 310, and it is conventional to **normalise** them so that they add up to 100.

Normalisation is achieved simply by dividing each weight by the sum of the weights (310) and multiplying by 100:

Attribute	**Original Weights**	**Normalised Weights**
Closeness to Customers	100	32
Visibility	80	26
Image	70	23
Size	30	10
Comfort	20	6
Car-Parking facilities.	10	3
	----	----
	310	100

The overall weight for the higher-level attribute "turnover" is : 32 + 26 + 23 = 81.
The overall weight for the higher-level attribute "working conditions" is : 10 + 6 + 3 = 19.

SMART stage 6: **Aggregating the Benefits.**

We now find out how each office location performs overall by combining the six value scores allocated to that location.

The simplest way to do this is by multiplying each value by the weight attached to that attribute, summing over the six attributes and finally dividing by 100.

This ***additive model*** is only appropriate when there is no interaction between the attribute values. In technical terms, we need to assume "***mutual preference independence***" between the attributes.

Suppose, for a moment, that the office location problem only involves two attributes "distance from customers" and "office size".

Suppose that the owner is offered two offices X and Y, both 1,000 ft^2 in size, but with X three miles from town centre and Y five miles from town centre. The owner prefers X to Y.

Now suppose that the size of the offices is changed to 400 ft^2. The owner still prefers X to Y.

If his preference remains true if we change the size of both offices to any other floor area, we can say that “distance from customers” is preference independent of “office size”.

If we also find that “office size” is preference independent of “distance from customers”, then these two attributes would be ***mutually preference independent***.

Notice that preference independence is not symmetric.

When choosing a holiday destination, you may prefer a warm climate to a cooler one, irrespective of whether or not the hotel has an indoor or open-air pool.

However, your preference between type of pool will probably depend on the whether the local climate was warm or cool.

The absence of mutual preference independence can often be identified when the decision maker responds to questions with “This depends on ...”. For example, asked whether you prefer an indoor or open-air pool you might respond with “This depends on the climate.”

If mutual preference independence does not exist, it is usually possible to return to the value tree and redefine the attributes so that a set of attributes which are mutually preference independent can be identified.

In the occasional problems where this is not possible, other models are available which can handle the interaction between attributes.

However, such models are more complex and difficult to administer, and are therefore not used widely.

The additive model calculations for Addison Square are:

Attribute	**Addison Square Values**	**Weights**	**Value x Weight**
Closeness to Customers	100	32	3200
Visibility	60	26	1560
Image	100	23	2300
Size	75	10	750
Comfort	0	6	0
Car-Parking facilities	90	3	270
		----	------
		100	8080

Therefore the aggregate value for Addison Square is:

8080/100 = 80.8

The benefit values obtained for all the locations and their aggregate values are:

Attribute	Weight	Office						
		A	B	C	D	E	F	G
Closeness	32	100	20	80	70	40	0	60
Visibility	26	60	80	70	50	60	0	100
Image	23	100	10	0	30	90	70	20
Size	10	75	30	0	55	100	0	50
Comfort	6	0	100	10	30	60	80	50
Car Parking	3	90	30	100	90	70	0	80
Aggregate Benefits		80.8	39.4	47.4	52.3	64.8	20.9	60.2

It can be seen that Addison Square has the highest value for benefits and Filton Village the lowest.

SMART stage 7: To make a provisional decision we should look at **Trading Benefits against Costs.**

The judgement involved in trading off costs against benefits can be extremely difficult and is often considered the least secure and most uncomfortable to make of all judgements involving multiple objectives.

Note that if a decision maker does **not** find this trade-off a problem then he could treat “cost” as just another attribute.

He could allocate values to various costs, with a value of 100 being given to the location with the lowest cost and 0 to the office with the highest.

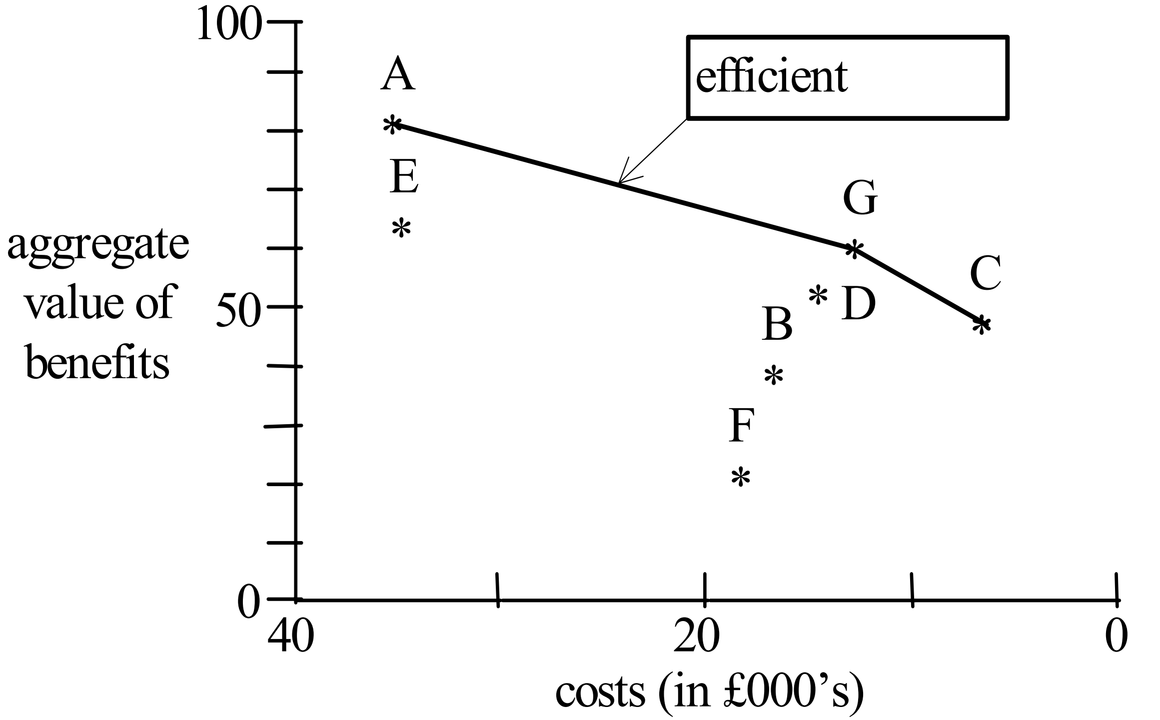
Weights could then be derived to compare swings from the least-preferred to the most-preferred level of benefits with swings from the worst to the best cost.

This would enable the value for cost to be included when the additive model is used to derive an overall value to measure the attractiveness of the different offices.

The office achieving the highest overall value would be the one the owner should choose.

In this case, the owner does have difficulty judging the cost-benefit trade-off.

We proceed by plotting the aggregate value of benefits against the annual cost, for each of the offices. the cost scale is “turned around” so that costs run from highest (least-preferable) to lowest (most-preferable).



A has roughly the same cost as E but higher benefits.

It is not worth considering E - we say that E is **dominated** by A.

G has lower costs, but higher benefits than B, D and F. B, D and F are thus also dominated.

Hence, the only alternatives worth considering are A, G and C.

These non-dominated locations are said to lie on the **efficient frontier**.

Choice between A, G and C depends on the relative weight that the owner attaches to costs and benefits.

If the owner is much more concerned about benefits Addison Square will be his choice.

If he is keen to keep costs down, he should choose Carlisle Walk.

Gorton Square would be an intermediate choice. It costs £5,300 more p.a., but offers slightly higher benefits.

In moving from C to G, each one-point increase in value of benefits would cost £414 (= £5,300/12.8).

A move from G to A would increase costs by £1,117 (=£23,000/20.6) for each value point increment.

If a value point is worth less than £414 he should choose C.

If it’s worth between £414 and £1,117 he should choose G, and if it’s worth more than £1,117 he should choose A.

We ask the owner to choose a low-level attribute that he could find fairly easy to evaluate in monetary terms.

He chooses “image”. We ask him what extra cost he would be willing to incur for a move from an office with the worst image to one with the best.

He says it would be worth £15,000 p.a. It’s worth paying £15,000 for a 100 point increase in image.

The weight of image is 23% of the total weight allocated to the benefit attributes.

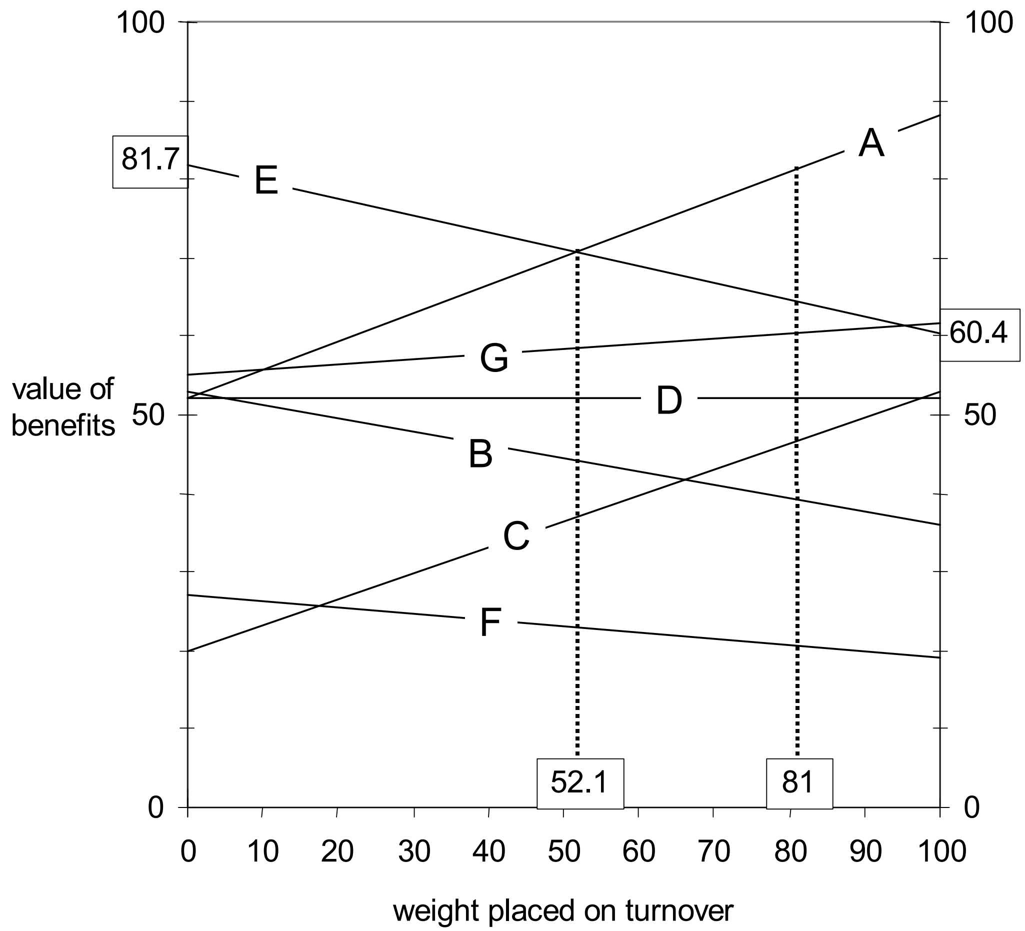
An increase of 100 image points would lead to a 23 point increase in aggregate benefits.

He is prepared to pay £652 (=£15,000/23) per point. So he should choose G.

SMART stage 8: Sensitivity Analysis.

Sensitivity analysis is used to examine how robust the choice of an alternative is to changes in the figures used in the analysis.

The owner is worried about the weight of "turnover" (i.e., 81) relative to that of "working conditions" (i.e., 19), and would like to know what would happen if the weights were changed.



This diagram shows how the value of benefits for the different locations varies with changes in the weight placed on turnover.

If turnover had a weight of zero, this would imply that the three turnover attributes would also have zero weights.

After re-normalisation, this would leave weights of 50, 33.3 and 16.7 for size, comfort and car parking, respectively.

This would mean, for example, that Elton Street (E) would have an aggregate benefit value of 81.7 .

At the other extreme, if turnover had a weight of 100 (and therefore working conditions a weight of zero) Elton Street would have an aggregate benefit value of 60.4 .

The line joining these points shows the value of benefits for Elton Street for turnover weights between 0 and 100.

Elton Street can be seen to have the highest value of benefits as long as the weight placed on turnover is less than 52.1 .

If the weight is above this level then Addison Square (A) has the highest level of benefits.

Since the owner assigned a weight of 81 to turnover, it would take a fairly large change in this weight before Elton Street was worth considering, and the owner can be reasonably confident that Addison Square should appear on the efficient frontier.

No changes in the weight attached to turnover would make the other offices achieve the highest value of benefits.

Carrying out sensitivity analysis should contribute to the decision maker's understanding of his problem and it may lead him to reconsider some of the figures he has supplied.

In many cases sensitivity analysis also shows that the data supplied do not need to be precise. Large changes in the figures are often required before one option becomes more attractive than another.

SMARTER

(SMART exploiting ranks)

The SMART technique involves

two quite tedious and demanding elicitation stages:

- **stage4** - assigning values for each attribute
  to measure the performance of the alternatives on that attribute
- **stage5** - determining a weight for each attribute.

The SMARTER technique, due to Edwards and Barron, attempts to:

- avoid difficult judgements on the part of the decision maker,
- produce recommended decisions that are

  almost as reliable as the more demanding SMART method.

SMARTER is motivated by the beliefs that:

- simpler tools are more likely to be useful, and
- the key to appropriate selection of methods is

  concern about the trade-off between modelling error and elicitation error.

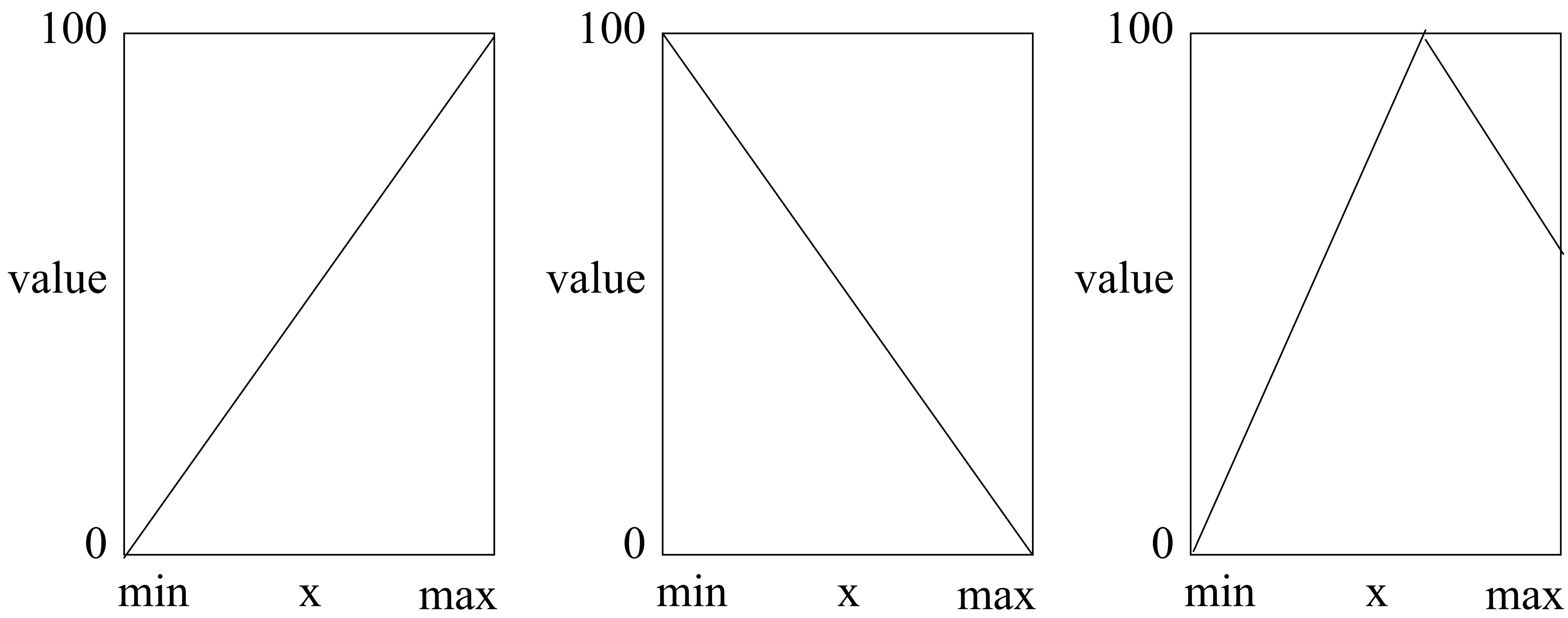
It is pointless building a highly rigorous model if it leads to making unrealistic demands of the decision maker, in terms of the judgements that need to be elicited.

In **stage 4** of the SMARTER technique

value functions of attributes

(which can be represented by easily quantifiable variables)

are normally assumed to be linear or piecewise linear:



In the first two cases:

the task of eliciting value functions

boils down to assessing the two extreme values of attribute x –

its maximum and minimum in the context at hand.

In the third case:

you also need elicit the best value of x

a judgement as to which branch of the function reaches 0 value and

by how much the other branch does not.

Cases in which the value function

reaches an internal minimum rather than an internal maximum are

hardly ever encountered.

It is necessary to check that it is valid to use a linear value function.

Suppose that the decision maker were choosing between cars and that an attribute under consideration was engine power, whose value function appeared to correspond to the first case shown.

One could ask the decision maker whether a small fixed improvement in power (say of 10 horsepower) would be more appealing if it fell near the bottom of the scale, in the middle or near the top.

If it didn't matter, then a linear approximation is acceptable.

If a linear approximation is found not to be appropriate for a particular attribute then the elicitor can always fall back on the SMART method of finding its value function.

Stage 5 of the SMARTER deals with
producing swing weights for the various attributes.

It is based on the insights that

- it is far easier and quicker for the decision maker to rank the attributes in terms of swing from worst to best value than it is to come up with actual swing weights,

but that

- most of the numerical information about swing weights is actually implicit in the rankings themselves.

There are a variety of different ways that one can derive a set of attribute swing weights from the rankings of the attributes.

The **Rank-Sum** Method

assigns reverse ranks

(if there were four attributes the weights would be 4, 3, 2, 1)

and normalises the weights so that they sum to 1.

Another method

assigns weights equal to the reciprocal of the ranks

(i.e., weights of 1, 1/2, 1/3, etc.)

and again normalises these.

The most satisfactory method –

the least arbitrary and the one that usually seems to produce the best results

(particularly when the decision maker feels secure about his rankings) –

is called the **Rank Order Centroid (ROC)** method.

Suppose that there are just two attributes and that

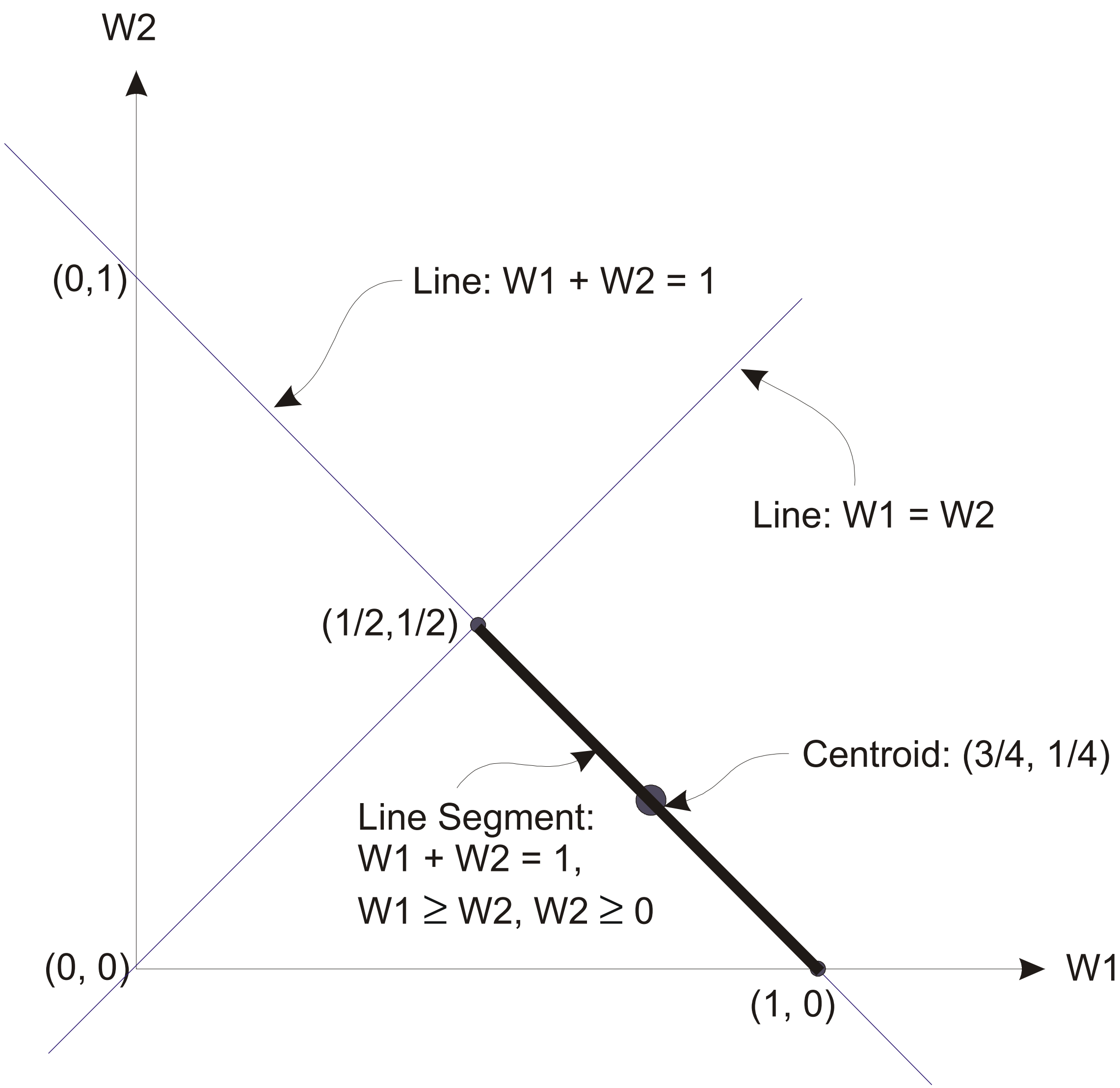
attribute 1 has been ranked ahead of attribute 2.

If W1 and W2 are the two swing weights to be given to the attributes, then

we know that:

$$0 \le W2 \le W1 \le 1 \quad \text{and} \quad W1 + W2 = 1$$

The pair of feasible values (W1, W2) that satisfy these constraints is restricted to the heavy line segment joining the points (1, 0) and (1/2, 1/2) in the next diagram:



If the only information we have are the rankings, then

we have no reason to prefer one point on this segment to any other.

It would then be natural (and error-minimizing) to

adopt the expected values for weights, W1 and W2,

given by the centroid (i.e., middle) of the segment.

So we choose: W1 = 3/4 and W2 = 1/4

In the case of three attributes,

ranked in descending order 1, 2, 3,

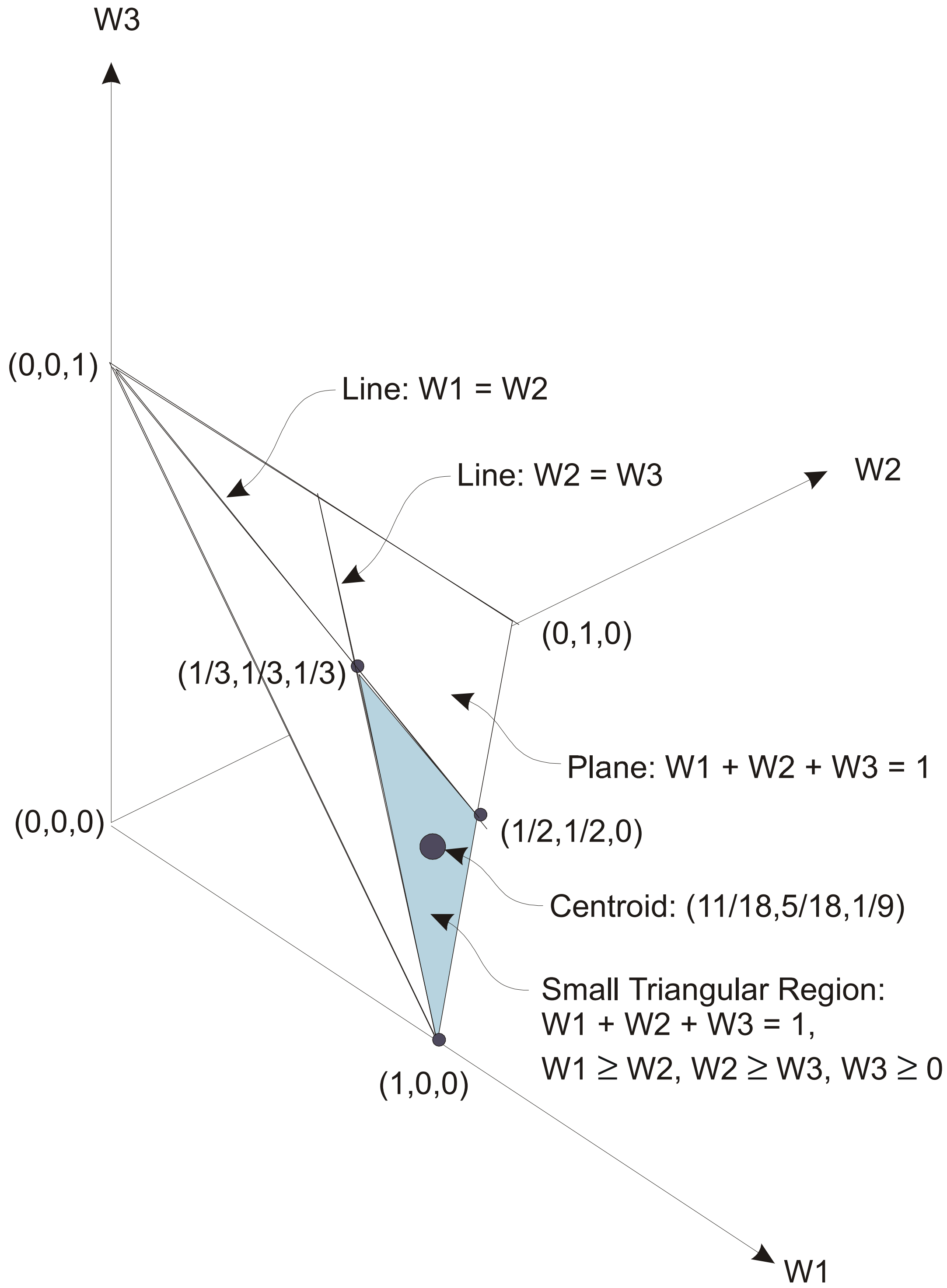
the swing weights W1, W2 and W3 satisfy the constraints:

$$0 \leq W3 \leq W2 \leq W1 \leq 1 \text{ and } W1 + W2 + W3 = 1$$

The feasible values (W1, W2, W3) that satisfy these constraints are

restricted to the small shaded triangular region

joining the points (1, 0, 0), (1/2, 1/2, 0) and (1/3, 1/3, 1/3):



W3
W2
W1
(0,0,1)
Line: W1 = W2
Line: W2 = W3
(0,1,0)
(1/3,1/3,1/3)
Plane: W1 + W2 + W3 = 1
(0,0,0)
(1/2,1/2,0)
Centroid: (11/18,5/18,1/9)
Small Triangular Region:
W1 + W2 + W3 = 1,
W1 ≥ W2, W2 ≥ W3, W3 ≥ 0
(1,0,0)

It would be natural to adopt the expected values of the weights W1, W2 and W3, given by the centroid of the region calculated by averaging the coordinates of the vertices of this region.

Thus:

$$W1 = (1 + 1/2 + 1/3) / 3 \qquad = 11/18$$

$$W2 = (0 + 1/2 + 1/3) / 3 \qquad = 5/18$$

$$W3 = (0 + 0 + 1/3) / 3 \qquad = 2/18$$

Generally, for any number, n, of attributes, the ROC weights are given by:

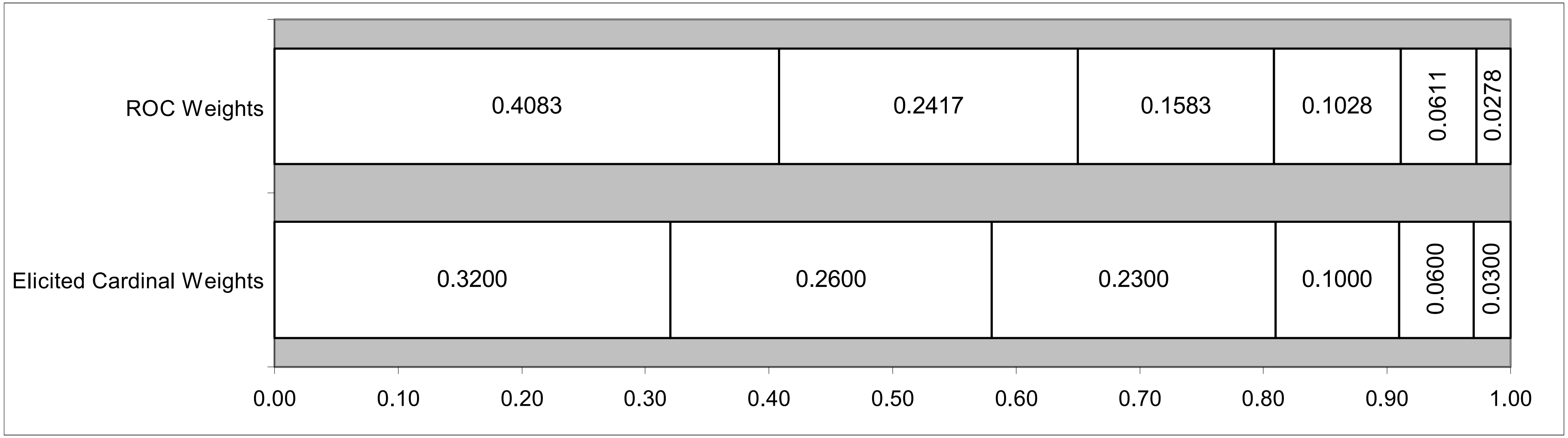
$$W_i = \frac{1}{n} \sum_{j=i}^{n} \frac{1}{j}$$

For example, when n = 8, the weights are:

W1	=	(1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7 + 1/8) / 8	= 0.3397
W2	=	(1/2 + 1/3 + 1/4 + 1/5 + 1/6+ 1/7 + 1/8) / 8	= 0.2147
W3	=	(1/3 + 1/4 + 1/5 + 1/6+ 1/7 + 1/8) / 8	= 0.1522
W4	=	(1/4 + 1/5 + 1/6+ 1/7 + 1/8) / 8	= 0.1106
W5	=	(1/5 + 1/6+ 1/7 + 1/8) / 8	= 0.0793
W6	=	(1/6+ 1/7 + 1/8) / 8	= 0.0543
W7	=	(1/7 + 1/8) / 8	= 0.0335
W8	=	(1/8) / 8	= 0.0156

The Office Location Problem:

Elicited Cardinal Weights v ROC Weights



Attribute	Elicited Weights	Office A	B	C	D	E	F	G	ROC Weights
Closeness	0.3200	100	20	80	70	40	0	60	0.4083
Visibility	0.2600	60	80	70	50	60	0	100	0.2417
Image	0.2300	100	10	0	30	90	70	20	0.1583
Size	0.1000	75	30	0	55	100	0	50	0.1028
Comfort	0.0600	0	100	10	30	60	80	50	0.0611
Car Parking	0.0300	90	30	100	90	70	0	80	0.0278
	Elicited Aggregates	80.8	39.4	47.4	52.3	64.8	20.9	60.2	
		81.4	39.1	53.0	55.4	61.0	16.0	62.3	*ROC Aggregates*
	Costs	35,000	17,800	6,700	14,100	34,800	18,600	12,000	

Move from C to G if an aggregate benefit point is worth more than:

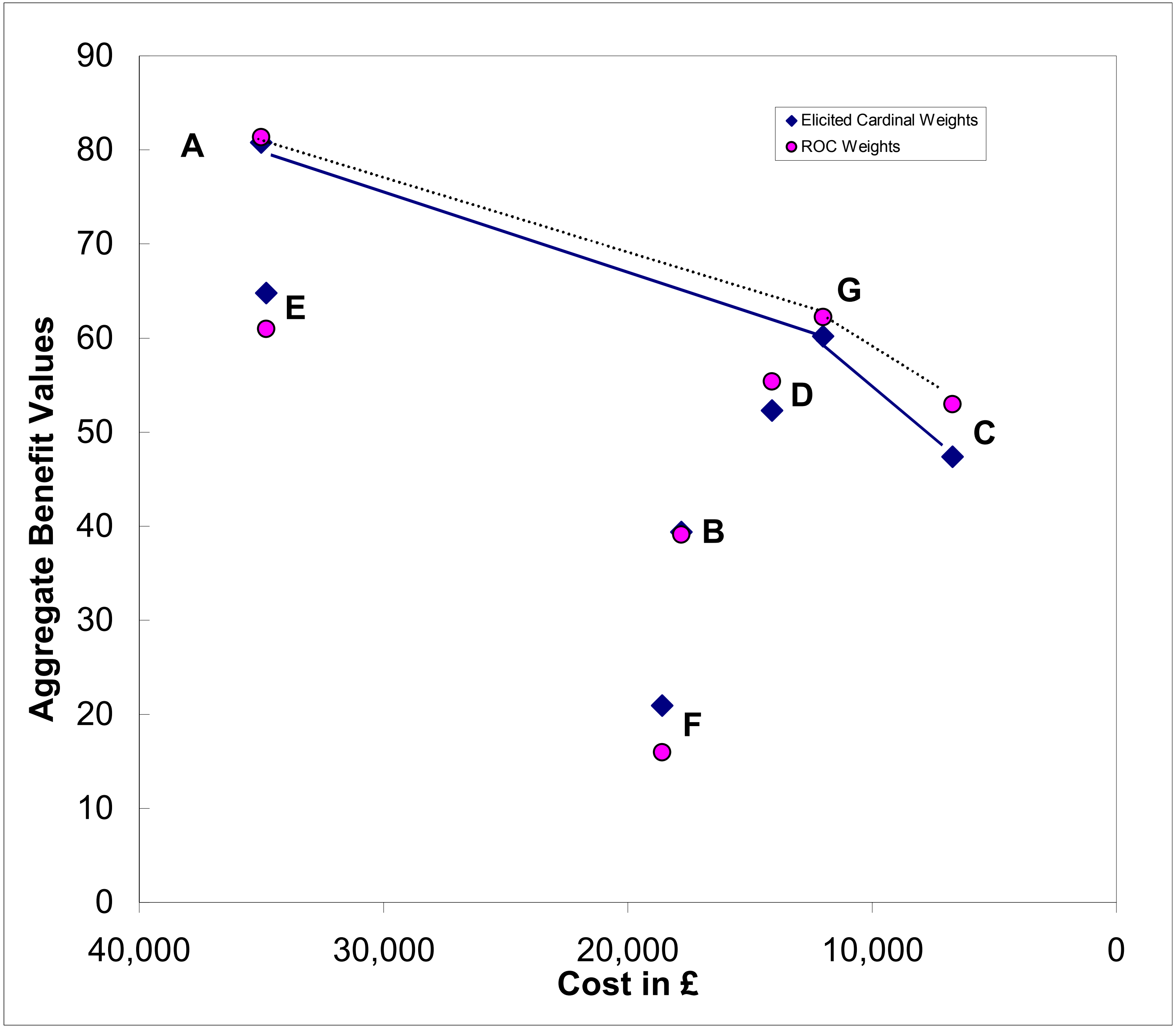
(12000 - 6700) / (60.2 - 47.4) = £414 (elicited)
(12000 - 6700) / (62.3 - 53) = £571 (ROC)

Move from G to A if an aggregate benefit point is worth more than:

(35000 - 12000) / (80.8 - 60.2) = £1,117 (elicited)
(35000 - 12000) / (81.4 - 62.3) = £1,203 (ROC)

In this (but NOT every) case The dominated alternatives and the efficient frontier are the same under SMARTER as they were under the original analysis using SMART.

However, care has to be taken when deciding between points on the efficient frontier as the critical values of aggregate benefit points may be significantly different.


90
80
70
60
50
40
30
20
10
0
Aggregate Benefit Values
40,000
30,000
20,000
10,000
0
Cost in £
Elicited Cardinal Weights
ROC Weights
A
E
G
D
C
B
F