Non-Monotonic Inference Properties for Assumption-Based Argumentation

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Abstract. Cumulative Transitivity and Cautious Monotonicity are widely considered as important properties of non-monotonic inference and equally as regards to information change. We propose three novel formulations of each of these properties for Assumption-Based Argumentation (ABA)—an established structured argumentation formalism, and investigate these properties under a variety of ABA semantics.

Keywords: Assumption-Based Argumentation · Non-monotonic inference · Argumentation dynamics

1 Introduction

In the 1980s, several non-monotonic reasoning formalisms were proposed (see [2] for an overview). Systemic investigations into aspects of Cautious Monotonicity and Cumulative Transitivity of non-monotonic inference followed (e.g. [24, 25]). Those works also contribute to the well-studied area of analysing non-monotonic reasoning with respect to information change (see e.g. [29]).

Since the early 1990s, argumentation (as overviewed in [28]) has emerged as a generic framework for non-monotonic reasoning, admitting existing non-monotonic reasoning formalisms as instances (see e.g. [7, 16]). Recently, some forms of structured argumentation (see [5] for an overview) have been investigated in terms of non-monotonic inference (see Sect. 4). Contributing to this area of research, we here analyse a well-established structured argumentation formalism, Assumption-Based Argumentation (ABA) [7, 30], against the non-monotonic inference properties of Cumulative Transitivity and Cautious Monotonicity in the spirit of [24, 25]. Since ABA is an instance of a well-known structured argumentation framework ASPIC$^+$ (see [26] for a tutorial), this work is potentially applicable to a wider array of argumentation systems.

Originally, the non-monotonic inference properties in question were defined with respect to non-monotonic entailment. Yet, ABA (as well as a significant portion of other structured argumentation formalisms) is defined in terms of *extensions* (e.g. sets of arguments). We thus first reformulate the properties to be applicable to extension-based non-monotonic reasoning formalisms (but see e.g. [11, 15] for different approaches). The essential idea is to characterize what happens to extensions when a certain change in knowledge occurs. The following will serve as an abstract pattern for producing the concrete instances of the properties (from now on, CUT and MON stand for Cumulative Transitivity and Cautious Monotonicity, respectively):

Let $\mathcal{K}$ be a knowledge base. Suppose that an 'entity' ψ 'belongs' to an extension E of $\mathcal{K}$, and let E' be an extension of the knowledge base $\mathcal{K}'$, which is obtained by 'adding' ψ to $\mathcal{K}$. Then

$$\texttt{CUT}: \ E \text{ 'contains' } E'; \qquad\qquad \texttt{MON}: \ E' \text{ 'contains' } E.$$

These properties concern what happens when a conclusion that is reached—which could have been already present as a hard fact, or inferred defeasibly—is added to the knowledge base and reasoned with anew. Arguably, there are many ways to interpret both properties, e.g. as checking that accepting a conclusion does not yield overwhelming changes in reasoning. One of our contributions is to provide three instantiations of both `CUT` and `MON` applicable to ABA. We will also discuss some possible interpretations of those instantiations.

The abstract formulation above, aiming to be universal, is informal: notions like 'entity' act as placeholders for alternative formal concepts (e.g. conclusion of an argument); 'containment' need not be understood in set-theoretic terms. For ABA, we will provide rigorously defined instances of the abstract formulation.

To ease the intuition behind the properties, consider the following illustration.

Example 1. Three prospective academic partners—*Al*, *Ben* and *Dan*—invite you to dine at a new restaurant. On the eve of the dinner it turns out that no one has booked a table in advance and, unfortunately, you will have to sit in pairs at two separate tables. You are the one invited, so you will have to choose whom to sit with. In a playful manner, your associates start competing for your company: both Ben and Dan claim that Al is *antisocial*, while Al retorts that Ben is *back-stabbing*. Somewhat puzzled, you casually inquire about the restaurant. Ben replies that it is a *gourmet* place. You then recall that Dan is a *disagreeable* person over fancy food. It is high time to decide, so what will be the verdict?

The reasoning may unfold as follows. Ben defends himself against Al by insisting that the latter is antisocial. Meanwhile, Al has nothing against his attacker Dan. The latter is not a good option, assuming that Ben is right about gourmet food. No more hesitating, and you decide to go for Ben.

Now, how would the information that you are really in a gourmet place change your reasoning, if at all? One can argue that, knowing as a matter of fact it is a gourmet restaurant immediately discards Dan as an option. So if Dan is out of consideration, then Al is attacked only by Ben, and in turn attacks him back. Thus, both Ben and Al defend themselves, and hence are acceptable choices. In terms of non-monotonic inference, `CUT` insists you should not draw any new conclusions, while `MON` demands not to lose previous inferences. Sticking to your first choice would satisfy both requirements, whereas choosing Al over Ben would violate both properties, indicating a revision of your previous decision.

In this work we investigate how ABA (background in Sect. 2) behaves when employed to formalize this sort of situations. In particular, in Sect. 3 we provide three instantiations of each of `CUT` and `MON`, and analyse their satisfaction under six extension-based ABA semantics. After discussing related work (Sect. 4), we conclude in Sect. 5.

2 Background

In this section, we provide background on ABA, following [30].

An **ABA framework** is a tuple $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ consisting of the following elements. $(\mathcal{L}, \mathcal{R})$ is a deductive system with a language $\mathcal{L}$ and a set $\mathcal{R}$ of rules: rules in $\mathcal{R}$ are assumed to be of the form $\varphi_0 \leftarrow \varphi_1, \ldots, \varphi_m$ with $m \geq 0$ and $\varphi_i \in \mathcal{L}$ for $i \in \{0, \ldots, m\}$; φ_0 is referred to as the *head*, and $\varphi_1, \ldots, \varphi_m$ is referred to as the *body* of the rule; if $m = 0$, then the rule is said to have an empty body and we write it as $\varphi_0 \leftarrow \top$. The set $\mathcal{A} \subseteq \mathcal{L}$ is non-empty, referred to as **assumptions**. The so called *contrary mapping* $\overline{} : \mathcal{A} \rightarrow \mathcal{L}$ is a total function and for $\alpha \in \mathcal{A}$, the $\mathcal{L}$-formula $\overline{\alpha}$ is referred to as the **contrary** of α.

We restrict the discussion to the so called *flat* ABA frameworks, where no assumption $\alpha \in \mathcal{A}$ can be the head of any rule from $\mathcal{R}$.

A ***deduction** for $\varphi \in \mathcal{L}$ supported by $S \subseteq \mathcal{L}$ and $R \subseteq \mathcal{R}$*, denoted by $S \vdash^R \varphi$, is a finite tree with the root labeled by φ, leaves labeled by $\top$ or elements from S, the children of non-leaf nodes ψ labeled by the elements of the body of some rule from $\mathcal{R}$ with the head ψ, and R being the set of all such rules. An **argument** A with *conclusion* $\varphi \in \mathcal{L}$ and *support* $A \subseteq \mathcal{A}$, written as $\mathsf{A} : A \vdash \varphi$, is a deduction for φ supported by A and some $R \subseteq \mathcal{R}$. We say that $\mathsf{A}' : A' \vdash \varphi'$ **attacks** $\mathsf{A} : A \vdash \varphi$ (on some $\alpha \in A$) just in case φ' is the contrary $\overline{\alpha}$ of some $\alpha \in A$.

Given an ABA framework $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$, we denote the set of constructible arguments by *Args*, the attack relation by $\rightsquigarrow$, and the corresponding *argument framework* by $(\mathit{Args}, \rightsquigarrow)$. For a set $S \subseteq \mathit{Args}$, we say that: S attacks an argument A', written $S \rightsquigarrow \mathsf{A}'$, if some $\mathsf{A} \in S$ attacks A'; S attacks a set $S' \subseteq \mathit{Args}$ of arguments, written $S \rightsquigarrow S'$, if S attacks some $\mathsf{A}' \in S'$; S is *conflict-free* if $S \not\rightsquigarrow S$; and S *defends* $\mathsf{A} \in \mathit{Args}$ if for each $\mathsf{A}' \rightsquigarrow \mathsf{A}$ we have $S \rightsquigarrow \mathsf{A}'$. For an argument A, let $\mathit{Cn}(\mathsf{A})$ be the conclusion of A and $\mathit{asm}(\mathsf{A})$ the support of A. We extend this notation so that for a set $S \subseteq \mathit{Args}$ of arguments, $\mathit{Cn}(S) = \{\mathit{Cn}(\mathsf{A}) : \mathsf{A} \in S\}$ and $\mathit{asm}(S) = \{\alpha \in \mathcal{A} : \alpha \in \mathit{asm}(\mathsf{A}),\ \mathsf{A} \in S\}$.

ABA semantics are defined as follows. A set $E \subseteq \mathit{Args}$, also called an **extension** (of $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ or $(\mathit{Args}, \rightsquigarrow)$), is: *admissible*, if E is conflict-free and defends all $\mathsf{A} \in E$; *preferred*, if E is $\subseteq$-maximally admissible; *sceptically preferred*, if E is the intersection of all the preferred extensions; *complete*, if E is admissible and contains all arguments it defends; *grounded*, if E is $\subseteq$-minimally complete; *stable*, if E is admissible and $E \rightsquigarrow \mathsf{A}$ for all $\mathsf{A} \in \mathit{Args} \setminus E$; and *ideal*, if E is $\subseteq$-maximal such that E is admissible and contained in all the preferred extensions.

Grounded, sceptically preferred and ideal semantics fall into the category of *sceptical* reasoning, whereby conclusions are drawn from a unique extension. Meanwhile stable, preferred and complete semantics represent *credulous* reasoning, in that multiple conflicting extensions can be present.

We also recall (see e.g. [16]) that the grounded extension G of any $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ always exists and is unique, and can be constructed inductively as $G = \bigcup_{i \geq 0} G_i$, where G_0 is the set of arguments that are not attacked at all, and for every $i \geq 0$, G_{i+1} is the set of arguments that are defended by G_i.

To simplify proofs of our results, we restrict to finite argument frameworks, as is common in literature.

3 Inference Properties for ABA

In this section we formulate and analyse non-monotonic inference properties regarding ABA. There will be three different settings of instantiations of CUT and MON. Each property will also have a strong and a weak version. The strong properties will quantify over all extensions, indicating the necessity to preserve the previously accepted conclusions after a change in information. Meanwhile, the weak properties, by quantifying existentially over extensions, will insist on the possibility, rather than necessity, to preserve the previously accepted conclusions. When referring to a property, we will have in mind its strong version, unless specified otherwise.

Throughout this section we use the following notation, unless stated otherwise. We take as given a fixed, but otherwise arbitrary (flat) ABA framework $\mathcal{F} = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$, and its corresponding argument framework $(Args, \rightsquigarrow)$. To instantiate the abstract formulations of CUT and MON given in the Introduction, we replace a knowledge base $\mathcal{K}$ with $\mathcal{F}$, fix an argumentation semantics σ and let E be an extension of $\mathcal{F}$ under $\sigma \in \{$grounded, ideal, sceptically preferred, stable, preferred, complete$\}$. An 'entity' ψ will come from the set $Cn(E)$ of conclusions of E. By default, the knowledge base $\mathcal{K}'$ will be represented by $\mathcal{F}'$, which will be the ABA framework obtained by 'adding' (to be formalized) ψ to $\mathcal{F}$. The corresponding argument framework of $\mathcal{F}'$ will be denoted by $(Args', \rightsquigarrow')$. Still further, E' will denote an extension of $\mathcal{F}'$ under the same fixed semantics σ. To avoid trivialities, we consider cases only where under a particular semantics σ, each of $\mathcal{F}$ and $\mathcal{F}'$ admits at least one extension, E and E', respectively.

3.1 Strict Cumulative Transitivity and Cautious Monotonicity

We now rigorously formulate the first type of properties for ABA. (Recall that E is an extension of $\mathcal{F}$ under a fixed semantics σ.) Initially, given some $\psi \in Cn(E) \setminus \mathcal{A}$, define $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\psi \leftarrow \top\}, \mathcal{A}, \overline{})$. The following then are the first concrete instances of non-monotonic inference properties that we consider.

STRONG STRICT CUT :	For all extensions E' of $\mathcal{F}'$ we have $Cn(E') \subseteq Cn(E)$;
WEAK STRICT CUT :	There is an extension E' of $\mathcal{F}'$ with $Cn(E') \subseteq Cn(E)$;
STRONG STRICT MON :	For all extensions E' of $\mathcal{F}'$ we have $Cn(E) \subseteq Cn(E')$;
WEAK STRICT MON :	There is an extension E' of $\mathcal{F}'$ with $Cn(E) \subseteq Cn(E')$.

STRICT CUT and STRICT MON concern what happens when a conclusion (not itself an assumption) is reached and then considered as a fact (i.e. a rule with empty body) to reason again. The conclusion may be learned as an objective truth, e.g. verifying that you are in a gourmet restaurant. In essence, STRICT properties regard *strengthening of information* and what effect it has on different ABA semantics in terms of extensions. A reasoner employing ABA semantics can utilize these properties to anticipate its behaviour regarding changes that strengthen knowledge.

The following remarks are in place. First, satisfaction of a strong property will always imply satisfaction of the corresponding weak property. Second, under sceptical semantics, weak and strong formulations actually coincide, because the extension is unique. Further, as grounded, ideal, stable and preferred extensions are complete [16, 18], a strong property satisfied under complete semantics holds for the other four. Similarly, if a strong property is violated under stable semantics, then it fails under both preferred and complete semantics, because stable extensions are also preferred [7].

Our first result shows that grounded semantics fulfils (the strong versions of) both `CUT` and `MON` in the `STRICT` setting.

Proposition 2. *Grounded semantics satisfies both* `STRICT CUT` *and* `STRICT MON`*.*

Proof. Let G be the grounded extension of $\mathcal{F}$. If $G = \emptyset$, then $\mathcal{F}' = \mathcal{F}$, so the properties are trivially satisfied. Otherwise, pick a conclusion $\psi \in \mathit{Cn}(G) \setminus \mathcal{A}$ and suppose that $\mathsf{B}_1 : B_1 \vdash \psi, \ \ldots, \ \mathsf{B}_n : B_n \vdash \psi$ are all the arguments in G that have conclusion ψ. Let G' be the grounded extension of $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\psi \leftarrow \top\}, \mathcal{A}, \bar{\ })$.

We prove $G \subseteq G'$ by induction on the construction of G.

For the basis step, let $G_0 \subseteq G$ be the set of arguments not attacked in $\mathcal{F}$. Since $\mathit{Cn}(\mathit{Args}') = \mathit{Cn}(\mathit{Args})$, arguments from G_0 are unattacked in $\mathcal{F}'$, so we get $G_0 \subseteq G'$.

For the inductive step, let $G_{i+1} \subseteq G$ be the set of arguments attacked in $\mathcal{F}$ but defended by $G_i \subseteq G$, assuming $G_i \subseteq G'$ as an induction hypothesis. Suppose that $\mathsf{A}' : A' \vdash \varphi$ attacks G_{i+1} in $\mathcal{F}'$. We split into cases.

- If $\mathsf{A}' \in \mathit{Args}$, then $\mathsf{A}' \rightsquigarrow G_{i+1}$, so that $G_i \rightsquigarrow \mathsf{A}'$, and so $G' \rightsquigarrow' \mathsf{A}'$ too.
- Else, if $\mathsf{A}' \notin \mathit{Args}$, then there is some $\mathsf{A} : A \vdash \varphi \in \mathit{Args}$ from which A' can be obtained by replacing occurrences of the deduction $B_j \vdash^{R_j} \psi$ (for some j) in A with the deduction $\emptyset \vdash^{\{\psi \leftarrow \top\}} \psi$. (Such A' and A are called **counterpart** arguments and satisfy $\mathit{asm}(\mathsf{A}) = \mathit{asm}(\mathsf{A}') \cup B_j$.) We then have $\mathsf{A} \rightsquigarrow G_{i+1}$, so that $G_i \rightsquigarrow \mathsf{A}$ on some $\alpha \in A \setminus B_j = A'$ (because $B_j \subseteq \mathit{asm}(G)$), which yields $G' \rightsquigarrow' \mathsf{A}'$.

In any event, G' defends G_{i+1}, so that $G_{i+1} \subseteq G'$.

By induction it holds that $G_i \subseteq G'$ for every $i \geq 0$, so that $G \subseteq G'$, and hence $\mathit{Cn}(G) \subseteq \mathit{Cn}(G')$, giving `STRICT MON`.

For `STRICT CUT`, given that we already have $G \subseteq G'$, it suffices to show that $\mathit{Cn}(G' \setminus G) \subseteq \mathit{Cn}(G)$. We prove this by induction on the construction of G'.

For the basis step, let $G'_0 \subseteq G' \setminus G$ be the set of arguments from $\mathit{Args}' \setminus \mathit{Args}$ unattacked in $\mathcal{F}'$. Pick $\mathsf{A}' \in G'_0$, if any. Consider a counterpart $\mathsf{A} \in \mathit{Args}$ with $\mathit{asm}(\mathsf{A}) = \mathit{asm}(\mathsf{A}') \cup B_j$ (for some j) and $\mathit{Cn}(\mathsf{A}) = \mathit{Cn}(\mathsf{A}')$ (so every occurrence of the deduction $\emptyset \vdash^{\{\psi \leftarrow \top\}} \psi$ in A' is replaced with the deduction $B_j \vdash^{R_j} \psi$ in A). Such an A can be attacked in $\mathcal{F}$ only on some $\beta \in B_j$, whereby G defends A, because $B_j \subseteq \mathit{asm}(G)$. Consequently, $\mathit{Cn}(\mathsf{A}') \in \mathit{Cn}(G)$, and therefore, $\mathit{Cn}(G'_0) \subseteq \mathit{Cn}(G)$.

For the inductive step, let $G'_{i+1} \subseteq G' \setminus G$ be the set of arguments attacked in $\mathcal{F}'$ but defended by $G \cup G'_i$, assuming $\mathit{Cn}(G'_i) \subseteq \mathit{Cn}(G)$ as an induction hypothesis. Pick $\mathsf{A}' \in G'_{i+1}$, if any, and consider a counterpart $\mathsf{A} \in \mathit{Args}$ with $\mathit{asm}(\mathsf{A}) = \mathit{asm}(\mathsf{A}') \cup B_j$ (for some j) and $\mathit{Cn}(\mathsf{A}) = \mathit{Cn}(\mathsf{A}')$. Then A can be attacked in $\mathcal{F}$ in two ways:

- either on some $\beta \in B_j$, whence G defends A in $\mathcal{F}$;
- or on some $\alpha \in \mathit{asm}(\mathsf{A}) \setminus B_j$, whence A' is attacked in $\mathcal{F}'$ (on α), and so defended in $\mathcal{F}'$ by $G \cup G'_i$, so that G defends A in $\mathcal{F}$, because $\mathit{Cn}(G \cup G'_i) \subseteq \mathit{Cn}(G)$.

In any case, $\mathsf{A} \in G$, and so $\mathit{Cn}(G'_{i+1}) \subseteq \mathit{Cn}(G)$.

By induction, $\mathit{Cn}(G') \subseteq \mathit{Cn}(G)$ holds as required to satisfy `STRICT CUT`. □

So we know that strong, and hence weak, `STRICT CUT` and `STRICT MON` hold for grounded semantics. What is more, weak versions of both properties are satisfied under complete semantics, as we see next.

Proposition 3. *Complete semantics satisfies both* `WEAK STRICT CUT` *and* `WEAK STRICT MON`*.*

Proof. We prove that for each complete extension E of $\mathcal{F}$, and for each conclusion $\psi \in Cn(E) \setminus \mathcal{A}$, there is a complete extension E' of $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\psi \leftarrow \top\}, \mathcal{A}, \overline{\ })$ such that $Cn(E') = Cn(E)$.

So let E be a complete extension of $\mathcal{F}$ and fix $\psi \in Cn(E) \setminus \mathcal{A}$. Suppose that $\mathsf{B}_1 : B_1 \vdash \psi, \ldots, \mathsf{B}_n : B_n \vdash \psi$ are all the arguments in E with conclusion ψ. Now, $Args' \setminus Args$ consists of arguments $\mathsf{A}' : A' \vdash \varphi$ which are constructed from arguments $\mathsf{A} : A \vdash \varphi$ in $Args$ that use some deduction(s) of the form $\Psi \vdash^R \psi$, by replacing (some) such deduction(s) with $\emptyset \vdash^{\{\psi \leftarrow \top\}} \psi$. (Such A and A' are said to be **corresponding** to each other.) Let E^+ be the collection of $\mathsf{A}' \in Args' \setminus Args$ whose corresponding A is in E. We claim that $E' = E \cup E^+$ is the required complete extension of $\mathcal{F}'$.

- First, E' is conflict-free, as $Cn(E^+) \subseteq Cn(E)$.
- Second, E' defends every argument it contains: if $\mathsf{A}' \in Args' \setminus Args$ attacks E' in $\mathcal{F}'$, but $E' \not\rightsquigarrow' \mathsf{A}'$, then a counterpart (as in the proof of Proposition 2) argument A attacks E in $\mathcal{F}$, but $E \not\rightsquigarrow \mathsf{A}$, contradicting admissibility of E.
- Finally, for completeness, assume E' defends $\mathsf{A}' \in Args'$. Then there are two cases.
  - If $\mathsf{A}' \in Args$, then, as $Cn(E^+) \subseteq Cn(E)$, we have that E defends A' in $\mathcal{F}'$.
  - Else, if $\mathsf{A}' \notin Args$, then assume $\mathsf{A}' \notin E^+$ for a contradiction. Then a counterpart $\mathsf{A} \in Args$ is not in E, and so some C attacks A in $\mathcal{F}$, but $E \not\rightsquigarrow \mathsf{C}$. As E defends all B_js, we have $\mathsf{C} \rightsquigarrow' \mathsf{A}'$, but $E' \not\rightsquigarrow' \mathsf{C}$, which is a contradiction.

  In any event, $\mathsf{A}' \in E'$. Hence, E' is complete.

Since clearly $Cn(E') = Cn(E)$, E' is the required complete extension of $\mathcal{F}'$. □

We can actually extend the proof above to be applicable to both preferred and stable semantics, as follows.

Proposition 4. *Preferred and stable semantics satisfy both* `WEAK STRICT CUT` *and* `WEAK STRICT MON`*.*

Proof. We first prove that for every preferred extension E of $\mathcal{F}$, there is a preferred extension E' of $\mathcal{F}'$ with $Cn(E') = Cn(E)$. Since preferred extensions are complete, it suffices to show that the corresponding complete extension $E' = E \cup E^+$ (as defined in the proof of Proposition 3) is preferred in $\mathcal{F}'$. And indeed, if E' were not $\subseteq$-maximally admissible, then some $\mathsf{A}' \in Args' \setminus E'$ could be added to E' without sacrificing admissibility. But then a counterpart $\mathsf{A} \in Args$ (possibly $\mathsf{A} = \mathsf{A}'$, if A' does not use ψ) could be added to E without losing its admissibility, whence E would not be preferred in $\mathcal{F}$.

Likewise, we show that if E is stable, then E' is also stable. Suppose $\mathsf{A}' \notin E'$. If $\mathsf{A}' \in Args$, then $\mathsf{A}' \notin E$, so $E \rightsquigarrow \mathsf{A}'$, and hence $E' \rightsquigarrow' \mathsf{A}'$. Else, if $\mathsf{A}' \notin Args$, then a counterpart A is not in E and $E \rightsquigarrow \mathsf{A}$, so that $E' \rightsquigarrow' \mathsf{A}'$ too. Consequently, E' is a stable extension of $\mathcal{F}'$. □

Having the results above, we conclude with the following.

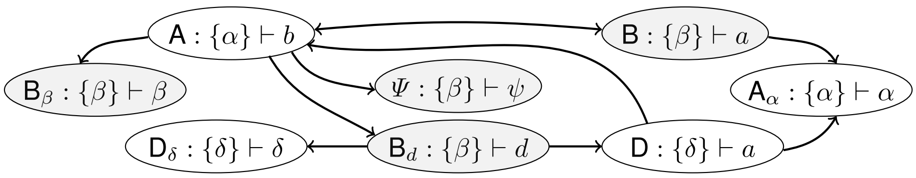
Corollary 5. *Sceptically preferred and ideal semantics satisfy* `STRICT CUT`*.*

Proof. Using notation from the proof of Proposition 3, let $S = \bigcap_i E_i$ be the intersection of all the preferred extensions E_i of $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$. Pick $\psi \in Cn(S) \setminus \mathcal{A}$ and consider $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\psi \leftarrow \top\}, \mathcal{A}, \overline{})$. Let $S' = \bigcap_j E'_j$ be the intersection of all the preferred extensions E'_j of $\mathcal{F}'$. We show $Cn(S') \subseteq Cn(S)$. According to Proposition 4, for every preferred extension E of $\mathcal{F}$, there is a preferred extension E' of $\mathcal{F}'$ such that $Cn(E') = Cn(E)$. Therefore, S' cannot contain arguments with conclusions not in $Cn(S)$. So `STRICT CUT` holds under sceptically preferred semantics.

Likewise, for the ideal extension I of $\mathcal{F}$ and $\psi \in Cn(I) \setminus \mathcal{A}$, if I' is the ideal extension of $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\psi \leftarrow \top\}, \mathcal{A}, \overline{})$, then, being contained in all preferred extensions of $\mathcal{F}'$, it has $Cn(I') \subseteq Cn(I)$. Thus, `STRICT CUT` holds under ideal semantics. □

The following formalization of the example from the Introduction reveals that neither of the (strong) properties holds for credulous reasoning. This violation is intuitive, as credulous semantics allow for multiple extensions, with different conclusions.

Example 6 (`STRICT CUT` *and* `STRICT MON` *violations).*
Let $\mathcal{L} = \{\alpha, \beta, \delta, a, b, d, \psi\}$, where: α, β, δ are the assumptions of choosing Al, Ben and Dan (resp.); a, b and d stand for 'antisocial', 'back-stabbing' and 'disagreeable' (resp.); and ψ expresses that we are in a gourmet place. So $\mathcal{A} = \{\alpha, \beta, \delta\}$, with contraries $\overline{\alpha} = a$, $\overline{\beta} = b$, $\overline{\delta} = d$. Then $\mathcal{R} = \{b \leftarrow \alpha,\ a \leftarrow \delta,\ a \leftarrow \beta,\ \psi \leftarrow \beta,\ d \leftarrow \psi\}$ completes the formalization: e.g. the rule $b \leftarrow \alpha$ represents Al's claim about Ben; the rule $d \leftarrow \psi$ indicates that Dan is a disagreeable company in a gourmet place. (In further examples, both $\mathcal{L}$ and $\mathcal{A}$ will be omitted, as they are implicit from $\mathcal{R}$ and the contrary relation.) The corresponding argument framework $(Args, \rightsquigarrow)$ can be represented graphically as follows (nodes hold arguments and directed edges indicate attacks):



Here, $\mathcal{F} = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{})$ has a unique preferred (also stable and ideal) extension $E = \{\mathsf{B}, \mathsf{B}_\beta, \mathsf{B}_d, \Psi\}$ (gray arguments) with $Cn(E) = \{a, \beta, d, \psi\}$. Now suppose that after deciding to sit with Ben, you check the menu and realize you are indeed in a gourmet restaurant. As knowledge changes—your belief that this is a gourmet place being strengthened—you wonder whether you would make the same decision now.

Consider thus $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\psi \leftarrow \top\}, \mathcal{A}, \overline{})$. In $Args'$, we get two new arguments: $\Psi' : \{\} \vdash \psi$ and $\mathsf{B}' : \{\} \vdash d$. While Ψ' neither attacks, nor is attacked by anything, B' is unattacked but attacks both D_δ and D. So $(Args', \rightsquigarrow')$ has two preferred extensions (which are also stable): $E_1 = \{\mathsf{B}, \mathsf{B}_\beta, \mathsf{B}_d, \mathsf{B}', \Psi, \Psi'\}$ (with $Cn(E_1) = Cn(E)$) and $E_2 = \{\mathsf{A}_\alpha, \mathsf{A}, \mathsf{B}', \Psi'\}$. Taking E_2 with $Cn(E_2) \nsubseteq Cn(E) \nsubseteq Cn(E_2)$ yields violations of `STRICT CUT` and `STRICT MON` under credulous reasoning. We also have $Cn(E) \nsubseteq Cn(\{\mathsf{B}', \Psi'\}) = Cn(E_1 \cap E_2)$, so `STRICT MON` is violated under both ideal and sceptically preferred semantics.

We see that a reasoner using ABA could find itself in a situation where adding credulously inferred information leads to a multitude of extensions. Even if the extension to begin with is unique, as in Example 6, strengthening some of its conclusions can result in more than one acceptable extension. Whether or not this behaviour is desirable depends on the application, anticipated changes in information and intended flexibility of the reasoner. For instance, one may wish for the reasoner to be credulous and try many different scenarios in order not to fixate on one particular decision. In contrast, sceptical semantics (except grounded) provide insurance that no new conclusions are attained—fulfil `STRICT CUT`, while ensuring that some are dropped (e.g. β, d). However, a sceptical reasoner may completely lose some previously acceptable choices (such as β in Example 6).

Example 6 also reveals contrast between `STRICT CUT` and `STRICT MON` under sceptically preferred and ideal semantics: adding a previously attained conclusion as a fact leaves all the original preferred extensions intact, yet allows for new ones, thus possibly shrinking their intersection. Hence, the sceptically preferred extension E' (as well as the ideal extension) of the ABA framework $\mathcal{F}'$ after the change in information will satisfy `STRICT CUT`; indeed, we have $Cn(E_1 \cap E_2) = Cn(\{\mathsf{B}', \Psi'\}) \subseteq Cn(E)$ in Example 6. For the same reason, `STRICT MON` is violated under both sceptically preferred and ideal semantics, as illustrated in Example 6.

We observe that under credulous semantics, the strong properties gain importance in settings where there is a unique credulous extension to begin with, such as in Example 6. Indeed, while the weak properties merely ask for the existence of an extension E' (of the framework $\mathcal{F}'$ after the knowledge change) with the same conclusions as the chosen extension E of the framework $\mathcal{F}$ to begin with, the strong properties require all new extensions to commit to the conclusions of E. The two properties together essentially insist that the new framework $\mathcal{F}'$ should admit a unique extension E' having the same conclusions as the original extension E.

The following table summarizes this subsection's results (as indicated, strong and weak versions coincide under sceptical reasoning, and for credulous semantics the status of the weak property is indicated in parentheses).

`STRICT` Cumulative Transitivity and Cautious Monotonicity

Property	Grounded	Ideal	Sceptically pref.	Stable	Preferred	Complete
`STRICT CUT`	✓	✓	✓	X (✓)	X (✓)	X (✓)
`STRICT MON`	✓	X	X	X (✓)	X (✓)	X (✓)

Only grounded semantics allows for safely strengthening information. However, as the grounded extension of a given ABA framework can be empty (e.g. Example 6), other semantics may be needed to make decisions. In that case, ideal and sceptically preferred semantics, for instance, guarantee that no new conclusions will be attained after strengthening information, yet some important ones may be lost: in Example 6, neither semantics allows to decide whom to dine with, because $\alpha, \beta, \delta \notin Cn(E_1 \cap E_2)$. Credulous semantics provide even less certainty (or more flexibility—depending on the way one intends to use it) unless one has a procedure allowing to pick the extension with the same conclusions as the extension to begin with (such an extension is guaranteed to exist due to satisfaction of the weak properties).

3.2 Defeasible Cumulative Transitivity and Cautious Monotonicity

We now formulate another type of variants of `CUT` and `MON`. Given $\psi \in Cn(E) \setminus \mathcal{A}$, define $\mathcal{F}' = (\mathcal{L} \cup \{y\}, \mathcal{R} \setminus \{r \in \mathcal{R} \ : \ \text{head of } r \text{ is } \psi\}, \mathcal{A} \cup \{\psi\}, \overline{})$.[1] Then

`STRONG DEF CUT` :	For all extensions E' of $\mathcal{F}'$ we have $Cn(E') \subseteq Cn(E)$;
`WEAK DEF CUT` :	There is an extension E' of $\mathcal{F}'$ with $Cn(E') \subseteq Cn(E)$;
`STRONG DEF MON` :	For all extensions E' of $\mathcal{F}'$ we have $Cn(E) \subseteq Cn(E')$;
`WEAK DEF MON` :	There is an extension E' of $\mathcal{F}'$ with $Cn(E) \subseteq Cn(E')$.

Unlike the `STRICT` setting, `DEF CUT` and `DEF MON` regard situations where a previously accepted conclusion (inferred possibly *defeasibly* using assumptions) is converted into an assumption itself, and can afterwards be drawn only *defeasibly*. For instance, instead of relying on Ben's claim about gourmet food, you may initially guess that you are in a gourmet place.

The same results (as in Sect. 3.1) hold in the defeasible (`DEF`) setting, and proofs follow a similar pattern.

Proposition 7. *Grounded semantics satisfies both* `DEF CUT` *and* `DEF MON`*.*

Proof. Let G be the grounded extension of $\mathcal{F}$. If $G = \emptyset$, then $\mathcal{F}' = \mathcal{F}$, so the properties are trivially satisfied. Otherwise, pick $\psi \in Cn(G) \setminus \mathcal{A}$ and let $\mathsf{B}_1 : B_1 \vdash \psi, \ \ldots, \mathsf{B}_n : B_n \vdash \psi \in G$ be all the arguments in G that conclude ψ. Let G' be the grounded extension of $\mathcal{F}' = (\mathcal{L} \cup \{y\}, \mathcal{R} \setminus \{r \in \mathcal{R} \ : \ \text{head of } r \text{ is } \psi\}, \mathcal{A} \cup \{\psi\}, \overline{})$ (where $\overline{\psi} = y$).

We first prove $Cn(G) \subseteq Cn(G')$ by induction on the construction of G.

For the basis step, let $G_0 \subseteq G$ be the set of arguments that are not attacked in $\mathcal{F}$ and pick $\mathsf{A} \in G_0$. There are two cases, as follows.

- If $\mathsf{A} \in Args \cap Args'$, then it is not attacked in $\mathcal{F}'$, because $Cn(Args') = Cn(Args)$.
- If $\mathsf{A} \in Args \setminus Args'$, then it uses some deduction(s) of the form $\Psi \vdash^{R} \psi$. Hence, there is a **corresponding** argument $\mathsf{A}' \in Args' \setminus Args$ (having $Cn(\mathsf{A}') = Cn(\mathsf{A})$) with (all) the deduction(s) $\Psi \vdash^{R} \psi$ replaced by the deduction $\{\psi\} \vdash^{\emptyset} \psi$. Note that A' cannot be attacked in $\mathcal{F}'$ on ψ, since $\overline{\psi} = y$ is new to the language.

In any case, we get that $Cn(\mathsf{A}) \in Cn(G')$.

For the inductive step, let $G_{i+1} \subseteq G$ be the set of arguments that are attacked in $\mathcal{F}$ but defended by $G_i \subseteq G$, where $Cn(G_i) \subseteq Cn(G')$ is assumed as an induction hypothesis. Pick $\mathsf{A} \in G_{i+1}$, if any. We split into cases.

- If $\mathsf{A} \in Args \cap Args'$, then it is defended by G_i in $\mathcal{F}$. So, on the one hand, G' defends A in $\mathcal{F}$ too, as $Cn(G_i) \subseteq Cn(G')$. On the other hand, if $\mathsf{C}' \in Args' \setminus Args$ attacks A in $\mathcal{F}'$ and is not attacked by G', then a **counterpart** argument $\mathsf{C} \in Args \setminus Args'$ (which uses some fixed deduction $B_j \vdash^{R_j} \psi$ instead of $\{\psi\} \vdash^{\emptyset} \psi$) attacks A in $\mathcal{F}$ and is not attacked by G_i (because $Cn(G_i) \subseteq Cn(G')$ and $B_j \subseteq asm(G)$), which is a contradiction.

[1] The modification of the rules in $\mathcal{F}'$ is required to preserve flatness. We also slightly abuse the notation by using $\overline{}$ for both contrary mappings: the implicit presumption is that the original contrary mapping $\overline{}$ is extended with the assignment $\overline{\psi} = y$, where y is new to $\mathcal{L}$.

- Else, if $\mathsf{A} \in \mathit{Args} \setminus \mathit{Args}'$, then like in the basis case, a corresponding argument $\mathsf{A}' \in \mathit{Args} \setminus \mathit{Args}$ (with deduction(s) $\Psi \vdash^R \psi$ replaced by the deduction $\{\psi\} \vdash^\emptyset \psi$) satisfying $\mathit{Cn}(\mathsf{A}') = \mathit{Cn}(\mathsf{A})$ is defended in $\mathcal{F}'$ by G' (as $\mathit{asm}(\mathsf{A}') \setminus \{\psi\} \subseteq \mathit{asm}(\mathsf{A})$, G_i defends A in $\mathcal{F}$, $\mathit{Cn}(G_i) \subseteq \mathit{Cn}(G')$ and $\overline{\psi} = y$ is new).

In any event, $\mathit{Cn}(G_{i+1}) \subseteq \mathit{Cn}(G')$.

By induction, $\mathit{Cn}(G) \subseteq \mathit{Cn}(G')$, as required for `DEF MON`.

For the satisfaction of `DEF CUT` under grounded semantics, we next show that $\mathit{Cn}(G' \setminus G) \subseteq \mathit{Cn}(G)$ holds by induction on the construction of G'.

For the basis step, let $G'_0 \subseteq G'$ be the set of arguments from Args' that are not attacked in $\mathcal{F}'$, and pick $\mathsf{A}' \in G'_0$, if any.

- If $\mathsf{A}' \in \mathit{Args}' \cap \mathit{Args}$, then it is not attacked in $\mathcal{F}$ either, so $\mathsf{A} \in G$.
- If $\mathsf{A}' \in \mathit{Args}' \setminus \mathit{Args}$, then a counterpart argument $\mathsf{A} \in \mathit{Args} \setminus \mathit{Args}'$ (having $\mathit{Cn}(\mathsf{A}) = \mathit{Cn}(\mathsf{A}')$ and every occurrence of the deduction $\{\psi\} \vdash^\emptyset \psi$ in A' replaced by some deduction $B_j \vdash^{R_j} \psi$ in A) is defended by G in $\mathcal{F}$, because $\mathit{Cn}(\mathit{Args}) = \mathit{Cn}(\mathit{Args}')$ (so that A cannot be attacked in $\mathcal{F}$ on $\mathit{asm}(\mathsf{A}) \setminus B_j$) and $B_j \subseteq \mathit{asm}(G)$ (so that G defends A in $\mathcal{F}$ from attacks on B_j).

In any case, $\mathit{Cn}(\mathsf{A}') \in \mathit{Cn}(G)$, and so $\mathit{Cn}(G'_0) \subseteq \mathit{Cn}(G)$.

For the inductive step, let $G'_{i+1} \subseteq G'$ be the set of arguments from Args' that are attacked in $\mathcal{F}'$ but defended by G'_i, where $\mathit{Cn}(G'_i) \subseteq \mathit{Cn}(G)$. Pick $\mathsf{A}' \in G'_{i+1}$, if any.

- If $\mathsf{A}' \in \mathit{Args}' \cap \mathit{Args}$, then G defends it in $\mathcal{F}$.
- If $\mathsf{A}' \in \mathit{Args}' \setminus \mathit{Args}$, then a counterpart argument $\mathsf{A} \in \mathit{Args} \setminus \mathit{Args}'$ can be attacked in $\mathcal{F}$ in two ways:
  - either on some $\beta \in B_j$: such attacks G defends against;
  - or on some $\alpha \in \mathit{asm}(\mathsf{A}) \setminus B_j$, in which case A' is attacked in $\mathcal{F}'$ (on α), and so defended by G'_i, so that G defends A in $\mathcal{F}$.

  In any event, $\mathit{Cn}(\mathsf{A}') \in \mathit{Cn}(G)$, and so $\mathit{Cn}(G'_{i+1}) \subseteq \mathit{Cn}(G)$.

By induction, $\mathit{Cn}(G') \subseteq \mathit{Cn}(G)$, as required for `DEF CUT`. □

Proposition 8. *Complete semantics satisfies* `WEAK DEF CUT` *and* `WEAK DEF MON`*.*

Proof. We show for every complete extension E of $\mathcal{F}$, for each $\psi \in \mathit{Cn}(E) \setminus \mathcal{A}$, there is a complete extension E' of $\mathcal{F}' = (\mathcal{L} \cup \{y\}, \mathcal{R} \setminus \{r \in \mathcal{R} : \text{ head of } r \text{ is } \psi\}, \mathcal{A} \cup \{\psi\}, \overline{})$ (where $\overline{\psi} = y$) such that $\mathit{Cn}(E') = \mathit{Cn}(E)$.

Let E be a complete extension of $\mathcal{F}$ and fix $\psi \in \mathit{Cn}(E) \setminus \mathcal{A}$ (assuming again that $\mathsf{B}_1 : B_1 \vdash \psi, \ \ldots, \ \mathsf{B}_n : B_n \vdash \psi \in E$ are all the arguments in E concluding ψ). Now, $\mathit{Args}' \setminus \mathit{Args}$ consists of arguments $\mathsf{A}' : A' \vdash \varphi$ constructed from the corresponding arguments $\mathsf{A} : A \vdash \varphi \in \mathit{Args}$ that use some deduction $\Psi \vdash^R \psi$. Let E^+ be the set of all such arguments A' for which $\mathsf{A} \in E$, and put $E' = (E \cap \mathit{Args}') \cup E^+$ (note that the argument $\{\psi\} \vdash \psi$ is in E' too, because $\mathsf{B}_j \in E$ for all j). Then $\mathit{Cn}(E) = \mathit{Cn}(E')$, so it suffices to prove that such E' is a complete extension of $\mathcal{F}'$.

- First, E' is conflict-free, because $\mathit{Cn}(E^+) \subseteq \mathit{Cn}(E)$ and $\overline{\psi} = y \notin \mathcal{L}$.
- Second, E' defends itself. Indeed, any $\mathsf{C} \in \mathit{Args} \cap \mathit{Args}'$ that attacks E' in $\mathcal{F}$ on some $\alpha \in \mathit{asm}(E') \setminus \{\psi\} \subseteq \mathit{asm}(E)$ is attacked by E', because $\mathit{Cn}(E') = \mathit{Cn}(E)$ and E is complete. On the other hand, if $\mathsf{C}' \in \mathit{Args}' \setminus \mathit{Args}$ attacks E' in $\mathcal{F}'$, but $E' \not\rightsquigarrow' \mathsf{C}'$, then a counterpart argument C with $\mathit{Cn}(\mathsf{C}) = \mathit{Cn}(\mathsf{C}')$ and some deduction $B_j \vdash^{R_j} \psi$ replacing (all) the deduction(s) $\{\psi\} \vdash^\emptyset \psi$ attacks E in $\mathcal{F}$, and we have $E \not\rightsquigarrow \mathsf{C}$ (because $B_j \subseteq \mathit{asm}(E)$), contradicting admissibility of E.

– Finally, E' is complete. For suppose towards a contradiction that E' defends some $\mathsf{A}' \in \mathit{Args}' \setminus \mathit{Args}$, but $\mathsf{A}' \notin E^+$ (as in the proof of Proposition 3, we do not consider $\mathsf{A}' \in \mathit{Args}$, for it would be defended by E and hence would belong to E). Consider thus a corresponding argument $\mathsf{A} \in \mathit{Args}$ of A'. Then there are two cases.
  - Either A has some deduction(s) $B_j \vdash^{R_j} \psi$ replacing (all) the deduction(s) $\{\psi\} \vdash^{\emptyset} \psi$ (so A is also a counterpart of A') and $\mathsf{A} \notin E$, in which case A is not defended by E against some attack $\mathsf{C} \rightsquigarrow \mathsf{A}$. As E defends B_j (for all j), we have $\mathsf{C}' \rightsquigarrow' \mathsf{A}'$, for a counterpart C' of C. But as $E \not\rightsquigarrow \mathsf{C}$ and $\psi \in \mathit{asm}(E')$, we get $E' \not\rightsquigarrow' \mathsf{C}'$, which is a contradiction to A' being defended by E'.
  - Or else, A uses deduction(s) of the form $\Psi \vdash^{R} \psi$, where $\Psi \neq B_j$ for any j. But then $E \rightsquigarrow \mathsf{A}$, and so E', being conflict-free, cannot defend A.

  We obtain a contradiction in any case, so that $\mathsf{A}' \in E^+$ after all.

Consequently, E', as defined above, is the required complete extension. □

Like with Proposition 4 and Corollary 5 (resp.), we have the following results.

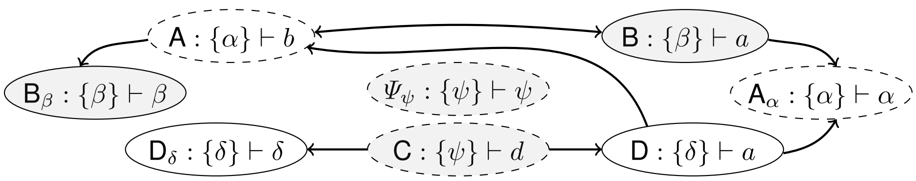
Proposition 9. *Preferred and stable semantics satisfy* `WEAK DEF CUT` *and* `WEAK DEF MON`.

Proof. The proof is *verbatim* to the proof of Proposition 4, with $E' = (E \cap \mathit{Args}') \cup E^+$ as in the proof of Proposition 8. □

Corollary 10. *Sceptically preferred and ideal semantics satisfy* `DEF CUT`.

The following example exhibits a violation of both `DEF CUT` and `DEF MON` under the remaining semantics.

Example 11 (`DEF CUT` *and* `DEF MON` *violations. Based on Example 6).*
Suppose that instead of relying on Ben about the restaurant (remove $\psi \leftarrow \beta$), you guess it to be a gourmet place to begin with (add ψ to assumptions). Reason then according to $(\mathcal{L} \cup \{y\}, \mathcal{R} \setminus \{\psi \leftarrow \beta\}, \mathcal{A} \cup \{\psi\}, \overline{})$ (where $\overline{\psi} = y$), with $(\mathit{Args}', \rightsquigarrow')$ as follows:



There are two preferred extensions (which are also stable): $E'_1 = \{\mathsf{B}, \mathsf{B}_\beta, \mathsf{C}, \Psi_\psi\}$ (gray) and $E'_2 = \{\mathsf{A}_\alpha, \mathsf{A}, \mathsf{C}, \Psi_\psi\}$ (dashed). The sceptically preferred (also ideal) extension is $E' = \{\mathsf{C}, \Psi_\psi\}$ with $\mathit{Cn}(E') \not\supseteq \{a, \beta, \psi, d\} = \mathit{Cn}(E)$, where E is as in Example 6. So `DEF MON` fails under both sceptically preferred and ideal semantics. `DEF CUT` and `DEF MON` fail in credulous reasoning, as $\mathit{Cn}(E) \not\subseteq \mathit{Cn}(E'_2) \not\subseteq \mathit{Cn}(E)$.

We see that even when starting with a unique credulous extension, assuming a previously defeasibly inferred conclusion opens up space for multiple credulous extensions. This may be desirable in situations where revision of decisions based on defeasible assumptions (β in Example 11) is important. At the same time, such behaviour results into possibly losing conclusions in sceptical reasoning (except, as before, under grounded semantics). This nevertheless may be sensible, if, for instance, differentiating defeasible information is needed (e.g. ψ versus $\psi \leftarrow \beta$).

Below is a summary of results in this subsection (using the same notational conventions as at the end of Sect. 3.1).

`DEFEASIBLE` Cumulative Transitivity and Cautious Monotonicity

Property	Grounded	Ideal	Sceptically pref.	Stable	Preferred	Complete
`DEF CUT`	✓	✓	✓	X (✓)	X (✓)	X (✓)
`DEF MON`	✓	X	X	X (✓)	X (✓)	X (✓)

Conclusions drawn using grounded semantics can be safely turned into assumptions and inferred defeasibly instead. However, such a change would not allow for new conclusions under the other two sceptical semantics, yet could lead to a decision vacuum: neither of α, β, δ belongs to $Cn(E')$ in Example 11. Credulous semantics, meanwhile, allow for greater dynamicity, which could be desirable: if independently from what Ben says a reasoner believes to be in a gourmet place and thus does not care about Dan, then Al can be as likely a choice as Ben, and so the conclusions may need revision.

Naturally, somewhat different formulations of the properties in the defeasible setting could be investigated. For example, the contrary of the new assumption ψ could instead be one of the existing symbols in $\mathcal{L}$, based on the rules and contraries of the assumptions that allowed to derive ψ in the first place. However, such behaviour need not be desirable in general: if you assume to begin with that you are about to dine in a gourmet place, then, arguably, this assumption should not be contingent on the objections against Ben. We chose the formulation above, readily applicable to all ABA frameworks, as the first step in our analysis. Different and more complex settings are left for future work.

3.3 Assumption Cumulative Transitivity and Cautious Monotonicity

Previously discussed properties focused on non-assumption conclusions. We now turn to conclusions that are also assumptions, as follows. Given $\psi \in Cn(E) \cap \mathcal{A}$, define $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\psi \leftarrow \top\}, \mathcal{A} \setminus \{\psi\}, \overline{})$.[2] Then

`STRONG ASM CUT`: For all extensions E' of $\mathcal{F}'$ we have $Cn(E') \subseteq Cn(E)$;

`WEAK ASM CUT`: There is an extension E' of $\mathcal{F}'$ with $Cn(E') \subseteq Cn(E)$;

`STRONG ASM MON`: For all extensions E' of $\mathcal{F}'$ we have $Cn(E) \subseteq Cn(E')$;

`WEAK ASM MON`: There is an extension E' of $\mathcal{F}'$ with $Cn(E) \subseteq Cn(E')$.

`ASM CUT` and `ASM MON` focus on previously accepted assumptions *being confirmed* and made into facts to reason again. For instance, you might have guessed that you are in a gourmet restaurant, and after deciding whom to sit with you may check the menu to confirm your guess and scrutinize your decision.

As for satisfaction of the properties, the same results (as in Sect. 3.1, 3.2) hold with proofs following the same pattern.

[2] Again, for brevity reasons, the same symbol $\overline{}$ is used for both contrary mappings: in $\mathcal{F}'$, the original contrary mapping $\overline{}$ is implicitly restricted to a diminished set of assumptions.

Proposition 12. *Grounded semantics satisfies* `ASM CUT` *and* `ASM MON`*.*

Proof. Let G be the grounded extension of $\mathcal{F}$. If $G = \emptyset$, then $\mathcal{F}' = \mathcal{F}$, so the properties are trivially satisfied. Otherwise, pick $\psi \in Cn(G) \cap \mathcal{A}$ and let G' be the grounded extension of $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\psi \leftarrow \top\}, \mathcal{A} \setminus \{\psi\}, \overline{})$.

First show $Cn(G) \subseteq Cn(G')$ by induction on the construction of G.

For the basis step, let $G_0 \subseteq G$ be the set of arguments that are not attacked in $\mathcal{F}$. Pick $\mathsf{A} \in G_0$, if any. We split into two cases.

- If $\psi \notin asm(\mathsf{A})$, then A remains unattacked in $\mathcal{F}'$. Hence $\mathsf{A} \in G'$.
- Otherwise, if $\psi \in asm(\mathsf{A})$, then in $Args'$, A is replaced by its **counterpart** A' with $asm(\mathsf{A}) = asm(\mathsf{A}') \cup \{\psi\}$ and $Cn(\mathsf{A}') = Cn(\mathsf{A})$ (the deduction $\emptyset \vdash^{\{\psi \leftarrow \top\}} \psi$ replaces (all) the deduction(s) $\{\psi\} \vdash^{\emptyset} \psi$). Since there were no attacks against A in $\mathcal{F}$, the counterpart A' is unattacked in $\mathcal{F}'$ either. Hence, $\mathsf{A}' \in G'$.

In any case, we have $Cn(G_0) \subseteq Cn(G')$.

For the inductive step, let $G_{i+1} \subseteq G$ be the set of arguments that are attacked in $\mathcal{F}$ but defended by G_i, where $Cn(G_i) \subseteq Cn(G')$. Suppose that in $\mathcal{F}'$, an argument $\mathsf{A}' \in Args'$ attacks the set $G'_{i+1} \subseteq Args'$ of arguments which are obtained from G_{i+1} by replacing the assumption ψ with the rule $\psi \leftarrow \top$.[3] We split into cases.

- If $\mathsf{A}' \in Args$, then $G_i \rightsquigarrow \mathsf{A}'$, so that $G' \rightsquigarrow' \mathsf{A}'$ too.
- Otherwise, if $\mathsf{A}' \notin Args$, then A' is constructed from the counterpart $\mathsf{A} \in Args$ such that $\mathsf{A} \rightsquigarrow G_{i+1}$. Now, if $G' \not\rightsquigarrow' \mathsf{A}'$, it means that $G_i \rightsquigarrow \mathsf{A}$ on ψ. This effectively yields $G \rightsquigarrow G$, contradicting conflict-freeness of G. Hence, $G' \rightsquigarrow' \mathsf{A}'$.

Thus, $Cn(G_{i+1}) \subseteq Cn(G')$, and so $Cn(G) \subseteq Cn(G')$ by induction, as required.

To show `ASM CUT` holds under grounded semantics, prove $Cn(G') \subseteq Cn(G)$ by induction on the construction of G'.

For the basis step, let $G'_0 \subseteq G'$ be the set of arguments that are not attacked in $\mathcal{F}'$, and pick $\mathsf{A}' \in G'_0$, if any. We split into cases.

- If $\mathsf{A}' \in Args$, then A' can be attacked in $\mathcal{F}$ only on ψ. But since $\psi \in Cn(G)$, we would then have A' defended by G, so that $\mathsf{A}' \in G$.
- Otherwise, if $\mathsf{A}' \notin Args$, then the counterpart $\mathsf{A} \in Args$ can be attacked in $\mathcal{F}$ only on $\psi \in Cn(G)$, and so is defended by G.

In any case, $Cn(\mathsf{A}') \in Cn(G)$ holds true.

For the inductive step, let $G'_{i+1} \subseteq G'$ be the set of arguments attacked in $\mathcal{F}'$ but defended by G'_i, where $Cn(G'_i) \subseteq Cn(G)$. Pick $\mathsf{A}' \in G'_{i+1}$, if any. We split into cases.

- If $\mathsf{A}' \in Args$, then A' can be attacked in $\mathcal{F}$ either on any $\alpha \in asm(\mathsf{A}') \setminus \{\psi\}$, or on ψ itself. Consider each case separately.
  - Suppose first that $\mathsf{B} \rightsquigarrow \mathsf{A}'$ on some $\alpha \in asm(\mathsf{A}') \setminus \{\psi\}$. Then either B or its counterpart $\mathsf{B}' \in Args'$ (if such can possibly be obtained from B) attacks A' in $\mathcal{F}'$ on α. In any event, G'_i defends against this attack, and since it holds that $Cn(G'_i) \subseteq Cn(G)$ by induction hypothesis, we get either $G \rightsquigarrow \mathsf{B}$, or $G \rightsquigarrow \mathsf{B}'$.
  - In the latter case, if $\mathsf{B} \rightsquigarrow \mathsf{A}'$ on ψ, then since $\psi \in Cn(G)$, we have $G \rightsquigarrow \mathsf{B}$.

  In any event $\mathsf{A}' \in G$.

[3] Deduction(s) $\Phi \vdash^{R} \varphi$ with $\psi \in \Phi$ are replaced with the deduction(s) $\Phi \setminus \{\psi\} \vdash^{R' \cup \{\psi \leftarrow \top\}} \varphi$ such that $R' \subseteq R$ is the set of rules from R that do not contain ψ in their bodies.

– Otherwise, suppose $\mathsf{A}' \notin \mathit{Args}$. Then consider its counterpart $\mathsf{A} \in \mathit{Args}$ and assume $\mathsf{B} \rightsquigarrow \mathsf{A}$ on some $\alpha \in \mathit{asm}(\mathsf{A})$. Then, like before:
  - either $\alpha = \psi$, in which case $G \rightsquigarrow \mathsf{B}$, so that $\mathsf{A} \in G$;
  - or $\alpha \neq \psi$, whence either B (or its counterpart $\mathsf{B}' \in \mathit{Args}'$) attacks A' in $\mathcal{F}'$, but as G'_i defends against this attack, we get $G \rightsquigarrow \mathsf{B}$ (or $G \rightsquigarrow \mathsf{B}'$), and so $\mathsf{A} \in G$.

Consequently, $\mathit{Cn}(\mathsf{A}') \in \mathit{Cn}(G)$, and by induction, $\mathit{Cn}(G') \subseteq \mathit{Cn}(G)$, as required. □

Proposition 13. *Complete semantics satisfies* `WEAK ASM CUT` *and* `WEAK ASM MON`.

Proof. Show for every complete extension E of $\mathcal{F}$, for each $\psi \in \mathit{Cn}(E) \setminus \mathcal{A}$, there is a complete extension E' of $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\psi \leftarrow \top\}, \mathcal{A} \setminus \{\psi\}, \bar{\ })$ with $\mathit{Cn}(E') = \mathit{Cn}(E)$.

Let E be a complete extension of $\mathcal{F}$. Now, $\mathit{Args}' \setminus \mathit{Args}$ consists of arguments A' that are counterpart to $\mathsf{A} \in \mathit{Args}$ with $\psi \in \mathit{asm}(\mathsf{A})$. Let $E^- \subseteq E$ be the set of arguments from E that use the assumption ψ and let $E^+ \subseteq \mathit{Args}'$ be the set of all the counterparts of arguments in E^-. Put $E' = (E \setminus E^-) \cup E^+$. The following then hold.

– E' is conflict-free, because $\mathit{Cn}(E^+) \subseteq \mathit{Cn}(E)$.
– E' defends itself: if $\mathsf{A}' \in \mathit{Args}' \setminus \mathit{Args}$ attacks E' in $\mathcal{F}'$, but $E' \not\rightsquigarrow' \mathsf{A}'$, then the counterpart argument $\mathsf{A} \in \mathit{Args}$ attacks E; yet, $E \not\rightsquigarrow \mathsf{A}$ (because $\psi \in \mathit{Cn}(E)$), contradicting admissibility of E.
– E' is complete. Suppose for a contradiction that E' defends $\mathsf{A}' \in \mathit{Args}' \setminus \mathit{Args}$, but $\mathsf{A}' \notin E^+$ (as in the proof of Proposition 3, we do not consider $\mathsf{A}' \in \mathit{Args}$). Then the counterpart argument $\mathsf{A} \in \mathit{Args}$ of A' does not belong to E, and hence is not defended by E against some attack $\mathsf{C} \rightsquigarrow \mathsf{A}$. As $\psi \in \mathit{Cn}(E)$, we have $\mathsf{C}' \rightsquigarrow' \mathsf{A}'$, for the counterpart C' of C. But since $E \not\rightsquigarrow \mathsf{C}$ and $\psi \in \mathit{Cn}(E')$, we get $E' \not\rightsquigarrow' \mathsf{C}'$, which is a contradiction to E' defending A'.

Then $\mathit{Cn}(E') = \mathit{Cn}(E)$ yields that E' is the required complete extension of $\mathcal{F}'$. □

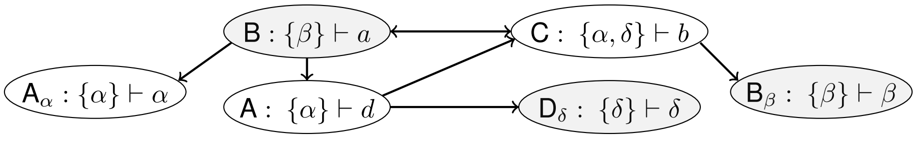
The next two results follow from the ones above, as with the other properties.

Proposition 14. *Preferred and stable semantics satisfy* `WEAK ASM CUT` *and* `WEAK MON`.

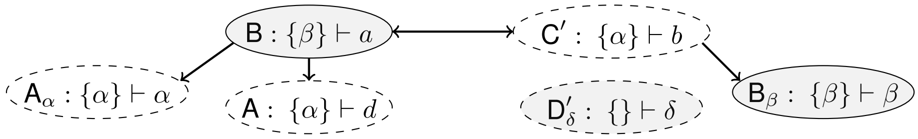
Corollary 15. *Sceptically preferred and ideal semantics satisfy* `ASM CUT`.

To show that the properties are violated under the remaining semantics, we consider a situation where, in contrast to Examples 6 and 11, one argument depends on two assumptions, one of which is to be turned into a fact, as follows.

Example 16 *(*`ASM CUT` *and* `ASM MON` *violations).*
Consider $\mathcal{R} = \{d \leftarrow \alpha,\ a \leftarrow \beta,\ b \leftarrow \alpha, \delta\}$ with $\overline{\alpha} = a,\ \overline{\beta} = b,\ \overline{\delta} = d$. This yields the following $(\mathit{Args}, \rightsquigarrow)$:



Here, $E = \{\mathsf{B}, \mathsf{B}_\beta, \mathsf{D}_\delta\}$ (gray) is a unique preferred (also stable and ideal) extension of $(\mathit{Args}, \rightsquigarrow)$. Taking $\delta \in \mathit{Cn}(E) \cap \mathcal{A}$ results in $\mathcal{F}' = (\mathcal{L}, \mathcal{R} \cup \{\delta \leftarrow \top\}, \mathcal{A} \setminus \{\delta\}, \bar{\ })$ in which C and D_δ are replaced by their counterparts $\mathsf{C}' : \{\alpha\} \vdash b$ and $\mathsf{D}'_\delta : \{\} \vdash \delta$:



Therefore, $\mathcal{F}'$ admits two preferred extensions: $E_1' = \{\mathsf{B}, \mathsf{B}_\beta, \mathsf{D}_\delta'\}$ (gray) and $E_2' = \{\mathsf{A}_\alpha, \mathsf{C}', \mathsf{D}_\delta', \mathsf{A}\}$ (dashed) with $Cn(E) \nsubseteq Cn(E_2') \nsubseteq Cn(E)$. The sceptically preferred and ideal extension is $E' = \{\mathsf{D}_\delta'\}$ with $Cn(E) \nsubseteq Cn(E')$.

Compared to sceptical semantics, credulous ones are more dynamic. Here, confirming δ results in retracting β (as well as a) under both ideal and sceptically preferred semantics. Meanwhile, the same change effectively removes A's attack on C, still leaving C defeasible, yet rendering B to lose its position as the sole defender against A, hence enabling mutual acceptability of α and δ, under, say, complete semantics. This allows for a possibly desirable revision of conclusions.

The following is a summary of this subsection's results (notation as before).

`ASSUMPTION` Cumulative Transitivity and Cautious Monotonicity

Property	Grounded	Ideal	Sceptically pref.	Stable	Preferred	Complete
`ASM CUT`	✓	✓	✓	X (✓)	X (✓)	X (✓)
`ASM MON`	✓	X	X	X (✓)	X (✓)	X (✓)

Confirmation of some defeasible information can lead to an increased number of options in credulous reasoning. This could be desirable if, for instance, one of the choices (like C with conclusion b in Example 16) depends on an assumption (δ) and is not considered acceptable to begin with (C has no defense against A), but becomes viable (via C′) as soon as the assumption is confirmed ($\delta \leftarrow \top$) and ceases to be questioned (D_δ'). Meanwhile, if confirming information widens the array of credulous choices, then a sceptical reasoner could opt for fewer—more certain—conclusions, as witnessed by the sceptical (bar grounded) semantics satisfying `ASM CUT` but failing `ASM MON`.

4 Related Work

The two most related works to ours are Hunter's [23] and Dung's [17]. The former investigates non-monotonic inference properties with respect to argument–claim entailment in logic-based argumentation systems. Given various base logics, Hunter defines argument construction-mimicking entailment operators to produce claims from knowledge bases, and examines those operators against non-monotonic inference properties (Cumulative Transitivity and Cautious Monotonicity among them). Meanwhile, Dung analyses, among other aspects of argumentation dynamics, Cumulativity (i.e. Cumulative Transitivity plus Cautious Monotonicity) of ASPIC$^+$ under stable extension semantics. The main concern there is that confirmation of some conclusions in an extension should strengthen other conclusions in that extension. To formalize this, Dung introduces two axioms—a variant of Cumulativity and another one regarding attack monotonicity. Stable extension semantics with respect to either of the main four ASPIC$^+$ attack relations are shown not to satisfy at least one of those axioms.

Other related works can be seen to fall under two broad research topics in argumentation: (i) analysing desirable properties of argumentation formalisms, and (ii) relating belief change and argumentation. Regarding (i), with the exceptions of [17] and [23], existing works on properties of argumentation disregard the issues of argumentation dynamics: for example, [12] propose rationality postulates for rule-based argumentation systems; [19] provide guidelines for argumentation-based practical reasoning; [22] postulate and examine properties of attack relations (and the corresponding extensions under alternative semantics) over classical logic–based argument graphs. As far as (ii) is concerned, argumentation dynamics has recently been studied with respect to Abstract Argumentation [16] and some other argumentation-based approaches to non-monotonic reasoning, such as DeLP [21] (see e.g. [3, 8, 13, 14, 20]). To the best of our knowledge, [17] is the only work in the direction of investigating structured, extension-based argumentation with regards to non-monotonic inference properties *á la* [24].

Our work differs from [17] in several aspects. First, we consider Cumulative Transitivity and Cautious Monotonicity as two separate properties, rather than one. Also, our reformulations of the properties are not restricted to one particular semantics (stable), but allow for any semantics. Still further, we consider three types of information change, including strengthening (`STRICT`) and confirmation (`ASM`), and analyse their influence to argumentation processes in ABA. Finally, we do not insist that properties have to be necessarily fulfilled, but maintain that their satisfaction is conditional on applications.

5 Conclusions

This paper researches extension-based structured argumentation dynamics in the spirit of non-monotonic inference properties of [24, 25]. To this end, we offer reformulations of non-monotonic inference properties in terms of extensions. Particularly, we introduce (strong and weak versions of) six properties applicable to the well-known structured argumentation formalism Assumption-Based Argumentation (ABA) and investigate their satisfaction under six key ABA semantics. Three pairs of properties reflect different modifications of knowledge in ABA frameworks, and each item of a pair concerns either Cumulative Transitivity (`CUT`) or Cautious Monotonicity (`MON`) of extension-based non-monotonic inference. While conceptually the three types of information change are different, we show that technically they lead to the same outcomes in the sense of a property being satisfied in either all or none of the three settings, under a particular semantics. Consequently, irrespective of the knowledge representation in ABA and the nature of the anticipated changes in information, one can choose semantics best suited for the application, depending on the desirable properties of the reasoner.

Credulous semantics violate the strong properties. This is expected, due to presence of choice between extensions that share conclusions. Meanwhile, the weak properties are satisfied under credulous semantics. This essentially says that ABA frameworks do not lose the extension based on which a change in knowledge occurs. As for further results on credulous reasoning, we can also identify a certain provocative aspect of our findings: even when a stable/preferred extension to begin with is unique, changing (even strengthening) information in ABA can lead to more than one stable/preferred extension afterwards (Examples 6, 11, 16). We believe this phenomenon deserves further study in terms of characterization of ABA frameworks and/or semantics for which it occurs.

In terms of sceptical reasoning, intuitively, the most sceptical (grounded) semantics satisfies all the properties. This is because grounded extensions commit to the most certain conclusions to begin with, and changing the way they are represented in ABA frameworks does not influence their (and other arguments') acceptance. Somewhat surprisingly, the other two sceptical semantics—sceptically preferred and ideal—fail MON, yet fulfill CUT. Such a behaviour is present because changes in information can increase the number of, particularly, preferred extensions, whence their intersection shrinks, resulting in violation of MON, at the same time satisfying CUT.

The results can serve as guidelines regarding argumentation dynamics for modeling common-sense reasoning using ABA. Due to the same property satisfaction outcomes, irrespective of knowledge representation in ABA, one has a range of differently behaving semantics to choose among, contingent on the intended behaviour of the reasoner. Depending on application, one may wish to rely on the static grounded semantics to prevent overwhelming changes in reasoning, or use a much more dynamic credulous semantics to be flexible about revising decisions.

This work serves as one of the first steps towards investigating extension-based structured argumentation dynamics. Current results cover ABA, and hence (by virtue of results in [27]) ASPIC$^+$ without preferences, with regards to CUT and MON. Future work directions include different formulations of the properties, as well as analysis of extension-based formalisms of argumentation with preferences against variants of the non-monotonic inference properties in question. As to the latter, ABA Equipped with Preferences (known as p_ABA [31]) is of particular interest, as well as other formalisms, such as ASPIC$^+$, Value-Based Argumentation [4] or PAFs [1]. It may also be possible to use the abstract formulations of the properties to analyse other non-monotonic reasoning formalisms, such as default logic and logic programming (see e.g. [9, 10]), from a slightly different perspective than in the existing work (e.g. [6, 11, 15]).

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