CO405H

Computing in Space with OpenSPL
Topic 10: Correlation Example

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http://www.doc.ic.ac.uk/~oskar/
http://www.doc.ic.ac.uk/~georgig/

CO405H course page: http://cc.doc.ic.ac.uk/openspl15/
WebIDE: http://openspl.doc.ic.ac.uk
OpenSPL consortium page: http://www.openspl.org

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This Lecture: Correlation Example

Let's create a spatial implementation of correlation:

$$r_{xy} = \frac{\sum^{n} x_i y_i - n\bar{x}\bar{y}}{(n-1)s_x s_y} = \frac{n\sum^{n} x_i y_i - \sum^{n} x_i \sum^{n} y_i}{\sqrt{n\sum^{n} x_i^2 - (\sum^{n} x_i)^2}\ \sqrt{n\sum^{n} y_i^2 - (\sum^{n} y_i)^2}}.$$

src: http://en.wikipedia.org/wiki/Correlation_and_dependence

r is the correlation coefficient for a pair of data timeseries *x,y* over a window *n*

Our example computes correlation for *N*=200-6000 data timeseries and returns the top 10 correlations at any point in time.

DFE Input: Interleaved stream of N timeseries, plus precalculations from the CPU

Pearson product-moment correlation

- Product-moment correlation: Calculate mean (i.e. first moment) of product of mean-adjusted values:

$$\rho_{X,Y} = \frac{E\left[(X-\mu_X)(Y-\mu_Y)\right]}{\sigma_X \sigma_Y}$$

with

E	expected value
μ	mean value
σ	standard deviation

- For sample data, calculate as:

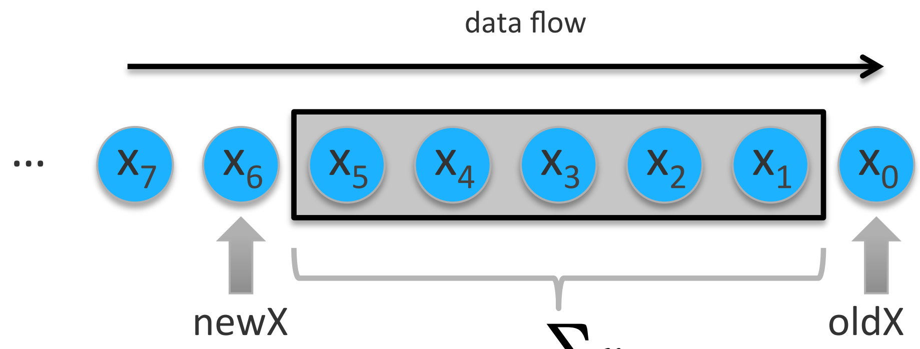
$$r_{x,y} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}} = \frac{n\sum_{i=1}^{n}x_i y_i - \sum_{i=1}^{n}x_i \sum_{i=1}^{n}y_i}{\sqrt{n\sum_{i=1}^{n}x_i^2 - \left(\sum_{i=1}^{n}x_i\right)^2}\sqrt{n\sum_{i=1}^{n}y_i^2 - \left(\sum_{i=1}^{n}y_i\right)^2}}$$

How to compute a correlation in practice

- Computation of correlation is based on 5 sums:

$$\text{sumXY: } \sum_n x_i y_i \quad \text{sumX: } \sum_n x_i \quad \text{sumY: } \sum_n y_i \quad \text{sumX2: } \left(\sum_n x_i\right)^2 \quad \text{sumY2: } \left(\sum_n y_i\right)^2$$

- Typical application involves continuous correlation of time series: use sliding window approach.



$$\sum_n x_i$$

No need to recompute entire sum, add next element and subtract previous:

```
sumX += newX – oldX;
```

For every time step, complexity of computing sums = O(1) regardless of window size!

Correlation between many time series

- Application in finance requires correlations between 200 to 6000 time series.
  - Complexity arises from number of time series *m* to be correlated, not from number of data elements *n* to be summed in sliding window.

O(m^2) sums_xy O(m) sums

O(m^2) correlations

$$r_{x,y} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}\sqrt{n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2}}$$

O(m) sums_sq

Original CPU Version for Correlation

```
for (uint64_t s=0; s<numTimesteps; s++) {
    index_correlation = 0;
    for (uint64_t i=0; i<numTimeseries; i++) {
        double old = (s>=windowSize ? data[i][s-windowSize] : 0);
        double new = data [i][s];
        sums[i] += new - old;
        sums_sq[i] += new*new - old*old; // start and end of window => let's call them data pairs
    }
    for (uint64_t i=0; i<numTimeseries; i++) {
        double old_x = (s>=windowSize ? data[i][s-windowSize] : 0);
        double new_x = data [i][s];
        for (uint64_t j=i+1; j<numTimeseries; j++) {
            double old_y = (s>=windowSize ? data[j][s-windowSize] : 0);
            double new_y = data [j][s];

            sums_xy[index_correlation] += new_x*new_y - old_x*old_y;
            correlations_step[index_correlation] = (windowSize*sums_xy[index_correlation]-sums[i]*sums[j])/
                                                   (sqrt(windowSize*sums_sq[i]sums[i]*sums[i])*
                                                   sqrt(windowSize*sums_sq[j]-sums[j]*sums[j]));
            indices_step[2*index_correlation] = j;
            indices_step[2*index_correlation+1] = i;
            index_correlation++;
        }
    }
```

Move to DFE LMEM?

Precompute?

Opportunities for acceleration 1

```
for (uint64_t s=0; s<numTimesteps; s++) {
    index_correlation = 0;
    for (uint64_t i=0; i<numTimeseries; i++) {
        double old = (s>=windowSize ? data[i][s-windowSize] : 0);
        double new = data[i][s];
        sums[i] += new - old;
        sums_sq[i] += new*new - old*old;
    }
    for (uint64_t i=0; i<numTimeseries; i++) {
        double old_x = (s>=windowSize ? data[i][s-windowSize] : 0);
        double new_x = data [i][s];
        for (uint64_t j=i+1; j<numTimeseries; j++) {
            double old_y = (s>=windowSize ? data[j][s-windowSize] : 0);
            double new_y = data[j][s];
            sums_xy[index_correlation] += new_x*new_y - old_x*old_y;
            correlations_step[index_correlation] = (windowSize*sums_xy[index_correlation] - sums[i]*sums[j]) /
(sqrt(windowSize*sums_sq[i] - sums[i]*sums[i])* sqrt(windowSize*sums_sq[j] -sums[j]*sums[j]));
            indices_step[2*index_correlation] = j;
            indices_step[2*index_correlation+1] = i;
            index_correlation++;
        }
    }
}
```

O(numTimeseries)

O(numTimeseries^2)

Opportunities for acceleration 2

```
for (uint64_t s=0; s<numTimesteps; s++) {
    index_correlation = 0;
    for (uint64_t i=0; i<numTimeseries; i++) {
        double old = (s>=windowSize ? data[i][s-windowSize] : 0);
        double new = data[i][s];
        sums[i] += new - old;
        sums_sq[i] += new*new - old*old;
    }
    for (uint64_t i=0; i<numTimeseries; i++) {
        double old_x = (s>=windowSize ? data[i][s-windowSize] : 0);
        double new_x = data [i][s];
        for (uint64_t j=i+1; j<numTimeseries; j++) {
            double old_y = (s>=windowSize ? data[j][s-windowSize] : 0);
            double new_y = data[j][s];
            sums_xy[index_correlation] += new_x*new_y - old_x*old_y;
            correlations_step[index_correlation] = (windowSize*sums_xy[index_correlation] - sums[i]*sums[j]) /
(sqrt(windowSize*sums_sq[i] - sums[i]*sums[i])* sqrt(windowSize*sums_sq[j] -sums[j]*sums[j]));
            indices_step[2*index_correlation] = j;
            indices_step[2*index_correlation+1] = i;
            index_correlation++;
        }
    }
}
```

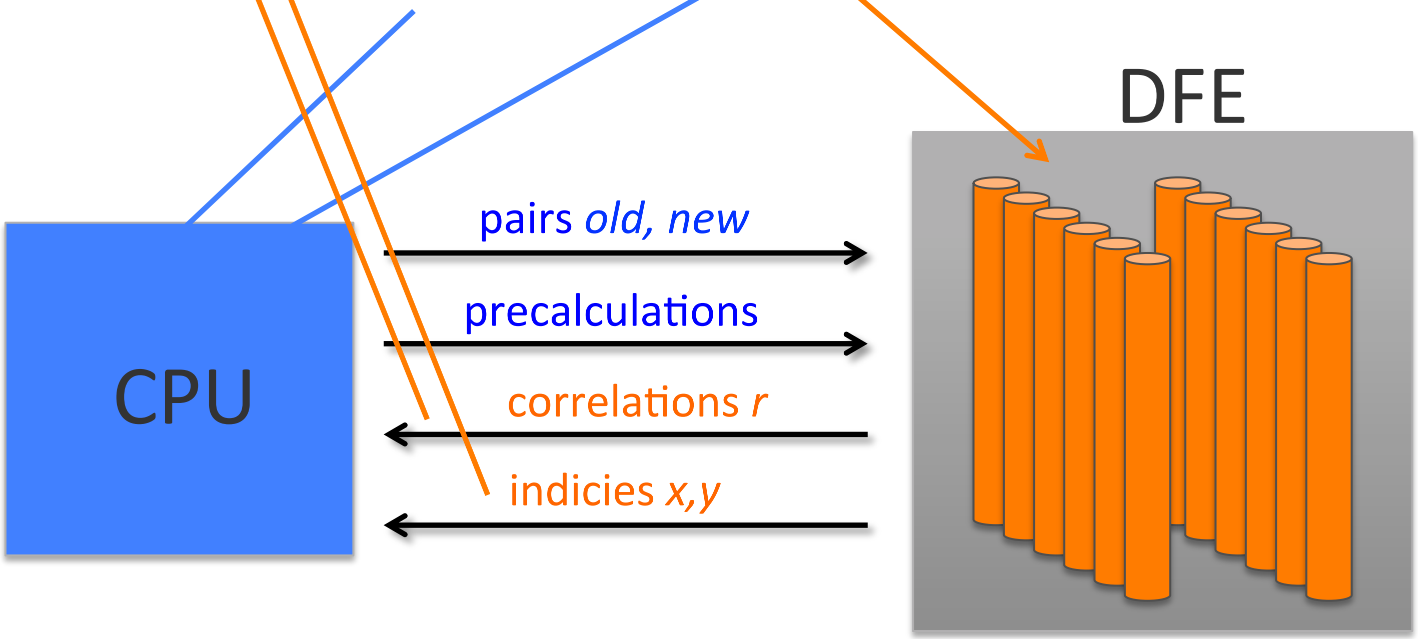
precompute

reorder data

split code

Split correlation on CPU and DFE

$$r_{x,y} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}\sqrt{n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2}}$$



12 pipes fill the chip

each pipe has a feedback loop of 12 ticks => 144 calculations running at the same time

DFE implementation

```
for (uint64_t s=0; s<numTimesteps; s++) {
    index_correlation = 0;
    for (uint64_t i=0; i<numTimeseries; i++) {
        double old = (s>=windowSize ? data[i][s-windowSize] : 0);
        double new = data[i][s];
        sums[i] += new - old;
        sums_sq[i] += new*new - old*old;
    }
    for (uint64_t i=0; i<numTimeseries; i++) {
        double old_x = (s>=windowSize ? data[i][s-windowSize] : 0);
        double new_x = data [i][s];
        for (uint64_t j=i+1; j<numTimeseries; j++) {
            double old_y = (s>=windowSize ? data[j][s-windowSize] : 0);
            double new_y = data[j][s];
            sums_xy[index_correlation] += new_x*new_y - old_x*old_y;
            correlations_step[index_correlation] = (windowSize*sums_xy[index_correlation] - sums[i]*sums[j]) /
(sqrt(windowSize*sums_sq[i] - sums[i]*sums[i])* sqrt(windowSize*sums_sq[j] -sums[j]*sums[j]));
            indices_step[2*index_correlation] = j;
            indices_step[2*index_correlation+1] = i;
            index_correlation++;
        }
    }
}
```

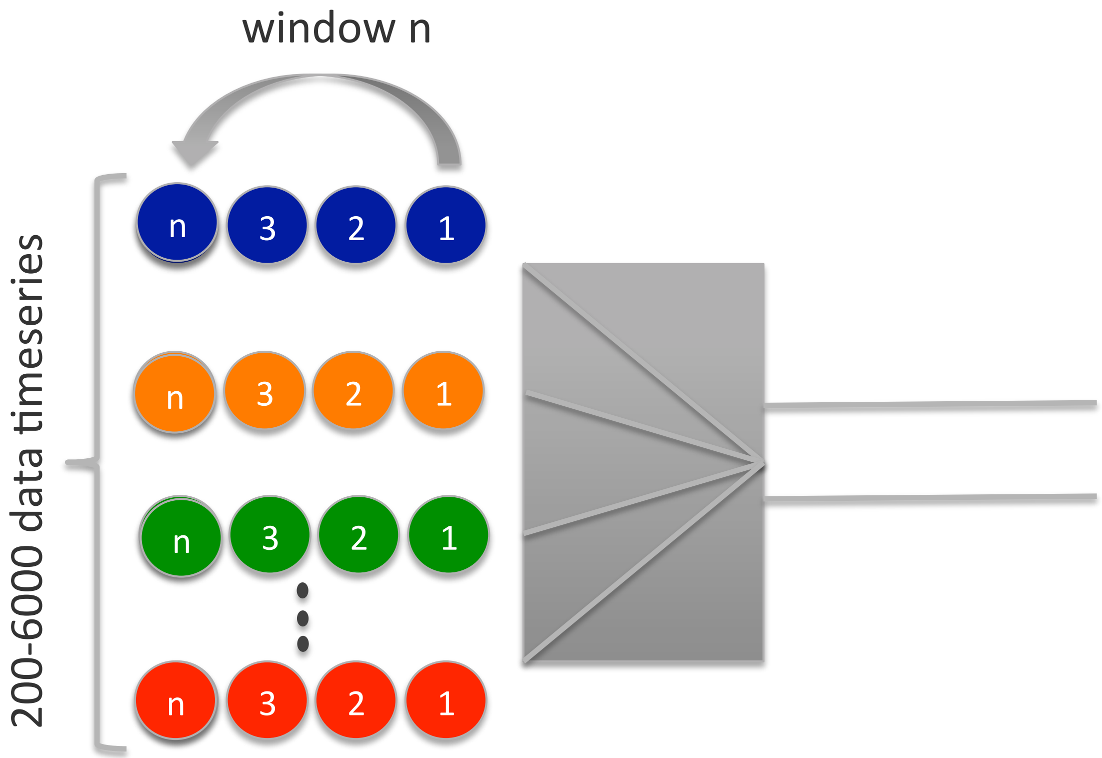
store pairs of *new* and *old* in LMEM

accelerate with DFE

precompute sums and inverse of sqrt on CPU

prepare_data_for_dfe() – The Movie

reordering for data_pairs

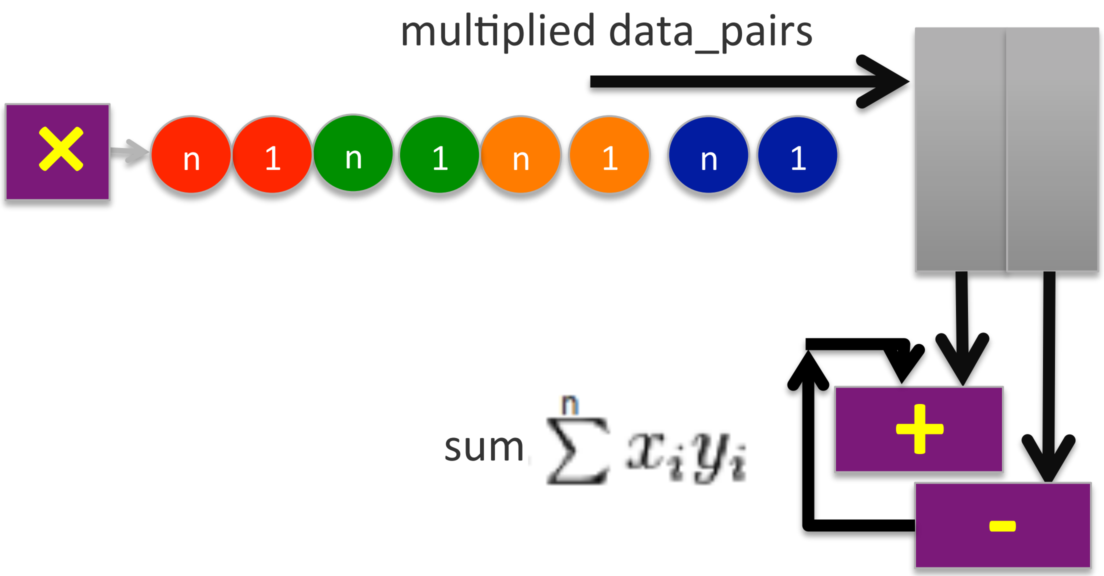


first and last element of each window creates a data_pair, then go round robin…

Example: Sum of a Moving Window

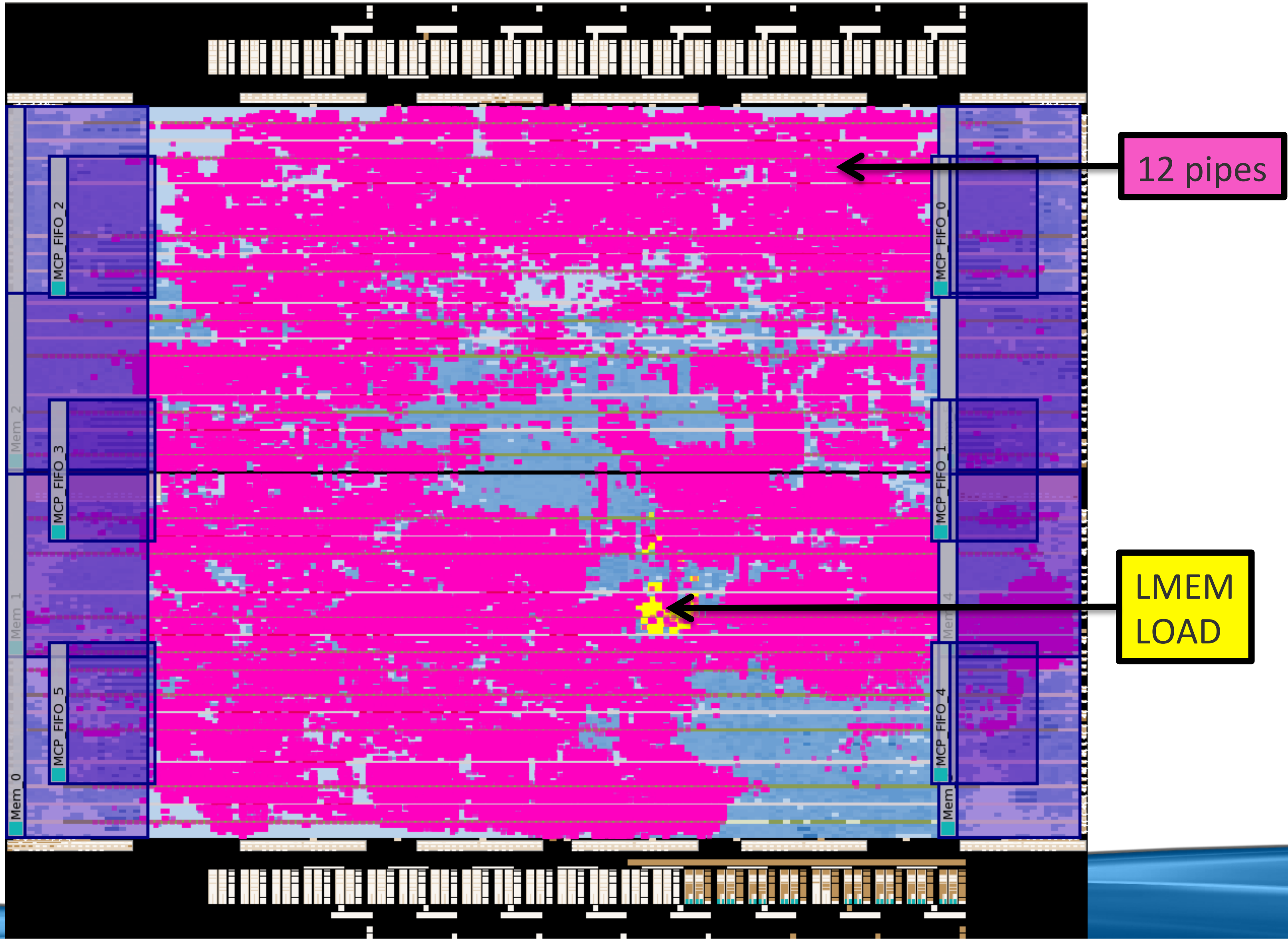
keep the window sum by subtracting the top and adding the new entrant

Note: this is a Case 2 Loop (Lecture 9), with a sequential streaming loop (par_loop)

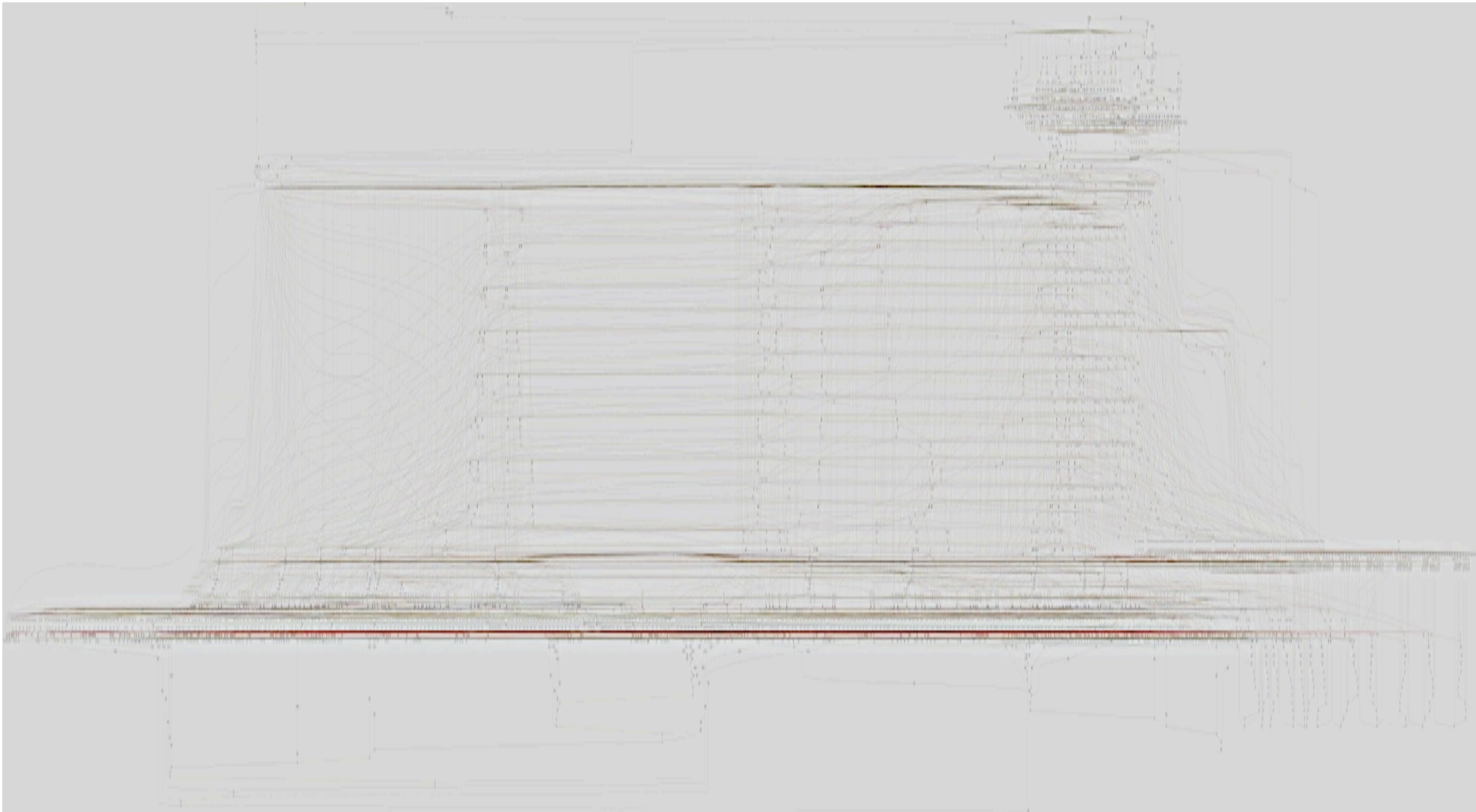



```
DFEParLoop lp = new DFEParLoop(this,"lp");
DFEVector<DFEVar> data_pairs = io.input("dp", dfeVector(...), lp.ndone);
lp.set_input(sum.getType(), 0.0);
  DFEVar sum = (lp.feedback + data_pairs[0]) - data_pairs[1];
lp.set_output(sum);
io.output("sum", sum, sum.getType(), lp.done);
```

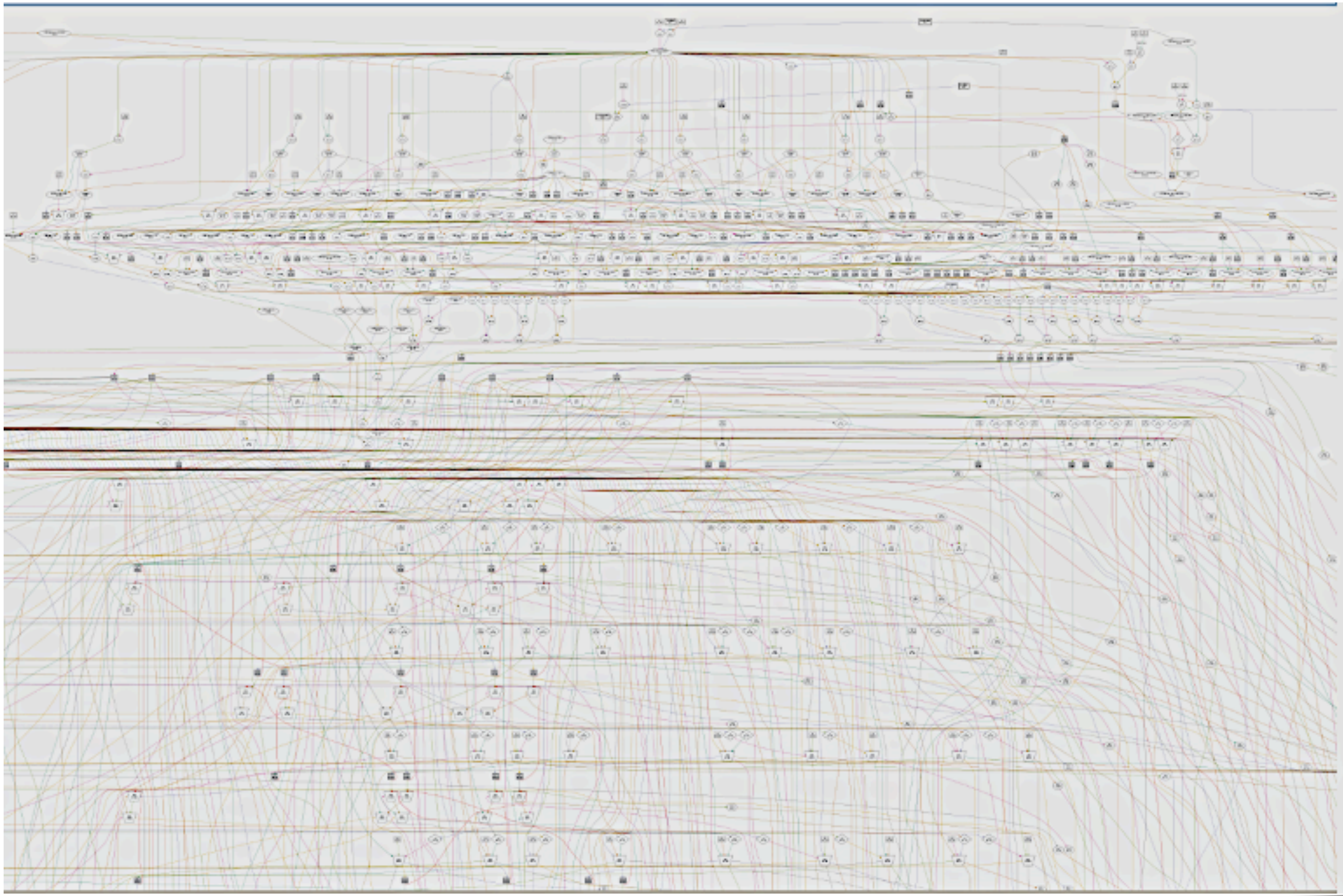
Correlation .max file plotted in Space



Correlation Pipes: Dataflow Graph



Correlation Dataflow Graph: Zoom In



File: correlationCPUCode.c
Purpose: calling correlationSAPI.h for correlation.max

Correlation formula:

scalar r(x,y) = (n*SUM(x,y) - SUM(x)*SUM(y))*SQRT_INVERSE(x)*SQRT_INVERSE(y)

where:

x,y, ...	- Time series data to be correlated
n	- window for correlation (minimum size of 2)
SUM(x)	- sum of all elements inside a window
SQRT_INVERSE(x)	- 1/sqrt(n*SUM(x^2)- (SUM(x)^2))

Action 'loadLMem':

[in] memLoad - initializeLMem, used as temporary storage

Action 'default':

[in] precalculations: {SUM(x), SQRT_INVERSE(x)} for all timeseries for every timestep

[in] data_pair: {..., x[i], x[i-n], y[i], y[i-n], ... , x[i+1], x[i-n+1], y[i+1], y[i-n+1], ...} for all timeseries for every timestep

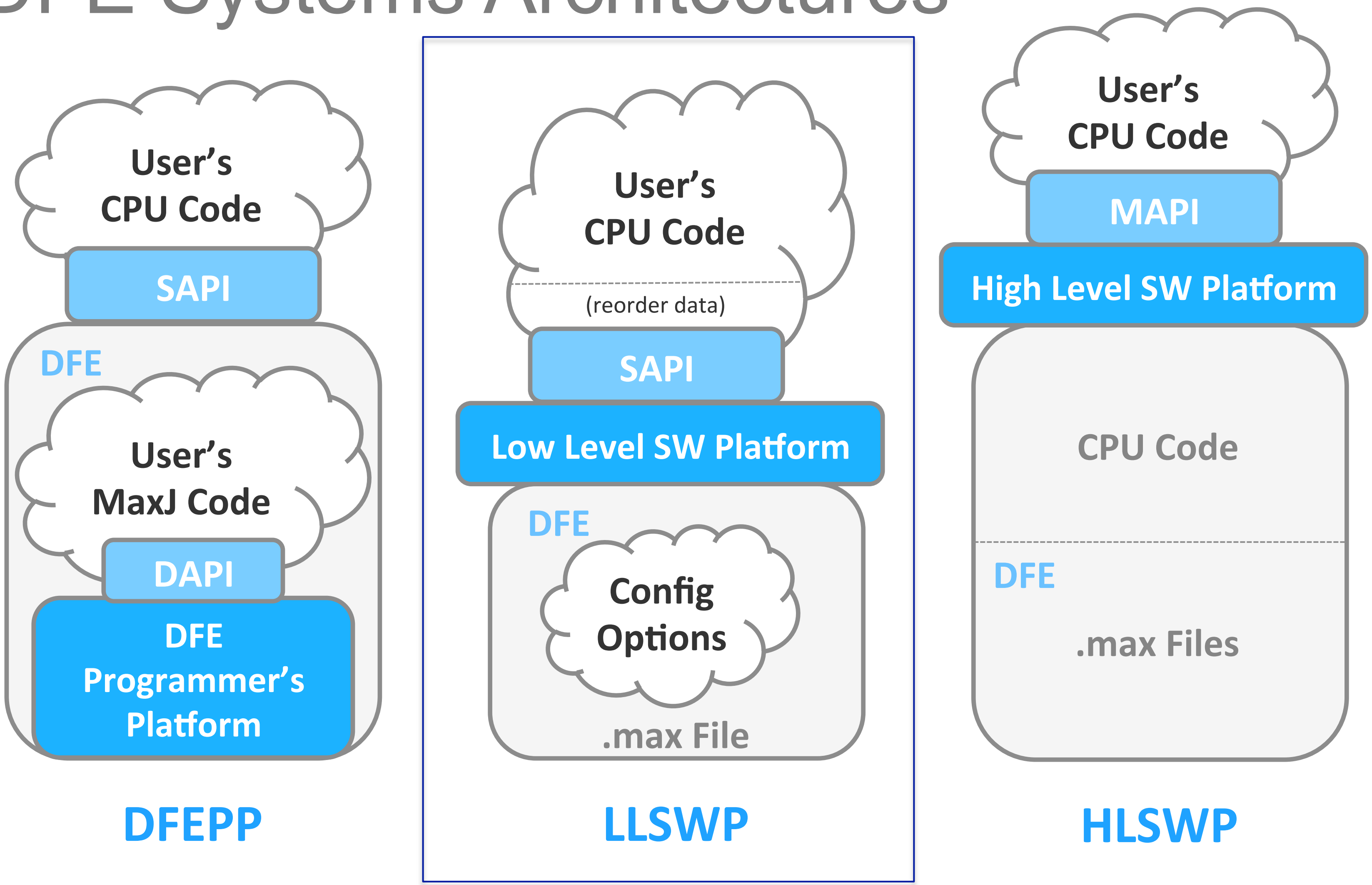
[out] correlation r: numPipes * CorrelationKernel_loopLength * topScores correlations for every timestep

[out] indices x,y: pair of timeseries indices for each result r in the correlation stream

prepare_data_for_dfe() – The Code

```
// 2 DFE input streams: precalculations and data pairs
for (uint64_t i=0; i<numTimesteps; i++) {
        old_index = i - (uint64_t)windowSize;

        for (uint64_t j=0; j<numTimeseries; j++) {
                if (old_index<0) old = 0; else old = data [j][old_index];
                new = data [j][i];

                if (i==0) {
                        sums [i][j] = new;
                        sums_sq [i][j] = new*new;
                }else {
                        sums [i][j] = sums [i-1][j] + new - old;
                        sums_sq [i][j] = sums_sq [i-1][j] + new*new - old*old;
                }
                inv [i][j] = 1/sqrt((uint64_t)windowSize*sums_sq[i][j] - sums[i][j]*sums[i][j]);

                // precalculations REORDERED in DFE ORDER
                precalculations [2*i*numTimeseries + 2*j] = sums[i][j];
                precalculations [2*i*numTimeseries + 2*j + 1] = inv [i][j];

                // data_pairs REORDERED in DFE ORDER
                data_pairs[2*i*numTimeseries + 2*j] = new;
                data_pairs[2*i*numTimeseries + 2*j + 1] = old;
        }
}
```

DFE Systems Architectures



EXAMPLES

Max MPT
Video transcoding/processing

Correlation App

Risk Analytics Library
Video Encoding

Correlation Lab Exercise (due 4 Dec 2015)

- Open correlation project at https://openspl.doc.ic.ac.uk/
- Full exercise specification on CATe
- Four versions of correlation:
    1. **Original:** Original software in C
    2. **Split Control and Data:** C software split into dataflow and controlflow
    3. **Single Pipe:** CPU code + DFE code. Runs in simulation and on DFE.
    4. **Multi Pipe:** CPU code + DFE binary. Runs in simulation and on DFE.

Task 1: Run the Original C code for correlating 256, 512, 1024 and 2048 time series for 10, 100 and 1000 time steps.

Task 2: Run the Split C code, correlating 256, 512, 1024 and 2048 time series for 1000 time steps.

Task 3: Run the Single Pipe and Multi Pipe dataflow designs for correlating 256 time series and 10 time steps in simulation mode (SIM).

Task 4: Run the Single Pipe and Multi Pipe dataflow designs for correlating 265, 512, 1024, 2048, and 4096 time series for 10, 100 and 100 time steps on the DFE.