Program Analysis (470)

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Program Analysis

Program analysis is an automated technique for finding out properties of programs without having to execute them.

Static Analysis vs **Dynamic Testing**

- Compiler Optimisation
- Program Verification
- Security Analysis

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Techniques

Some techniques used in program analysis include:

- Data Flow Analysis
- Control Flow Analysis
- Types and Effects Systems
- Abstract Interpretation

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A First Example

Consider the following fragment in *some* procedural language.

```
$m \leftarrow 2$;
while $n > 1$ do
    $m \leftarrow m \times n$;
    $n \leftarrow n - 1$
end while
stop
```

```
$[m \leftarrow 2]^1$;
while $[n > 1]^2$ do
    $[m \leftarrow m \times n]^3$;
    $[n \leftarrow n - 1]^4$
end while
$[\text{stop}]^5$
```

We annotate a program such that it becomes clear about what program point we are talking about.

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We annotate a program such that it becomes clear about what program point we are talking about.

A Parity Analysis

Claim: This program fragment always returns an **even** m, idependently of the initial values of m and n.

We can statically determine that in any circumstances the value of m at the last statement will be **even** for any input n.

A program analysis, so-called parity analysis, can determine this by propagating the even/odd or *parity* information *forwards* form the start of the program.

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Properties

We will assign to each variable one of three properties:

- **even** — the value is known to be even
- **odd** — the value is known to be odd
- **unknown** — the parity of the value is unknown

For both variables m and n we record its parity at each stage of the computation (beginning of each statement).

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A First Example

Executing the program with *abstract* values, parity, for m and n.

```
$m \leftarrow 2$;
while $n > 1$ do
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end while
stop
```

Important: We can restart the loop!

A First Example

Executing the program with *abstract* values, parity, for m and n.

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$m \leftarrow 2$;                    ▷ unknown(m) – unknown(n)
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The first program computes 2 times the factorial for any positive value of n. Replacing ‘2’ by ‘1’ in the first statement gives:

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$m \leftarrow 1$;
while $n > 1$ do
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i.e. the factorial – but then the program analysis is unable to tell us anything about the parity of m at the end of the execution.

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Loss of Precision

The analysis of the new program does not give a satisfying result because:

- m could be **even** — if the input $n > 1$, or
- m could be **odd** — if the input $n \leq 1$.

However, even if we fix/require the input to be positive and **even** — e.g. by some suitable conditional assignment — the program analysis still might not be able to accurately predict that m will be **even** at statement **5**.

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Safe Approximations

Such a loss of precession is a common feature of program analysis: many properties that we are interested in are essentially undecidable and therefore we cannot hope to detect (all of) them accurately.

We only aim to ensure that the answers/results we obtain by program analysis are at least safe, i.e.

- yes means *definitely* yes,
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Facets of Program Analysis

We can identify the following facets of program analysis which we will mostely cover in this course:

- specification,
- implementation,
- correctness,
- *applications*.

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The starting point for data flow analysis is a representation of the control flow graph of the program: the nodes of such a graph may represent individual statements – as in a flowchart – or sequences of statements; arcs specify how control may be passed during program execution.

The data flow analysis is usually specified as a set of equations which associate analysis information with program points which correspond to the nodes in the control flow graph. This information may be propagated *forwards* through the program (e.g. parity analysis) or *backwards*.

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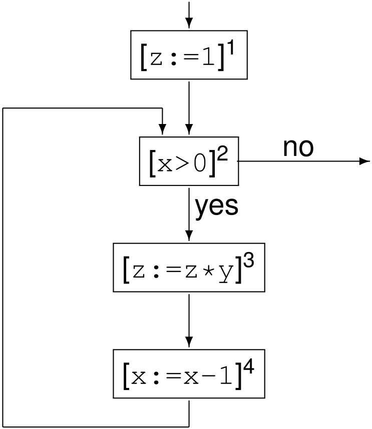
Control Flow Information



This allows us to determine the predecessors *pred* and successors *succ* of each statement, e.g. $\mathit{pred}(2) = \{1, 4\}$.

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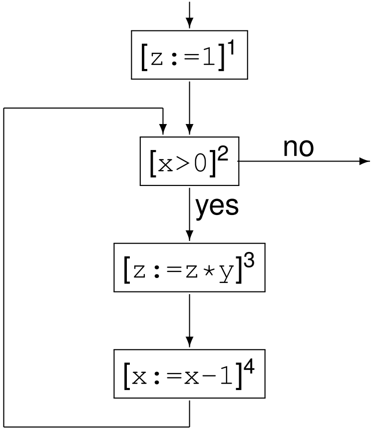

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Reaching Definition

Reaching Definition (*RD*) analysis determines which set of definitions (i.e. assignments) are current when control reaches a certain program point *p*.

The analysis can be specified by equations of the form:

$$\mathrm{RD}_{entry}(p) = \begin{cases} \mathrm{RD}_{init} & \text{if } p \text{ is initial} \\ \bigcup\limits_{p' \in pred(p)} \mathrm{RD}_{exit}(p') & \text{otherwise} \end{cases}$$

$$\mathrm{RD}_{exit}(p) = (\mathrm{RD}_{entry}(p) \setminus kill_{\mathrm{RD}}(p)) \cup gen_{\mathrm{RD}}(p)$$

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Analysis Information

At each program point some definitions get "killed" (those which define the same variable as at the program point) while others are "generated".

A suitable representation for properties are sets of pairs, where each pair contains a variable x and a program point p: the meaning of the pair (x, p) is that the assignment to x at point p is the current one. The initial value in this case is:

$$\mathrm{RD}_{init} = \{(x, ?) \mid x \text{ is a variable in the program}\}$$

Reaching Definitions is a forward analysis and we require the least (most precise) solutions to the set of equations.

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Equations & Solutions

For our initial program fragment

```
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**while** $[n > 1]^2$ **do**
    $[m \leftarrow m \times n]^3$;
    $[n \leftarrow n - 1]^4$
**end while**
$[\textbf{stop}]^5$
```

some of the *RD* equations we get are:

$$\begin{aligned} \mathrm{RD}_{entry}(1) &= \{(m, ?), (n, ?)\} \\ \mathrm{RD}_{entry}(2) &= \mathrm{RD}_{exit}(1) \cup \mathrm{RD}_{exit}(4) \end{aligned}$$

Equations & Solutions

For our initial program fragment

```
$[m \leftarrow 2]^1$;
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	RD_{entry}	RD_{exit}
1	$\{(m,?),(n,?)\}$	$\{(m,1),(n,?)\}$
2	$\{(m,1),(m,3),(n,?),(n,4)\}$	$\{(m,1),(m,3),(n,?),(n,4)\}$
3	$\{(m,1),(m,3),(n,?),(n,4)\}$	$\{(m,3),(n,?),(n,4)\}$
4	$\{(m,3),(n,?),(n,4)\}$	$\{(m,3),(n,4)\}$
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Solving Equations

How can we construct solution to the data flow equations?
Answer: Iteratively, by improving approximations/guesses.

INPUT: Control Flow Graph
i.e. initial(p), pred(p).

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METHOD: Step 1: Initialisation
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Some Examples

Some examples of data flow analyses — and the possible applications and optimisations they allow for — are:

- Reaching Definitions — Constant Folding
- Available Expressions — Avoid Re-computations
- Very Busy Expressions — Hoisting
- Live Variables — Dead Code Elimination

- *Information Flow — Computer Security*
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Code Optimisation

To illustrate the ideas we shall show how Reaching Definitions can be used to perform Constant Folding.

There are two ingredients to this:

- Replace the use of a variable in some expression by a constant if it is known that the value of that variable will always be a constant.
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Constant Folding I

$$RD \vdash [\, x := a \,]^{\ell} \;\triangleright\; [\, x := a[y \mapsto n] \,]^{\ell}$$

$$\text{if} \begin{cases} y \in FV(a) \;\wedge\; (y, ?) \notin \mathrm{RD}_{entry}(\ell) \;\wedge \\ \forall (y', \ell') \in \mathrm{RD}_{entry}(\ell) : \\ y' = y \;\Rightarrow\; [\ldots]^{\ell'} = [\, y := n \,]^{\ell'} \end{cases}$$

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$$\frac{RD \vdash S_1 \;\rhd\; S_1'}{RD \vdash S_1 ; S_2 \;\rhd\; S_1' ; S_2}$$

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$$\frac{RD \vdash S_1 \;\rhd\; S_1'}{RD \vdash \textbf{if } [b]^\ell \textbf{ then } S_1 \textbf{ else } S_2 \;\rhd\; \textbf{if } [b]^\ell \textbf{ then } S_1' \textbf{ else } S_2}$$

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An Example

To illustrate the use of the transformation consider:

$$[\ x := 10\ ]^1;\ [\ y := x + 10\ ]^2;\ [\ z := y + 10\ ]^3$$

The (least) solution to the Reaching Definition analysis is:

$$\begin{aligned}
\mathrm{RD}_{entry}(1) &= \{(x,?),(y,?),(z,?)\}\\
\mathrm{RD}_{exit}(1) &= \{(x,1),(y,?),(z,?)\}\\[1ex]
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Constant Folding

We have for example the following:

$$RD \vdash [\, y := x + 10 \,]^2 \;\triangleright\; [\, y := 10 + 10 \,]^2$$

and therfore the rules for sequential composition allow us to do the following transformation:

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Transformation

We can continue this kind of transformation and obtain:

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after which no more steps are possible.

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Additional Issues

The above example shows that optimisation is in general the result of a number of successive transformations.

$$RD \vdash S_1 \;\triangleright\; S_2 \;\triangleright\; \ldots \;\triangleright\; S_n.$$

This could be costly because one S_1 has been transformed into S_2 we might have to *re-compute* the Reaching Definition analysis before the next transformation step can be done.

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Lecture: Milestones

Coursework I: Mid February – Test

Coursework II: Early March – Test

Examination: Sometime in Week 11

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