Program Analysis (470)
Further Topics
[Not for Exam]

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Further Topics in Program Analysis

Nearly all aspects or properties of programs (in different kind of language) can be topic for program analysis.

Relational Analysis: How are values of variables related?

Pointer Analysis: Where do pointers refer to (alias)?

Termination Analysis: Does program (never) terminate?

Quantitative Anlysis: Estimate probabilities of properties.

Security Analysis: Guarantee security properties.

Groundness Analysis, Shape Analysis, Information Flow, etc.

There are also various other techniques (e.g. type based analysis) and combinations of static and dynamic analysis.

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Why Probabilistic Analysis?

Probabilistic Program Analysis aims in understanding probabilistic programs and probabilistic properties of (deterministic) programs (e.g. average properties).

- Introducing probability in the problem formulation allows us to get approximate answers rather that a 'yes' or 'no' answers from a data-flow analysis.
- Optimisations can then be performed *with some probability* by taking into account the various costs associated to the program. cf. speculative optimisation

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Determine the necessary registration fee for 130 delegates.

	Costs	Overest.	Estimate	Underest.
Proceedings	3.345 £	4.000 £	3.500 £	3.000 £
Dinner	5.672 £	6.000 £	5.500 £	5.000 £
Excursion	1.813 £	2.000 £	2.000 £	1.000 £
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Relational Program Analysis

Factorial – Input/Output behaviour (I/O Semantics):

$$\begin{aligned} 1! &= 1 \\ n! &= n \cdot (n-1)! \end{aligned}$$

For random input $n \in \{1,2,3\}$ – i.e. $P(n=1) = P(n=2) = P(n=3) = \frac{1}{3}$ – determine the probability that $n!$ is **even** or **odd**:

$$P(\text{parity}(n!) = \textbf{even}) = \frac{2}{3} \text{ and } P(\text{parity}(n!) = \textbf{odd}) = \frac{1}{3}.$$

However – the probabilities are not independent – we have, e.g.

$$\begin{aligned} P(\text{parity}(n!) = \textbf{even} \ \wedge \ n=1) &= 0 \\ P(\text{parity}(n!) = \textbf{even}) \cdot P(n=1) &= \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9} \end{aligned}$$

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Expressing Dependencies/Correlations/etc.

Some joint probability distributions can be expressed as tensor product of two (independent) probability distributions $\mathbf{d}_1$ and $\mathbf{d}_2$:

$$\begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix} = (\frac{2}{3}, \frac{1}{3})^t \otimes (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$$

However, in general we can express any joint probability distribution as a linear combination of distributions.

$$\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 \end{pmatrix} = \frac{1}{3}(\mathbf{e}_2 \otimes \mathbf{f}_1) + \frac{1}{3}(\mathbf{e}_1 \otimes \mathbf{f}_2) + \frac{1}{3}(\mathbf{e}_1 \otimes \mathbf{f}_3)$$

with $\mathbf{e}_i \in \mathbb{R}^2$ and $\mathbf{f}_j \in \mathbb{R}^3$ (column and row) basis vectors

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But there are no two vectors $\mathbf{d}_1$ and $\mathbf{d}_2$ such that for example

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Tensor (Kronecker) Product

Every vector space has an (algebraic) base $\{\mathbf{e}_i\}$ such that

$$\mathbf{x} = x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + \dots$$

This allows to specify vectors via coordinates $\mathbf{x} = (x_1, x_2, \dots)$.
Base vectors are in this context simply of the form

$$\mathbf{e}_i = (e_{i1}, e_{i2}, \dots) \quad \text{with } e_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{otherwise} \end{cases}$$

The tensor product space $\mathcal{V} \otimes \mathcal{W}$ can be seen as generated by (formal) tensors of the form $\mathbf{v}_i \otimes \mathbf{w}_j$ with $\mathbf{v}_i \in \mathcal{V}$ and $\mathbf{w} \in \mathcal{W}$ base vectors, i.e. every $\mathbf{x} \in \mathcal{V} \otimes \mathcal{W}$ is of the form

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Markov Chain Semantics

The collecting semantics of a program P is given by:

$$\mathbf{T}(P) = \sum_{\langle i, p_{ij}, j \rangle \in \mathcal{F}(P)} p_{ij} \cdot \mathbf{T}(\ell_i, \ell_j)$$

i.e. as a (stochastic) linear operator (i.e. matrix) on the space of distributions over configurations.

Local effects $\mathbf{T}(\ell_i, \ell_j)$: Data Update $\mathbf{N}$ + Control Step $\mathbf{M}$

$$\mathbf{T}(\ell_i, \ell_j) = \mathbf{N}_i \otimes \mathbf{M}_{ij} = \mathbf{N}_{i1} \otimes \mathbf{N}_{i2} \otimes \ldots \otimes \mathbf{N}_{iv} \otimes \mathbf{M}_{ij}$$

Describes the effect of changes to the probabilistic state via a Discrete Time Markov Chain (DTMC)

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Modelling Tests

Consider conditional jumps or statements, e.g.

$$\textbf{if } \mathit{even}(x) \textbf{ then } x := x/2 \textbf{ else } y + 1 \textbf{ fi}$$

NB To avoid errors $a/b = \lceil a/b \rceil$ and $a + b = a + b \bmod n$.

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$$[\![x := x/2]\!] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \mathbf{I}$$

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$$[\![y := y + 1]\!] = \mathbf{I} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Tests and Distribution Splitting

We represent the filter for testing if x is even by a projection:

$$\mathbf{P}(\mathit{even}(x)) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \otimes \mathbf{I}$$

Its negation is represented by:

$$\mathbf{P}(\neg \mathit{even}(x)) = \mathbf{P}(\mathit{even}(x))^{\perp} = \mathbf{I} - \mathbf{P}(\mathit{even}(x)).$$

Conditional Statements

The semantics of a conditional is given by applying the semantics of the branches to the filtered (probabilistic) states and to combine the results. In our example:

$$[\![\textbf{if } \mathit{even}(x) \textbf{ then } x := x/2 \textbf{ else } y + 1 \textbf{ fi}]\!] =$$
$$= \quad \mathbf{P}(\mathit{even}(x)) \cdot [\![x := x/2]\!] + \mathbf{P}(\mathit{even}(x))^{\perp} \cdot [\![y := y + 1]\!]$$

Given state where x has with probability $\frac{1}{2}$ values 3 and 6, and y value 2, i.e. $\sigma_0 = (0, 0, \frac{1}{2}, 0, 0, \frac{1}{2}, 0, 0) \otimes (0, 1, 0, 0)$ then

$$\sigma_0 \cdot \mathbf{P}(\mathit{even}(x)) \quad = \quad (0, 0, 0, 0, 0, \frac{1}{2}, 0, 0) \otimes (0, 1, 0, 0)$$
$$\sigma_0 \cdot \mathbf{P}(\mathit{even}(x))^{\perp} \quad = \quad (0, 0, \frac{1}{2}, 0, 0, 0, 0, 0) \otimes (0, 1, 0, 0)$$

Semantics of Conditionals

Applying the semantics of both branches gives us:

$$\begin{aligned}
&\sigma_0 \cdot \mathbf{P}(\mathit{even}(x)) \cdot [\![x := x/2]\!] = \\
&\quad = \quad (0, 0, \frac{1}{2}, 0, 0, 0, 0) \otimes (0, 1, 0, 0) \\
&\sigma_0 \cdot \mathbf{P}(\mathit{even}(x))^{\perp} \cdot [\![y := y + 1]\!] = \\
&\quad = \quad (0, 0, \frac{1}{2}, 0, 0, 0, 0, 0) \otimes (0, 0, 1, 0)
\end{aligned}$$

The sum of both branches is now, maybe somewhat surprising:

$$\sigma = (0, 0, 1, 0, 0, 0, 0, 0) \otimes (0, \frac{1}{2}, \frac{1}{2}, 0)$$

Though we have started with a definitive value for y and a distribution for x, the opposite is now the case

Semantical Abstraction

Consider a concrete domain $\mathcal{C}$ and an abstract domain $\mathcal{D}$:

With an abstraction $\mathbf{A} : \mathbf{C} \to \mathbf{D}$ and a concretisation $\mathbf{G} : \mathbf{D} \to \mathbf{C}$:

$$\mathbf{T}^\# = \mathbf{GTA}$$

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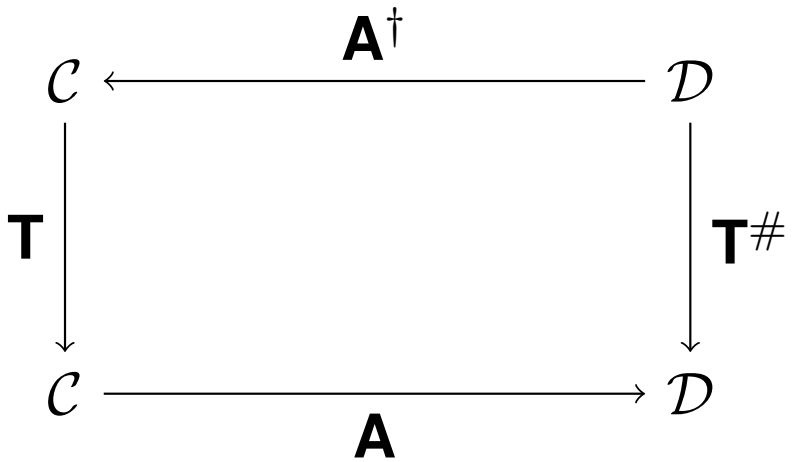

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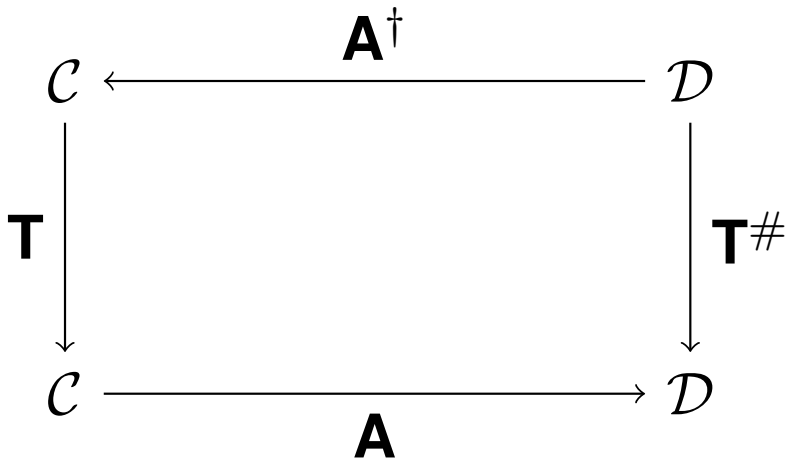

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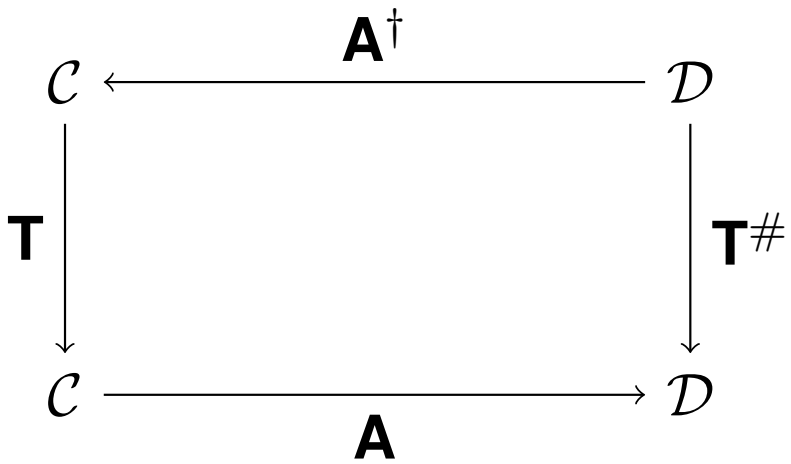

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Abstract Interpretation: $(\mathbf{A}, \mathbf{G})$ form a Galois Connection.

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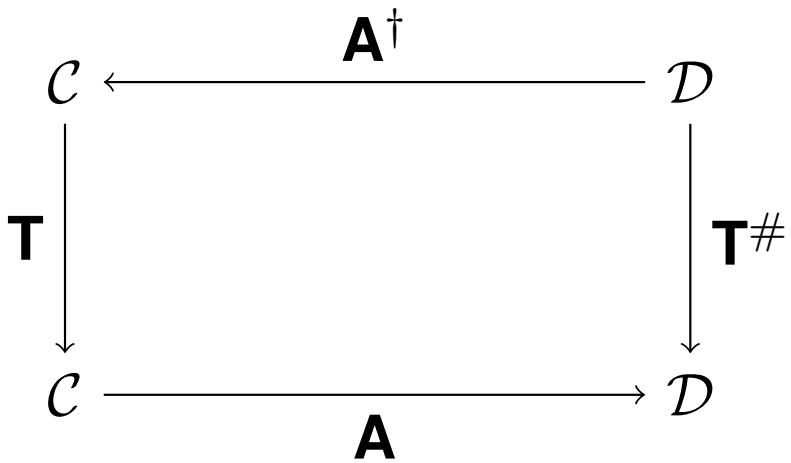

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Probabilistic Abst.Int.: $(\mathbf{A}, \mathbf{G})$ Moore-Penrose Pseudo-Inverse.

Moore-Penrose Pseudo-Inverse

Definition

Let $\mathcal{C}$ and $\mathcal{D}$ be two Hilbert spaces and $\mathbf{A} : \mathcal{C} \to \mathcal{D}$ a bounded linear map. A bounded linear map $\mathbf{A}^\dagger = \mathbf{G} : \mathcal{D} \to \mathcal{C}$ is the Moore-Penrose pseudo-inverse of $\mathbf{A}$ iff

$$\text{(i)} \quad \mathbf{A} \circ \mathbf{G} = \mathbf{P}_A,$$
$$\text{(ii)} \quad \mathbf{G} \circ \mathbf{A} = \mathbf{P}_G,$$

where $\mathbf{P}_A$ and $\mathbf{P}_G$ denote orthogonal projections onto the ranges of $\mathbf{A}$ and $\mathbf{G}$.

For a concrete linear operator $\mathbf{T}$ describing the semantics of a statement or test we have the induced abstract semantics as $\mathbf{T}^\# = \mathbf{A}^\dagger \mathbf{T} \mathbf{A}$ (post-multiplication).

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Examples of Abstractions

Parity Abstraction operator on $\mathcal{V}(\{1, \ldots, n\})$ (with n even):

$$\mathbf{A}_p = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{pmatrix} \qquad \mathbf{A}_p^\dagger = \begin{pmatrix} \frac{2}{n} & 0 & \frac{2}{n} & 0 & \cdots & 0 \\ 0 & \frac{2}{n} & 0 & \frac{2}{n} & \cdots & \frac{2}{n} \end{pmatrix}$$

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Sign Abstraction operator on $\mathcal{V}(\{-n, \ldots, 0, \ldots, n\})$:

$$\mathbf{A}_s = \begin{pmatrix} 1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{A}_s^\dagger = \begin{pmatrix} \frac{1}{n} & \ldots & \frac{1}{n} & 0 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 1 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 0 & \frac{1}{n} & \ldots & \frac{1}{n} \end{pmatrix}$$

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Example: Factorial

1: $[m \leftarrow 1]^1$;
2: **while** $[n > 1]^2$ **do**
3: $\quad [m \leftarrow m \times n]^3$;
4: $\quad [n \leftarrow n - 1]^4$
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$$
\begin{aligned}
\mathbf{T} &= \mathbf{U}(m \leftarrow 1) \otimes \mathbf{E}(1,2) \\
&+ \mathbf{P}(n > 1) \otimes \mathbf{E}(2,3) \\
&+ \mathbf{P}(n \leq 1) \otimes \mathbf{E}(2,5) \\
&+ \mathbf{U}(m \leftarrow m \times n) \otimes \mathbf{E}(3,4) \\
&+ \mathbf{U}(n \leftarrow n - 1) \otimes \mathbf{E}(4,2) \\
&+ \mathbf{I} \otimes \mathbf{E}(5,5)
\end{aligned}
$$

Abstract Semantics

Abstraction: $\mathbf{A} = \mathbf{A}_p \otimes \mathbf{I}$, i.e. *m* abstract (parity) but *n* concrete.

$$
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Abstract Semantics – Matrices

$$\mathbf{U}^{\#}(m \leftarrow 1) =$$

$$= \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & & \dots & 1 \end{pmatrix}$$

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$$\mathbf{U}^{\#}(n \leftarrow n-1) =$$

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Abstract Semantics – Matrices

$$\mathbf{P}^{\#}(n > 1))$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

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Implementation

Implementation of concrete and abstract semantics of Factorial using **octave**. Ranges: $n \in \{1, 2, \max\}$ and $m \in \{1, 2, \max!\}$.

n	$\dim(\mathbf{T}(F))$	$\dim(\mathbf{T}^{\#}(F))$
2	45	30
3	140	40
4	625	50
5	3630	60
6	25235	70
7	201640	80
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9	18144050	100

n	**even**	**odd**
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We investigate the problem of (guaranteeing) secure information flow, i.e. look at the question which objects can be handled by which subjects securely.

Subjects: People, officials, processes, ...

Objects: Papers, files, variables, ...

In order to simplify things we consider only two different types of interaction:

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In order to avoid non-authorised information flow:

Discretional Approach: The “owner” of sensitive information controls access.

Non-Discretional Approach: There are formal rules which guide information access.

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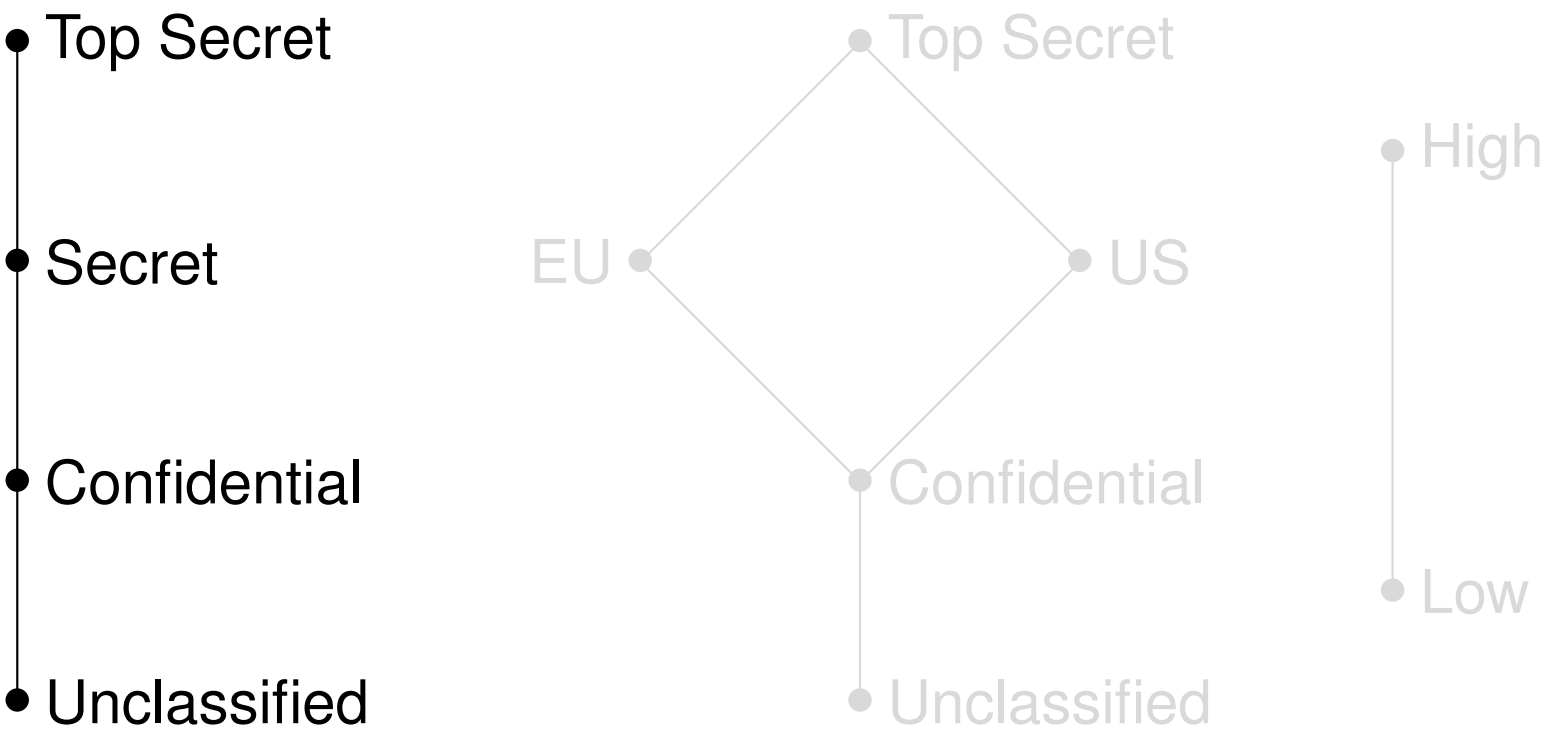

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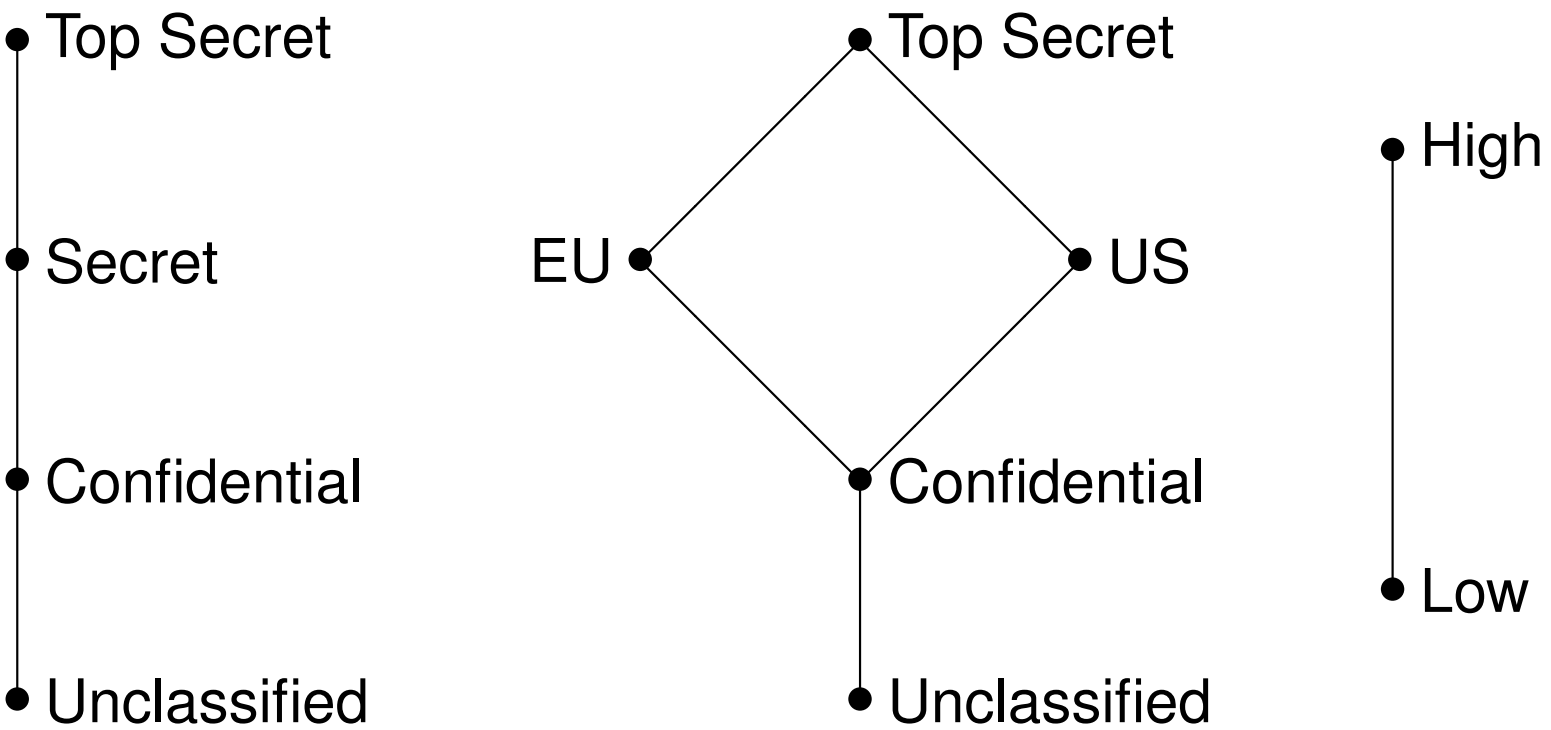

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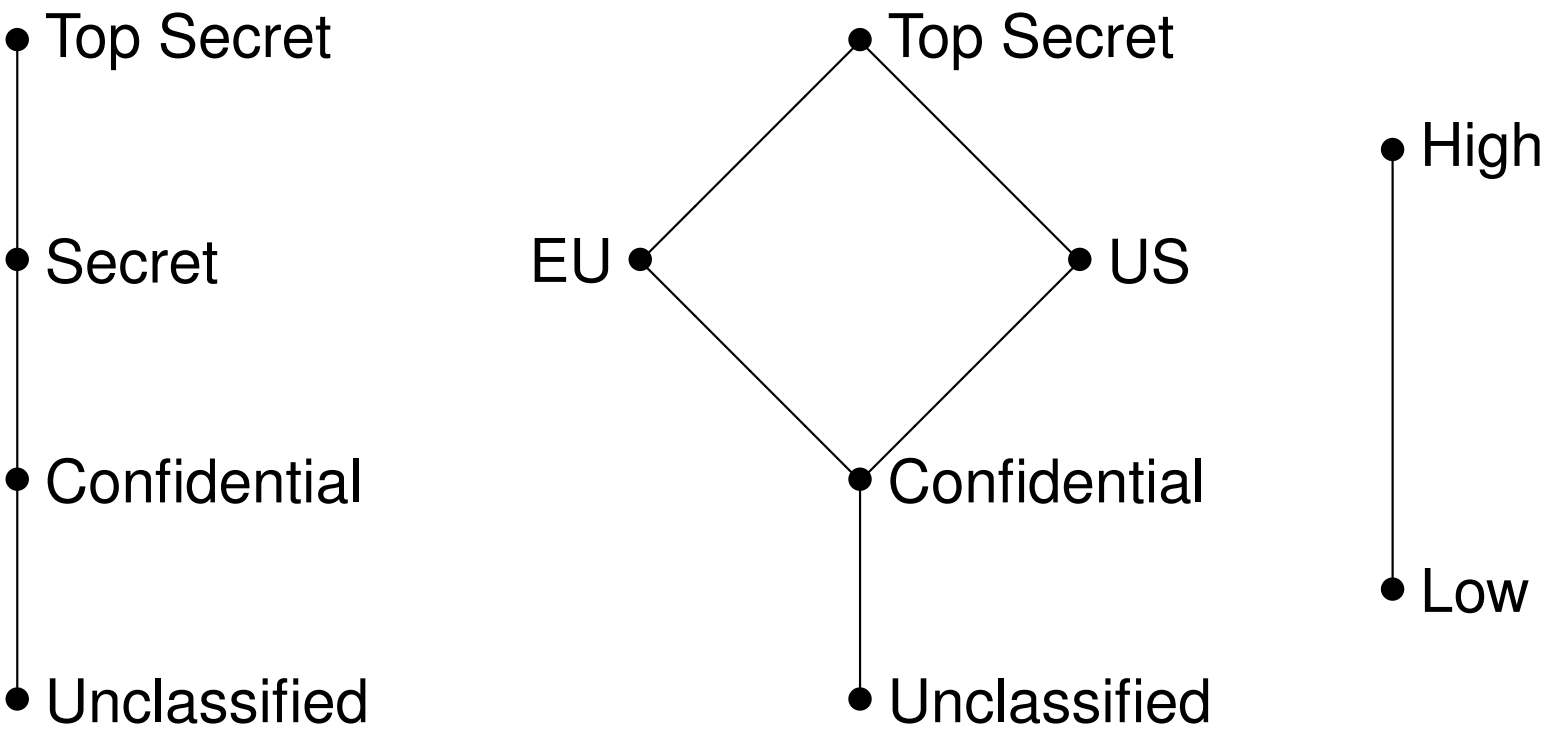

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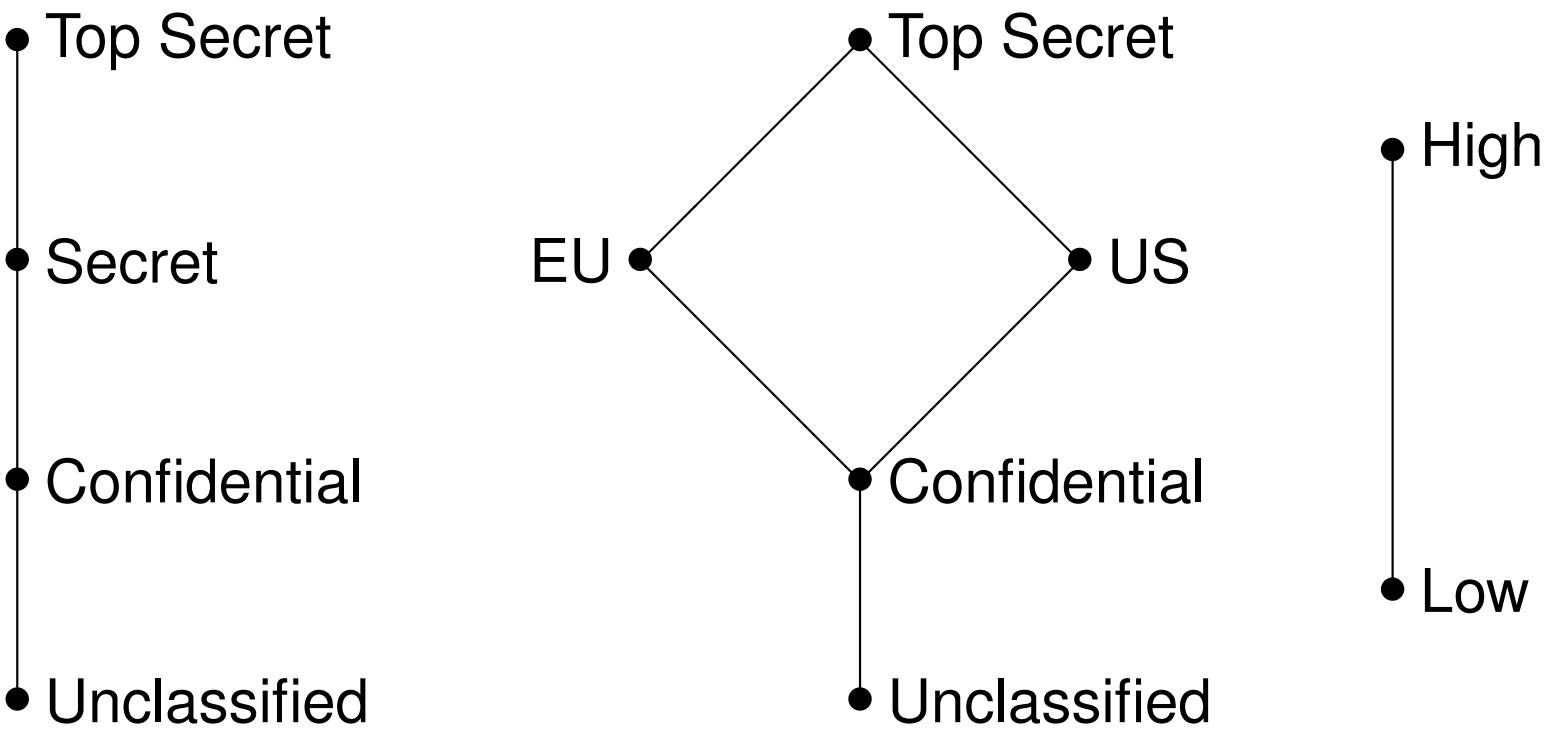

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There are two principles in the Bell-LaPadula approach which determine the security level of objects which can be accessed by subjects with a certain security clearance.

No Read-Up: A subject is allowed a read access to an object only if the security class of the subject dominates the security class of the object.

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Information Flow

Direct Information Flow

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high := low   vs   low := high
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Indirect Information Flow

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if high=0 then low:=0 else low:=1
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low:=0; while low<high do low:=low+1
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Type System for Security

Assume a lattice of *security classes* $(SC, \sqsubseteq)$.

$$\sigma \in SC$$

(data types) $\quad \tau ::= \sigma$

(phrase types) $\quad \rho ::= \tau \mid \tau\ var \mid \tau\ cmd$

Identifier Typing

$$SC : \mathbf{Var}_\star \to SC$$

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Security Typing I

$$\text{(CONST)} \quad SC \vdash n : \tau$$

$$\text{(VAR)} \quad SC \vdash x : \tau\ var \quad \text{if } SC(x) = \tau$$

$$\text{(EXPR)} \quad \frac{SC \vdash e_1 : \tau \qquad SC \vdash e_2 : \tau}{SC \vdash e_1\ op\ e_2 : \tau}$$

$$\text{(VAL)} \quad \frac{SC \vdash e : \tau\ var}{SC \vdash e : \tau}$$

$$\text{(ASSIGN)} \quad \frac{SC \vdash x : \tau\ var \qquad SC \vdash e : \tau}{SC \vdash x := e : \tau\ cmd}$$

Security Typing II

$$(\textsc{skp}) \quad SC \vdash \texttt{skip} : \tau\ cmd$$

$$(\textsc{seq}) \quad \frac{SC \vdash S_1 : \tau\ cmd \qquad SC \vdash S_2 : \tau\ cmd}{SC \vdash S_1;\ S_2 : \tau\ cmd}$$

$$(\textsc{if}) \quad \frac{SC \vdash e : \tau \qquad SC \vdash S_1 : \tau\ cmd \qquad SC \vdash S_2 : \tau\ cmd}{SC \vdash \texttt{if}\ e\ \texttt{then}\ S_1\ \texttt{else}\ S_2 : \tau\ cmd}$$

$$(\textsc{while}) \quad \frac{SC \vdash e : \tau \qquad SC \vdash S : \tau\ cmd}{SC \vdash \texttt{while}\ e\ \texttt{do}\ S : \tau\ cmd}$$

Type Soundness = Security

Theorem

If

- $SC \vdash S : \rho$
- $\langle S, s_1 \rangle \Rightarrow^* \langle S_1, s_1' \rangle$ *or* $\langle S, s_1 \rangle \Rightarrow^* s_1'$
- $\langle S, s_2 \rangle \Rightarrow^* \langle S_2, s_2' \rangle$ *or* $\langle S, s_2 \rangle \Rightarrow^* s_2'$
- $s_1(x) = s_2(x)$ *for all* x *with* $SC(x) \sqsubseteq \tau$

then

- $s_1'(x) = s_2'(x)$ *for all* x *with* $SC(x) \sqsubseteq \tau$.

For all typeable programs S and any given security level τ: If the initial states agree on “low” variables they will also be indistinguishable on “low” variables in the end; no leakage.

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