Information and Coding Theory CO349

Tutorial

Sheet 1 - Questions

Exercise 1 (Entropy) *Consider the Cheltenham horse race (slide 7). Calculate the entropy for winning chances:*

$$\mathbf{p}_1 = (1, 0, 0, 0, 0, 0, 0, 0)$$
$$(\frac{1}{2}, 0, 0, 0, 0, 0, 0, \frac{1}{2})$$
$$(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64})$$
$$(\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8})$$

Compute the corresponding entropies. What does that tell you about the property "entropy" describes?

Exercise 2 (Biggs, Example 1.5) *The following messages are coded versions of English sentences. Explain the codes and what is the original messages:*

7 15 15 4 27 12 21 3 11

00111 01111 01111 00100 11011 01100 10101 00011 01011

Exercise 3 (Biggs, Example 2.1) *Given a binary code* $c : \{s_1, s_2, s_3, s_4\} \to \mathbb{B}^*$ *defined by*

$$s_1 \mapsto 00, \quad s_2 \mapsto 010, \quad s_3 \mapsto 100, \quad s_4 \mapsto 111.$$

Is this code UD? Decode

$$1001110100011101010000.$$

Exercise 4 (Biggs, Example 2.9) *Do PF binary codes with the following parameters*

(i) $n_1 = 0, n_2 = 1, n_3 = 4, n_4 = 3$ *and*

(ii) $n_1 = 1, n_2 = 0, n_3 = 2, n_4 = 3, n_5 = 2.$

exist? If so state them. Could they be extended, i.e. more codewords added?

Exercise 5 (Biggs, Example 2.13) *If we want to construct a UD code* $c : S \to \mathbb{B}^*$ *with* $|S| = 12$ *with a maximal codeword length* 4. *For which parameters* n_i *does such a code exists?*

Exercise 6 (Product Probability) *Consider* $\Omega_1 = \{0, 1\}$ *and* $\Omega_2 = \{z, o\}$ *and a probability distribution* $Pr(\langle 0, z\rangle) = Pr(\langle 1, o\rangle) = \frac{1}{2}$ *and* $Pr(\langle 0, o\rangle) = Pr(\langle 1, z\rangle) = 0$. *This* **cannot** *be represented as a product* $\mathbf{p}_1 \otimes \mathbf{p}_2$.

In other words: Show that $(\frac{1}{2}, 0, 0, \frac{1}{2})$ *cannot be obtained as a (tensor) product of two two-dimensional vectors. Give an example of a probability distribution which could be represented this way.*

Given a $n \times m$ **matrix A** *and a* $k \times l$ *matrix* **B***:*

$$\mathbf{A} = \begin{pmatrix} a_{11} & \dots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nm} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & \dots & b_{1l} \\ \vdots & \ddots & \vdots \\ b_{k1} & \dots & b_{kl} \end{pmatrix}$$

The **tensor** *or* **Kronecker product** $\mathbf{A} \otimes \mathbf{B}$ *is a* $nk \times ml$ *matrix:*

$$\mathbf{A} \otimes \mathbf{B} = \begin{pmatrix} a_{11}\mathbf{B} & \dots & a_{1m}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{n1}\mathbf{B} & \dots & a_{nm}\mathbf{B} \end{pmatrix}$$

Special cases are **square matrices** *(*$n = m$ *and* $k = l$*) and* **vectors** *(row* $n = k = 1$*, column* $m = l = 1$*).*