Information and Coding Theory CO349

Tutorial

Sheet 3 - Questions

Exercise 1 (Biggs, Ex 5.15) *Consider the channel* Γ *which accepts as input* $1, 2, \ldots, 2n$ *and outputs* 0 *iff the input is* **even** *and* 1 *for* **odd** *input. Write down the channel matrix.*

Show that for any input distribution $\mathbf{p}$ *and* $\mathbf{q} = \mathbf{p}\Gamma$ *we have* $H(\mathbf{p}) - H(\Gamma; \mathbf{p}) = H(\mathbf{q})$.

Furthermore, show that the channel capacity $\gamma = 1$ *and explain this.*

Exercise 2 (Exam 2015) *Consider the following channel (matrix)* $\Gamma = (\Gamma_{ij})_{i,j=1}^{8}$ *with*

$$\Gamma_{ij} = \frac{1}{8}$$

for $i, j = 1, \ldots, 8$, *and the distributions* $\mathbf{p} = (1, 0, 0, 0, 0, 0, 0, 0)$ *and* $\mathbf{p}^* = (\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8})$. *Is* Γ *indeed a channel matrix?*

Describe briefly what effect this channel has on any message sent through it. Compute $\mathbf{H}(\Gamma; \mathbf{p})$ *and* $\mathbf{H}(\mathbf{p})$ *as well as* $\mathbf{H}(\Gamma; \mathbf{p}^*)$ *and* $\mathbf{H}(\mathbf{p}^*)$, *respectively.*

Exercise 3 (Biggs, Ex 6.15) *If require a (binary) code* $C \subseteq \mathbb{B}^6$ *with information rate* $\rho \geq 0.35$, *what is the smallest number of code-words in* C.