Quantum Computation (CO484)

Quantum Physics and Concepts

Herbert Wiklicky

herbert@doc.ic.ac.uk

Autumn 2018

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7. Grover's Search Algorithm
8. Shor's Quantum Factorisation
9. [Quantum Error Correction]

Practicalities

Two Lecturers

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Herbert Wiklicky `~herbert/teaching.html`

h.wiklicky@imperial.ac.uk

Teaching $3\frac{1}{2}$ weeks until 30 October

Open-book coursework test 30 October (or 26?)

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Different classes, different background, different applications.

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If in doubt, read the guidelines available at the link above ☺

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Simon Singh: *Code Book*, Forth Estate, 1999.

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Manjit Kumar: *Quantum – Einstein, Bohr and Their Great Debate about the Nature of Reality*, Icon Books 2009

Photoelectric Effect – Millikan Experiment

Experimental Setup:

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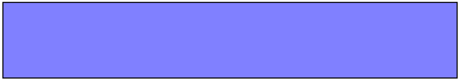

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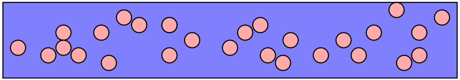

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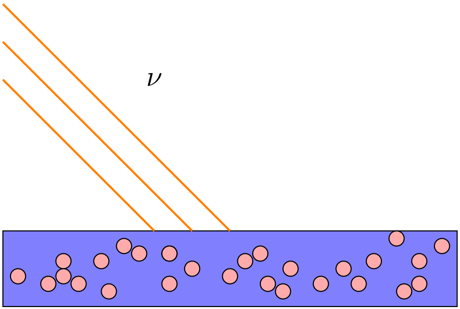

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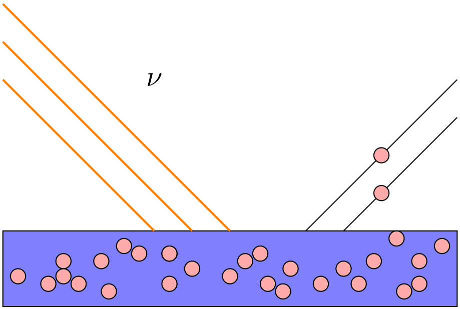

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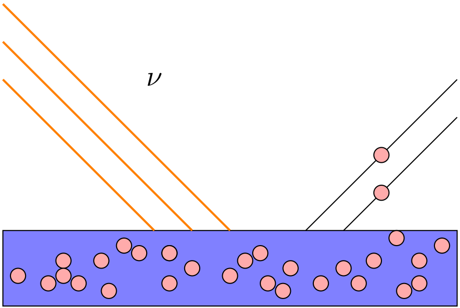

Observed: The velocity, and thus kinetic energy, of the emitted electrons depends not on the intensity of the incoming light but only on its “colour”, i.e. frequency ν.

Radiation Law

Observed relationship:

$$W_k = h\nu - W_e$$

W_k …Kinetic Energy of Electron
W_e …Escape Energy of Material
ν …Frequency of Light
h …Plank's Constant

$$\begin{aligned} h &= 6.62559 \cdot 10^{-34} Js \\ \hbar &= \frac{h}{2\pi} = 1.05449 \cdot 10^{-34} Js \end{aligned}$$

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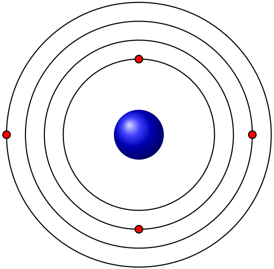

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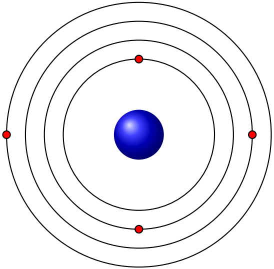

In this way one can also explain the spectral emissions (and absorption) of various elements, e.g. to analyse the material composition of stars (and to make great fireworks).

Quantum Paradoxes and Myths

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7. Whereof one cannot speak, thereof one must be silent.
Ludwig Wittgenstein: *Tractatus Logico-Philosophicus*, 1921

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Each area has its own language which however often applies only to classical entities – for the quantum world we often have simply the wrong vocabulary.

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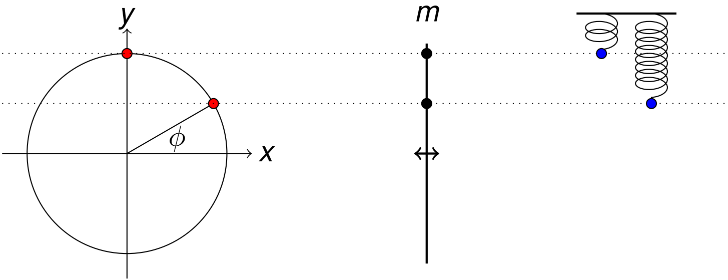
Related Questions: What is our knowledge of what? How do we obtain this information? What is a description on how the system changes?

Harmonic Oscillator or just a “Shadow”

One can observe the same “behaviour” of the shadow of a rotating object or an object on a spring.



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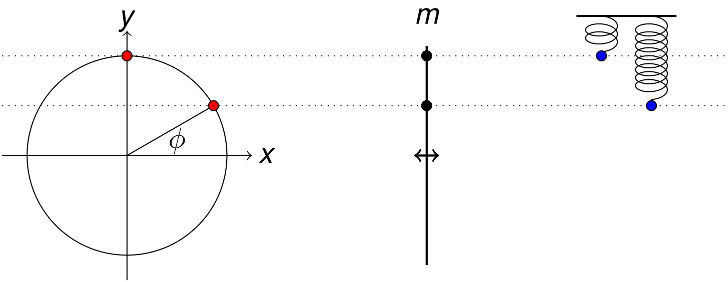

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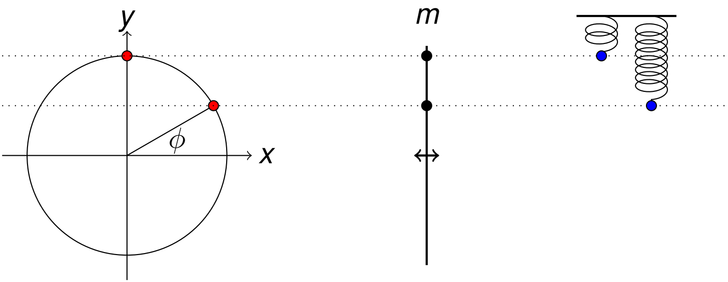

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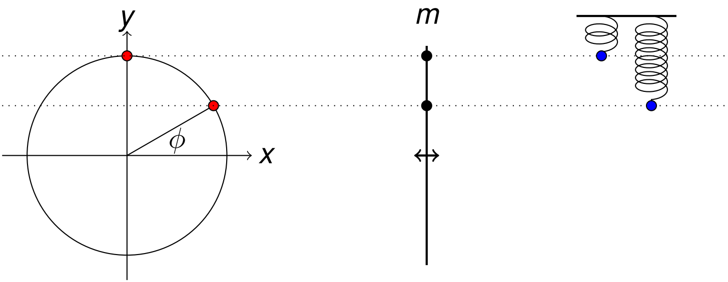

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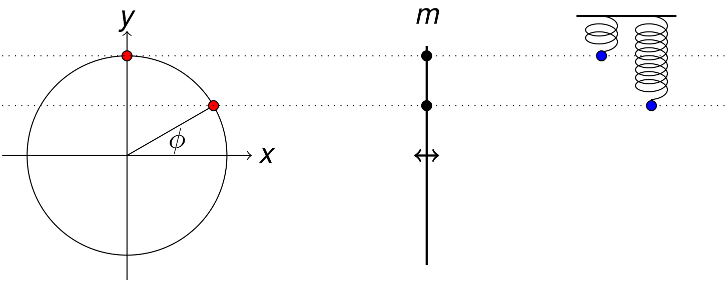

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Dynamics: $(x, y)(t) = (\cos(t), \sin(t))$ or also: $\phi(t) = t$

Postulates for Quantum Mechanics [$*$]

- Observables and states of a system are represented by *hermitian* (i.e. self-adjoint) elements *a* of a C*-algebra $\mathcal{A}$ and by *states* *w* (i.e. normalised linear functionals) over this algebra.

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Key Notions: A quantum systems is (may be) in a certain state, but physicists have to decide which properties they want to observe before a measurement is made (which instrument?).

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There are additional mathematical details in order to deal with “real” quantum physics, e.g. systems an infinite degree of freedom; for quantum computation it is however enough to study finite-dimensional Hilbert spaces.

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Two states can be combined to form a new state $\alpha |x\rangle + \beta |y\rangle$ as long as $|\alpha|^2 + |\beta|^2 = 1$, by **superposition**.

Consequence: We can compute with many inputs in parallel.

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P.A.M. Dirac "invented" the bra-ket notation (most likely inspired by the limitations of old mechanical type-writers); Simply "take the inner product apart" to denote vectors in $\mathcal{H}$:

$$\text{inner product } \langle x|y\rangle \quad = \quad \text{product } \langle x| \cdot |y\rangle$$

For indexed sets of vectors $\{\mathbf{x}_i\}$ (maybe because typographic "typing" was problematic) different notations are used:

$$\mathbf{x}_i = \vec{x}_i = \mathfrak{x}_i = |i\rangle$$

Quantum States and Vectors

Finite quantum states can be described by vectors in $\mathbb{C}^n$, e.g.

$$\vec{\psi} = |\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{or} \quad \langle\phi| = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

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- to enumerate coordinates of one vector, e.g. $\vec{\psi}_1 = 1/\sqrt{2}$, or better perhaps: $|0\rangle_1 = 0$.

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then we have: $\mathbf{A} = \sum_i \lambda_i \mathbf{P}_i$ (Spectral Theorem).

Heisenberg's Uncertainty Relation

Theorem

For two observables $\mathbf{A}_1$ *and* $\mathbf{A}_2$ *we have:*

$$(\Delta_{|x\rangle}\mathbf{A}_1)(\Delta_{|x\rangle}\mathbf{A}_2) \geq \frac{1}{2}\left|\left(\langle x|\ [\mathbf{A}_1, \mathbf{A}_2]\ |x\rangle\right)\right|$$

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In **classical** physics observables always commute, are *comensurable*, i.e. $[\mathbf{A}_1, \mathbf{A}_2] = 0$. In **quantum** physics for most observables $[\mathbf{A}_1, \mathbf{A}_2] \neq 0$, i.e. the observable algebra is typically non-commutative or non-abelian (cf. multiplication of (complex) numbers vs multiplication of matrices).

Quantum Dynamics

- The **dynamics** of a (closed) system is described by the Schrödinger Equation:

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Theorem

For any self-adjoint operator $\mathbf{A}$ *the operator*

$$\exp(i\mathbf{A}) = e^{i\mathbf{A}} = \sum_{n=0}^{\infty} \frac{(i\mathbf{A})^n}{n!}$$

is a unitary operator.

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There are a number of immediate consequence of the postulates.

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The mathematical structure has also consequences for any **Quantum Logic**, e.g. De Morgan fails, 'Tertium non datur' is not guaranteed, etc.

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Quantum Computation
Given a desired computation (dynamics).
What quantum device (e.g. circuit) is needed to obtain this?

Quantum Computation

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When will (cheap) quantum computers be available? What will be a **killer application** for quantum computation? When will we reach quantum supremacy?