Quantum Computation (CO484)

Quantum Cryptography with No Cloning

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Cloning of Qubits?

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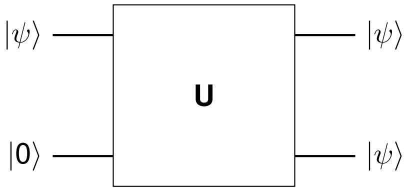

Theorem (No Cloning Theorem)

The exists no unitary transformation **U** *such that*

$$\mathbf{U}\,|\psi\rangle\,|0\rangle = |\psi\rangle\,|\psi\rangle$$

for all qubits $|\psi\rangle \in \mathbb{C}^2$.

Argument

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$$\begin{aligned}\mathbf{U}(\alpha|\psi\rangle + \beta|\phi\rangle)|0\rangle &= \alpha\mathbf{U}(|\psi\rangle)|0\rangle + \beta\mathbf{U}(|\phi\rangle)|0\rangle \\ &= \alpha|\psi\rangle|\psi\rangle + \beta|\phi\rangle|\phi\rangle\end{aligned}$$

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but also if **U** is a cloning operator:

$$\begin{aligned}\mathbf{U}(\alpha|\psi\rangle+\beta|\phi\rangle)|0\rangle &= (\alpha|\psi\rangle+\beta|\phi\rangle)(\alpha|\psi\rangle+\beta|\phi\rangle) \\ &= \alpha^2|\psi\rangle|\psi\rangle+\beta^2|\phi\rangle|\phi\rangle \\ &\quad +\alpha\beta|\psi\rangle|\phi\rangle+\alpha\beta|\phi\rangle|\psi\rangle\end{aligned}$$

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Only for $\alpha=0$ or $\beta=0$ we have

$$\begin{aligned}\alpha|\psi\rangle|\psi\rangle+\beta|\phi\rangle|\phi\rangle &= \alpha^2|\psi\rangle|\psi\rangle+\beta^2|\phi\rangle|\phi\rangle \\ &\quad +\alpha\beta|\psi\rangle|\phi\rangle+\alpha\beta|\phi\rangle|\psi\rangle\end{aligned}$$

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$$\mathbf{U}(|\psi\rangle \otimes |0\rangle) \approx |\psi\rangle \otimes |\psi\rangle \quad \text{and} \quad \mathbf{U}(|\phi\rangle \otimes |0\rangle) \approx |\phi\rangle \otimes |\phi\rangle$$

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By unitarity – **U** preserving inner products – we get

$$(|\psi\rangle\,|0\rangle)^\dagger(|\phi\rangle\,|0\rangle) = \langle\psi|\phi\rangle\,\langle 0|0\rangle = \langle\psi|\phi\rangle \approx \langle\psi|\phi\rangle^2$$

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Thus $\langle\psi|\phi\rangle \approx 0$ or $\langle\psi|\phi\rangle \approx 1$.

Communication on Insecure Channels

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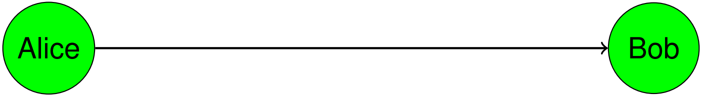

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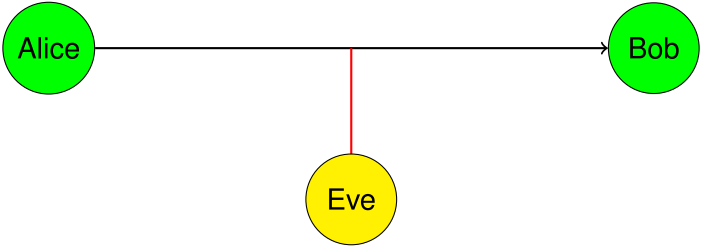

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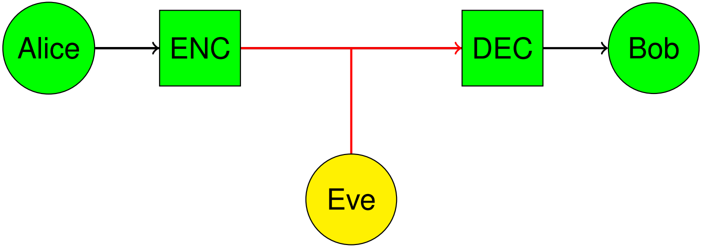

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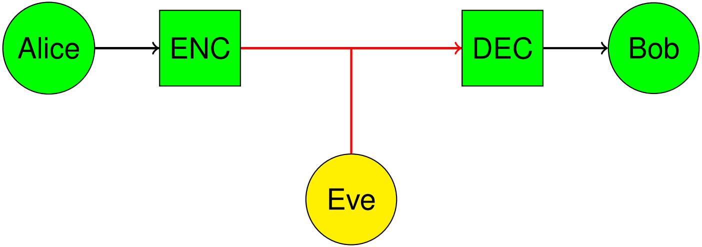

$$
\begin{aligned}
ENC(T, K_A) &= M \\
DEC(M, K_B) &= T \\
DEC(ENC(T, K_A), K_B) &= T
\end{aligned}
$$

One-Time-Pad or Vernam Cipher

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Step 0. Alice and Bob share a common, random key K.

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Caveat: Never ever reuse random key K!

Example

T	0	1	1	0	1	1

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K	$\oplus$	1	1	1	0	1	0

Example

$$\begin{array}{lcrrrrrr} T & & 0 & 1 & 1 & 0 & 1 & 1 \\ K & \oplus & 1 & 1 & 1 & 0 & 1 & 0 \\ \hline M & & 1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{cccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$

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Authentication. Is sender really Alice?

Intrusion Detection. Is Eve eavesdropping?

BB84

Charles Bennett and Gilles Brassard 1984

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The protocol is based on the use of two (computational) bases:

$$\Leftrightarrow\!\!\!\!\Updownarrow = \{|\updownarrow\rangle, |\leftrightarrow\rangle\} = \{(1,0)^T, (0,1)^T\}$$

$$\boxtimes = \{|\nwarrow\!\!\searrow\rangle, |\swarrow\!\!\nearrow\rangle\} = \{\frac{1}{\sqrt{2}}(-1,1)^T, \frac{1}{\sqrt{2}}(1,1)^T\}$$

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Interpretation of messages in both basis

M	$\leftrightarrow\!\!\!\!\!\!\updownarrow$	$\nwarrow\!\!\!\!\searrow\!\!\!\!\!\!\swarrow\!\!\!\!\nearrow$
0	$\vert\leftrightarrow\rangle$	$\vert\swarrow\!\!\!\!\nearrow\rangle$
1	$\vert\updownarrow\rangle$	$\vert\nwarrow\!\!\!\!\searrow\rangle$

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This is due to the following obvious facts that:

$$
\begin{aligned}
|\searrow\rangle &= \frac{1}{\sqrt{2}}(|\updownarrow\rangle - |\leftrightarrow\rangle) & \qquad |\updownarrow\rangle &= \frac{1}{\sqrt{2}}(|\nearrow\rangle + |\searrow\rangle) \\
|\nearrow\rangle &= \frac{1}{\sqrt{2}}(|\updownarrow\rangle + |\leftrightarrow\rangle) & \qquad |\leftrightarrow\rangle &= \frac{1}{\sqrt{2}}(|\nearrow\rangle - |\searrow\rangle)
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Step 3. Over the classical channel Alice and Bob compare which basis they used for each bit. If they agree they keep it otherwise they drop it.

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- **Step 2.a** Alice encodes the bits accordingly in the bases and sends the qubits to Bob.
- **Step 2.b** Bob randomly chooses *n* times whether to use  or  to measure the qubits he got and measures them.
- **Step 3.** Over the classical channel Alice and Bob compare which basis they used for each bit. If they agree they keep it otherwise they drop it.
- **Step 4.a** Bob choose a part (e.g. half) of the transmitted bits (drops them) and compares them openly with Alice.
- **Step 4.b** If these test bits do not agree (subject to transmission errors) Alice and Bob conclude that Eve was eavesdropping and abandon

Example

K_A	0	1	1	0	1	1	1	0	1	0	1	0

Example

K_A	0	1	1	0	1	1	1	0	1	0	1	0
B_A	+	+	×	+	+	+	×	+	×	×	×	+

Example

K_A	0	1	1	0	1	1	1	0	1	0	1	0
B_A	+	+	×	+	+	+	×	+	×	×	×	+
	↔	↕	⤡	↔	↕	↕	⤡	↔	⤡	⤢	⤡	↔

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K_A	0	1	1	0	1	1	1	0	1	0	1	0
B_A	✛	✛	✕	✛	✛	✛	✕	✛	✕	✕	✕	✛
	↔	↕	⤡	↔	↕	↕	⤡	↔	⤡	⤢	⤡	↔
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B_B	✕	✛	✕	✕	✛	✕	✛	✛	✕	✕	✕	✛

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	↔	↕	⤡	↔	↕	↕	⤡	↔	⤡	⤢	⤡	↔
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B_B	×	+	×	×	+	×	+	+	×	×	×	+
obs	⤢	↕	⤡	⤡	↕	⤢	↕	↔	⤡	⤢	⤡	↔

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	↔	↕	⤡	↔	↕	↕	⤡	↔	⤡	⤢	⤡	↔
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B_B	✕	✛	✕	✕	✛	✕	✛	✛	✕	✕	✕	✛
obs	⤢	↕	⤡	⤡	↕	⤢	↕	↔	⤡	⤢	⤡	↔
K_B	0	1	1	1	1	0	1	0	1	0	1	0

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B_A	✛	✛	✕	✛	✛	✛	✕	✛	✕	✕	✕	✛
	↔	↕	⤡	↔	↕	↕	⤡	↔	⤡	⤢	⤡	↔
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B_B	✕	✛	✕	✕	✛	✕	✛	✛	✕	✕	✕	✛
obs	⤢	↕	⤡	⤡	↕	⤢	↕	↔	⤡	⤢	⤡	↔
K_B	0	1	1	1	1	0	1	0	1	0	1	0
		√	√		√			√	√	√	√	√

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B_A	✛	✛	✕	✛	✛	✛	✕	✛	✕	✕	✕	✛
	$\leftrightarrow$	$\updownarrow$	⤡	$\leftrightarrow$	$\updownarrow$	$\updownarrow$	⤡	$\leftrightarrow$	⤡	⤢	⤡	$\leftrightarrow$
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
B_B	✕	✛	✕	✕	✛	✕	✛	✛	✕	✕	✕	✛
obs	⤢	$\updownarrow$	⤡	⤡	$\updownarrow$	⤢	$\updownarrow$	$\leftrightarrow$	⤡	⤢	⤡	$\leftrightarrow$
K_B	0	1	1	1	1	0	1	0	1	0	1	0
		$\surd$	$\surd$		$\surd$			$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
K		1	1		1			0	1	0	1	0

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The idea is to use a **non-orthogonal** basis to encode 0 and 1, e.g.

$$B = \{|\leftrightarrow\rangle, |\swarrow\!\!\!\nearrow\rangle\} = \{(1,0)^T, \frac{1}{\sqrt{2}}(1,1)^T\}$$

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Again – as in BB84 – some bits can be sacrificed to see if an extensive number of “transmission errors” indicates that Eve was eavesdropping and abandon transmission.

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In the average three quarters of the qubits have to be discarded.

Example

K_A	0	0	1	0	1	0	1	0	1	1	1	0

Example

K_A	0	0	1	0	1	0	1	0	1	1	1	0
	$\leftrightarrow$	$\leftrightarrow$	$\swarrow\!\!\!\nearrow$	$\leftrightarrow$	$\swarrow\!\!\!\nearrow$	$\leftrightarrow$	$\swarrow\!\!\!\nearrow$	$\leftrightarrow$	$\swarrow\!\!\!\nearrow$	$\swarrow\!\!\!\nearrow$	$\swarrow\!\!\!\nearrow$	$\leftrightarrow$

Example

K_A	0	0	1	0	1	0	1	0	1	1	1	0
	$\leftrightarrow$	$\leftrightarrow$	⤢	$\leftrightarrow$	⤢	$\leftrightarrow$	⤢	$\leftrightarrow$	⤢	⤢	⤢	$\leftrightarrow$
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$

Example

K_A	0	0	1	0	1	0	1	0	1	1	1	0
	↔	↔	⤢	↔	⤢	↔	⤢	↔	⤢	⤢	⤢	↔
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B_B	×	+	×	×	+	×	+	+	×	+	×	+

Example

K_A	0	0	1	0	1	0	1	0	1	1	1	0
	↔	↔	⤢	↔	⤢	↔	⤢	↔	⤢	⤢	⤢	↔
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B_B	×	+	×	×	+	×	+	+	×	+	×	+
obs	⤡	↔	⤢	⤡	↕	⤡	↔	↔	⤢	↕	⤢	↔

Example

K_A	0	0	1	0	1	0	1	0	1	1	1	0
	↔	↔	⤢	↔	⤢	↔	⤢	↔	⤢	⤢	⤢	↔
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B_B	×	+	×	×	+	×	+	+	×	+	×	+
obs	⤡	↔	⤢	⤡	↕	⤡	↔	↔	⤢	↕	⤢	↔
K_B	0	?	?	0	1	0	?	?	?	1	?	?

Example

K_A	0	0	1	0	1	0	1	0	1	1	1	0
	↔	↔	⤢	↔	⤢	↔	⤢	↔	⤢	⤢	⤢	↔
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B_B	×	+	×	×	+	×	+	+	×	+	×	+
obs	⤡	↔	⤢	⤡	↕	⤡	↔	↔	⤢	↕	⤢	↔
K_B	0	?	?	0	1	0	?	?	?	1	?	?
	√			√	√	√				√		

Example

K_A	0	0	1	0	1	0	1	0	1	1	1	0
	↔	↔	⤢	↔	⤢	↔	⤢	↔	⤢	⤢	⤢	↔
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B_B	⤡⤢	↔↕	⤡⤢	⤡⤢	↔↕	⤡⤢	↔↕	↔↕	⤡⤢	↔↕	⤡⤢	↔↕
obs	⤡	↔	⤢	⤡	↕	⤡	↔	↔	⤢	↕	⤢	↔
K_B	0	?	?	0	1	0	?	?	?	1	?	?
	√			√	√	√				√		
K	0			0	1	0				1		

EPR

Artur Ekert 1991

The idea is to distribute a key K via pairs of entangled states, for example the **Bell states**:

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This protocol is inspired by the Einstein-Podolsky-Rosen (EPR, 1935) Gedanken-Experiment.

EPR Protocol

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Step 1. A random sequence of entangled 2-qubit states – e.g. $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ – is created. For each such state one of the qubits is given to Alice and Bob, respectively.

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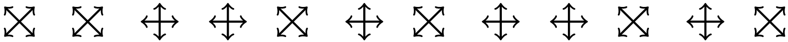
As in BB84 too many “transmission errors” indicate that Eve was eavesdropping and the transmission is abandoned. Ekert proposed a more sophisticated eavesdropping detection (Bell’s theorem).

Example

B_A 

Example

B_A	⤢⤡	⤢⤡	↔↕	↔↕	⤢⤡	↔↕	⤢⤡	↔↕	↔↕	⤢⤡	↔↕	⤢⤡
obs	⤢	⤡	↔	↕	⤢	↔	⤡	↔	↔	⤢	↔	⤢

Example

B_A	$\times$	$\times$	$+$	$+$	$\times$	$+$	$\times$	$+$	$+$	$\times$	$+$	$\times$
obs	⤢	⤡	↔	↕	⤢	↔	⤡	↔	↔	⤢	↔	⤢
K_A	0	1	0	1	0	0	1	0	0	0	0	0

Example

B_A	×	×	+	+	×	+	×	+	+	×	+	×
obs	⤢	⤡	↔	↕	⤢	↔	⤡	↔	↔	⤢	↔	⤢
K_A	0	1	0	1	0	0	1	0	0	0	0	0
B_B	×	+	+	×	×	+	+	+	+	×	×	+

Example

B_A	✕	✕	✛	✛	✕	✛	✕	✛	✛	✕	✛	✕
obs	⤢	⤡	↔	↕	⤢	↔	⤡	↔	↔	⤢	↔	⤢
K_A	0	1	0	1	0	0	1	0	0	0	0	0
B_B	✕	✛	✛	✕	✕	✛	✛	✛	✛	✕	✕	✛
obs	⤢	↔	↔	⤢	⤢	↔	↕	↔	↔	⤢	⤡	↔

Example

B_A	×	×	+	+	×	+	×	+	+	×	+	×
obs	⤢	⤡	↔	↕	⤢	↔	⤡	↔	↔	⤢	↔	⤢
K_A	0	1	0	1	0	0	1	0	0	0	0	0

B_B	×	+	+	×	×	+	+	+	+	×	×	+
obs	⤢	↔	↔	⤢	⤢	↔	↕	↔	↔	⤢	⤡	↔
K_B	0	0	0	0	0	0	1	0	0	0	1	0

Example

B_A	×	×	+	+	×	+	×	+	+	×	+	×
obs	⤢	⤡	↔	↕	⤢	↔	⤡	↔	↔	⤢	↔	⤢
K_A	0	1	0	1	0	0	1	0	0	0	0	0
B_B	×	+	+	×	×	+	+	+	+	×	×	+
obs	⤢	↔	↔	⤢	⤢	↔	↕	↔	↔	⤢	⤡	↔
K_B	0	0	0	0	0	0	1	0	0	0	1	0
	√		√		√	√		√	√	√		

Example

B_A	$\times$	$\times$	$+$	$+$	$\times$	$+$	$\times$	$+$	$+$	$\times$	$+$	$\times$
obs	⤢	⤡	↔	↕	⤢	↔	⤡	↔	↔	⤢	↔	⤢
K_A	0	1	0	1	0	0	1	0	0	0	0	0
B_B	$\times$	$+$	$+$	$\times$	$\times$	$+$	$+$	$+$	$+$	$\times$	$\times$	$+$
obs	⤢	↔	↔	⤢	⤢	↔	↕	↔	↔	⤢	⤡	↔
K_B	0	0	0	0	0	0	1	0	0	0	1	0
	√		√		√	√		√	√	√		
K	0		0		0	0		0	0	0		