Addendum to: Finite variable logics

Ian Hodkinson

Department of Computing, Imperial College, London SW7 2BZ, England.

Email: imh@doc.ic.ac.uk

I am most grateful for the response of readers to the article *Finite Variable Logics* (Bull. EATCS, Oct. 1993, 111–140). Following their comments, I would like to add some remarks and corrections. References refer to those in the original article.

1. pp 112,113,123. The reference [V] should be [Va].

2. p 114. The result that FO+WHILE(<) = PSPACE is due to Vardi [Va, Remarks 2, 4]. The proof is sketched; many of the ideas needed can be found in Chandra & Harel's [CH, Theorem 6.3].

3. p 114. LFP, PFP et. al. are not subsumed by $L^\omega_{\infty\omega}$ on arbitrary classes of structures: for example, LFP can express well-foundedness, but $L^\omega_{\infty\omega}$ cannot. Rubin showed that LFP is expressible by $L^\omega_{\infty\omega}$ on a fixed structure; Kolaitis and Vardi observed that the same holds for the class of all finite structures.

4. p 115. The notation $L^3_{\infty\omega}$ means the 3-variable fragment of $L^\omega_{\infty\omega}$: the $L^\omega_{\infty\omega}$-formulas that are written with at most three variables. Similarly, $L^2_{\infty\omega}$.

5. p 115. I should have mentioned that the 0–1 law for $L^\omega_{\infty\omega}$ was established in [KV].

6. p 119. For a finite structure M, the bound of $|M|^{2k}$ that I gave on the Scott height of M can be trivially improved to $|M|^k$. (For infinite M, $|M|^{2k} = |M|^k$.)

7. p 122. The result of section 6 on ordering the types should be credited to Abiteboul and Vianu [AV2, §3.2]; the proof I gave is adapted from the one in [DLW].

8. p 133, k-variable property. Related results are given in the paper *On bounded theories* by J. Flum (in Proc. Computer Science Logic 91, Berne, eds. E. Boerger, G. Jaeger, H. Kleine Buening, M.M. Richter, Springer LNCS 626, 111–118). There, a first-order theory T in signature L is said to be *k-bounded* if (essentially) every first-order L-formula is T-equivalent to one where at most k distinct variables are bound in any branch of its formation tree. If L is relational, of arity $< k$, then it can be shown (cf. [HS, §3.2]) that T is k-bounded iff for all $n \geq k$, every formula $\varphi(x_1, \ldots, x_n)$ is T-equivalent to a formula $\varphi^*(x_1, \ldots, x_n)$ written with only n variables ('T has the non-monadic n-variable property for all $n \geq k$'). Flum also gives an example (suggested by Ziegler) of a theory T that is not k-bounded for any k, but such that any formula can be equivalently rewritten over T using only one bound variable. (The example of [HS] mentioned on p 133 is in some ways similar.)

9. When the article was written, the status of [DLW] was 'submitted', not 'to appear'.