Addendum to: Finite variable logics

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I am most grateful for the response of readers to the article Finite Variable Logics (Bull. EATCS, Oct. 1993, 111–140). Following their comments, I would like to add some remarks and corrections. References refer to those in the original article.

1. pp 112, 113, 123. The reference [V] should be [Va].

2. p 114. The result that FO+WHILE(<) = PSPACE is due to Vardi [Va, Remarks 2, 4]. The proof is sketched; many of the ideas needed can be found in Chandra & Harel’s [CH, Theorem 6.3].

3. p 114. LFP, PFP et. al. are not subsumed by \( L_{\omega}^{\omega} \) on arbitrary classes of structures: for example, LFP can express well-foundedness, but \( L_{\omega}^{\omega} \) cannot. Rubin showed that LFP is expressible by \( L_{\omega}^{\omega} \) on a fixed structure; Kolaitis and Vardi observed that the same holds for the class of all finite structures.

4. p 115. The notation \( L_{\omega}^{3} \) means the 3-variable fragment of \( L_{\omega}^{\omega} \); the \( L_{\omega}^{\omega} \)-formulas that are written with at most three variables. Similarly, \( L_{\omega}^{2} \).

5. p 115. I should have mentioned that the 0–1 law for \( L_{\omega}^{\omega} \) was established in [KV].

6. p 119. For a finite structure \( M \), the bound of \( |M|^{2k} \) that I gave on the Scott height of \( M \) can be trivially improved to \( |M|^k \). (For infinite \( M \), \( |M|^{2k} = |M|^k \).

7. p 122. The result of section 6 on ordering the types should be credited to Abiteboul and Vianu [AV2, §3.2]; the proof I gave is adapted from the one in [DLW].

8. p 133, \( k \)-variable property. Related results are given in the paper On bounded theories by J. Flum (in Proc. Computer Science Logic 91, Berne, eds. E. Boerger, G. Jaeger, H. Kleine Buening, M.M. Richter, Springer LNCS 626, 111–118). There, a first-order theory \( T \) in signature \( L \) is said to be \( k \)-bounded if (essentially) every first-order \( L \)-formula is \( T \)-equivalent to one where at most \( k \) distinct variables are bound in any branch of its formation tree. If \( L \) is relational, of arity \( < k \), then it can be shown (cf. [HS, §3.2]) that \( T \) is \( k \)-bounded iff for all \( n \geq k \), every formula \( \varphi(x_1, \ldots, x_n) \) is \( T \)-equivalent to a formula \( \varphi'(x_1, \ldots, x_n) \) written with only \( n \) variables (‘\( T \) has the non-monadic \( n \)-variable property for all \( n \geq k \).’) Flum also gives an example (suggested by Ziegler) of a theory \( T \) that is not \( k \)-bounded for any \( k \), but such that any formula can be equivalently rewritten over \( T \) using only one bound variable. (The example of [HS] mentioned on p 133 is in some ways similar.)

9. When the article was written, the status of [DLW] was ‘submitted’, not ‘to appear’. 