Proving Probabilistic Properties of the Itai Rodeh leader election protocol for any Number of Processes

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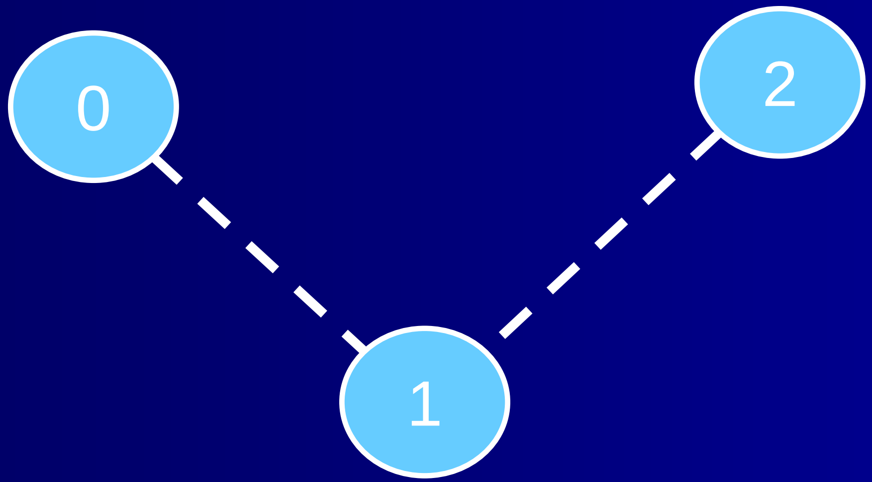
Overview

- Parameterised model checking
  - Classical parameterised model checking problem
  - Proof by induction: Firewire example
  - Extending Firewire & proof probabilistically
- Itai Rodeh leader election protocol
  - Application of induction proof to Itai Rodeh

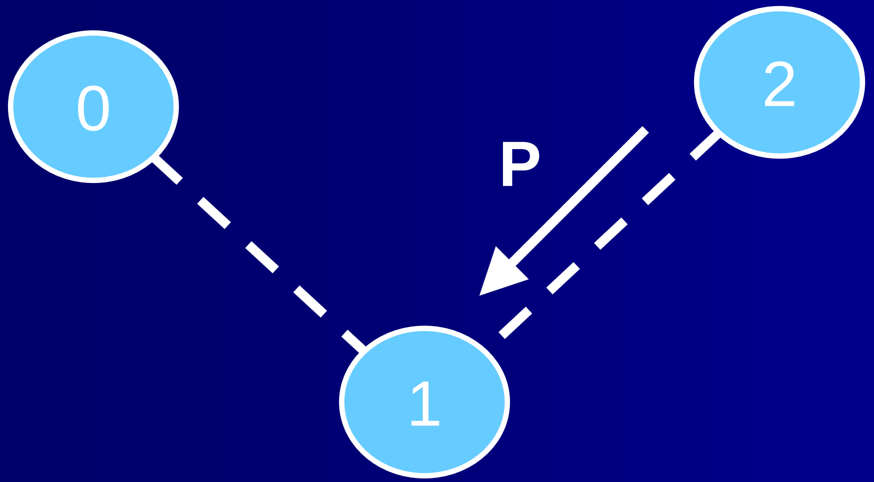
Parameterised Model Checking

- For system $M(N)=p(1) \mathbin{//} p(2) \mathbin{//} \ldots \mathbin{//} p(N)$ can only model check property P for fixed N
- What if we want to verify for any N?
- Undecidable in general but techniques apply for subclasses of system
- E.g. proof by induction [Miller & Calder]
  - Firewire leader election protocol

Parameterised Model Checking



Parameterised Model Checking



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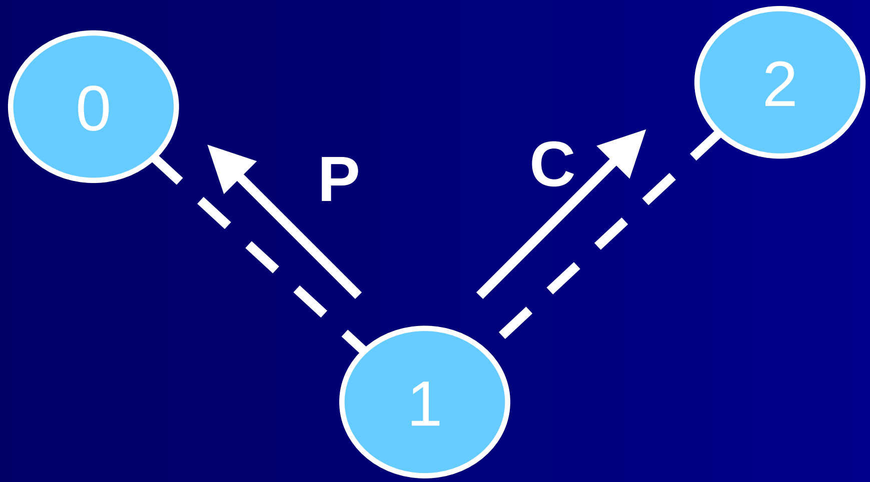

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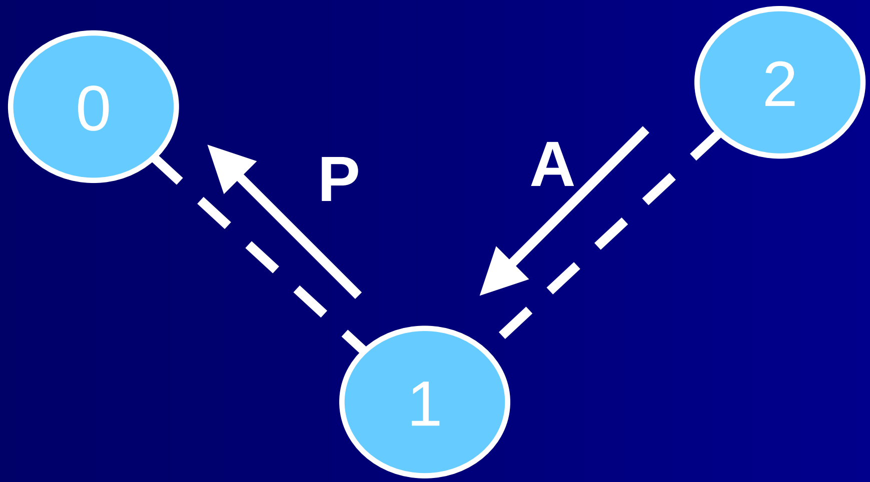

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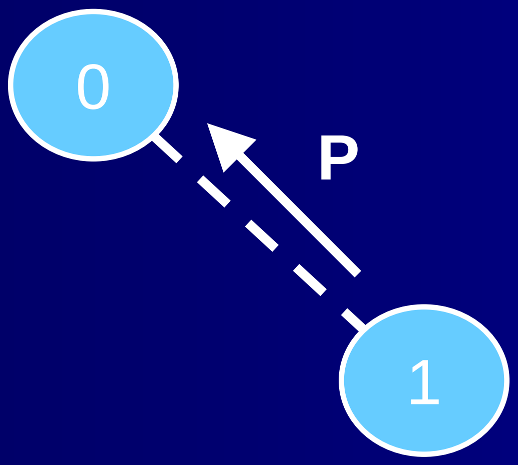

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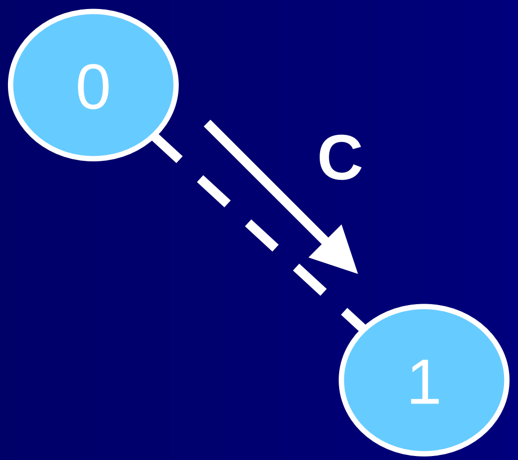

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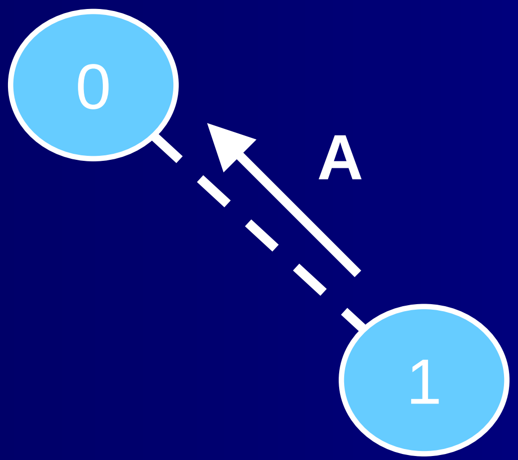


Parameterised Model Checking

Parameterised Model Checking

- Notice that once child node has sent *ack* it no longer takes part
- System is described as *degenerative*
- Can exploit this behaviour
- Prove by induction that certain types of property hold for any number of nodes [Miller & Calder]

Parameterised Model Checking

- Show property holds for `base' system – star topology e.g. "leader will always be elected"
- For any configuration and size of system every execution of model is related (*stutter equivalent)* to execution in model of smaller system

Probabilistic Parameterised Model Checking

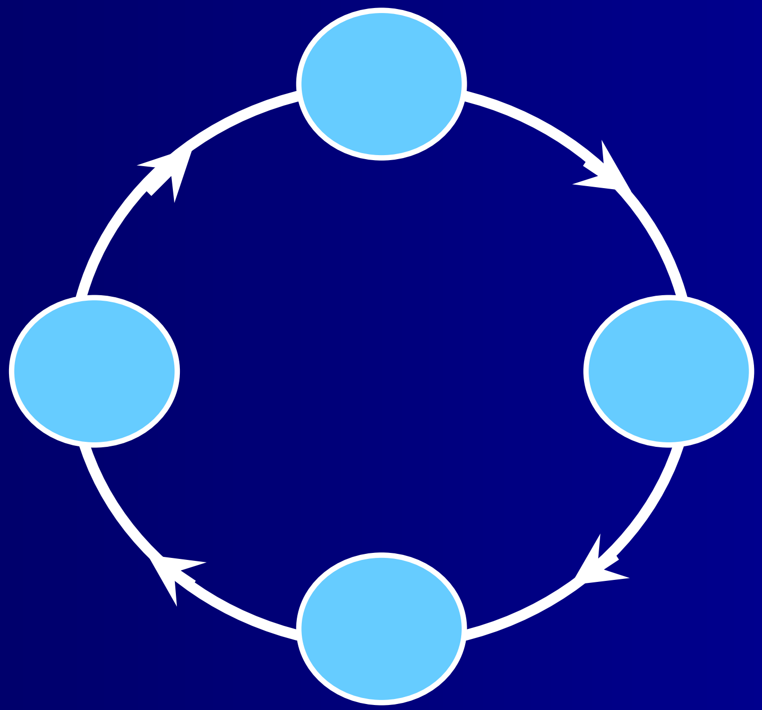
- Can we apply degenerative approach to probabilistic systems?
- Extend Firewire probabilistically
  - Resolve “contention” situations with coin flip
  - Model as MDP in PRISM
- Extend induction proof
  - “Executions” are DTMCs not paths
  - Weak bisimulation instead of stutter equivalence

Probabilistic Parameterised Model Checking

- Can we apply induction approach to any other degenerative probabilistic systems?
- Itai Rodeh leader election protocol for rings?

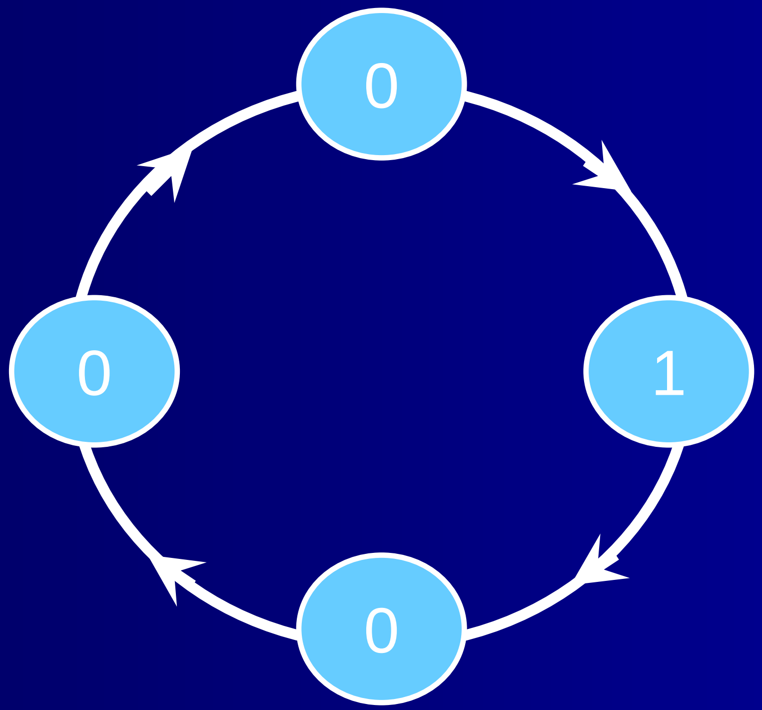
Probabilistic Parameterised Model Checking

- Unidirectional ring of processes:



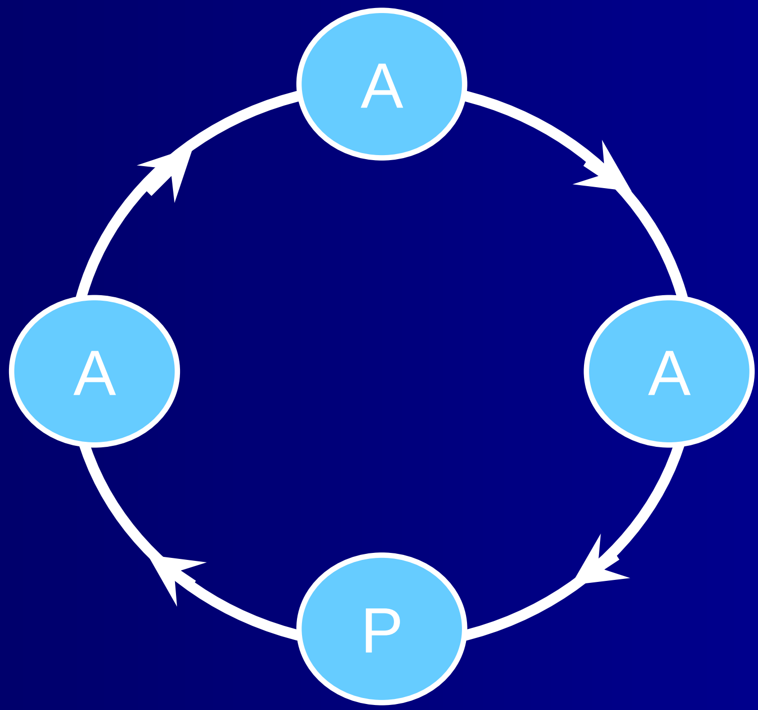
Probabilistic Parameterised Model Checking

- Each process flips coin and chooses 0 or 1 with equal probability



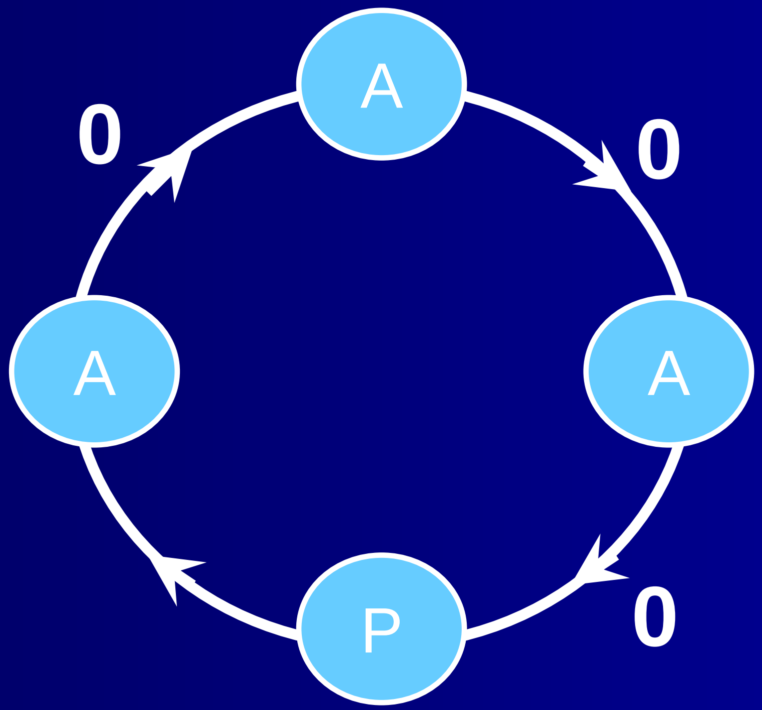
Probabilistic Parameterised Model Checking

- Each process then passes choice to neighbour; if chosen 0 and receive 1 become passive



Probabilistic Parameterised Model Checking

- Counter is then passed around ring by each active process; counter is incremented by any passive process



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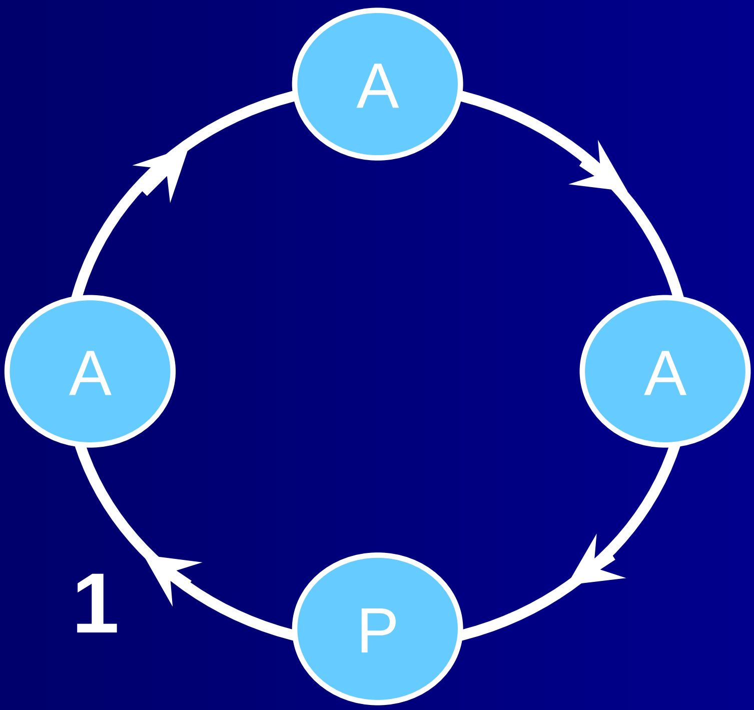
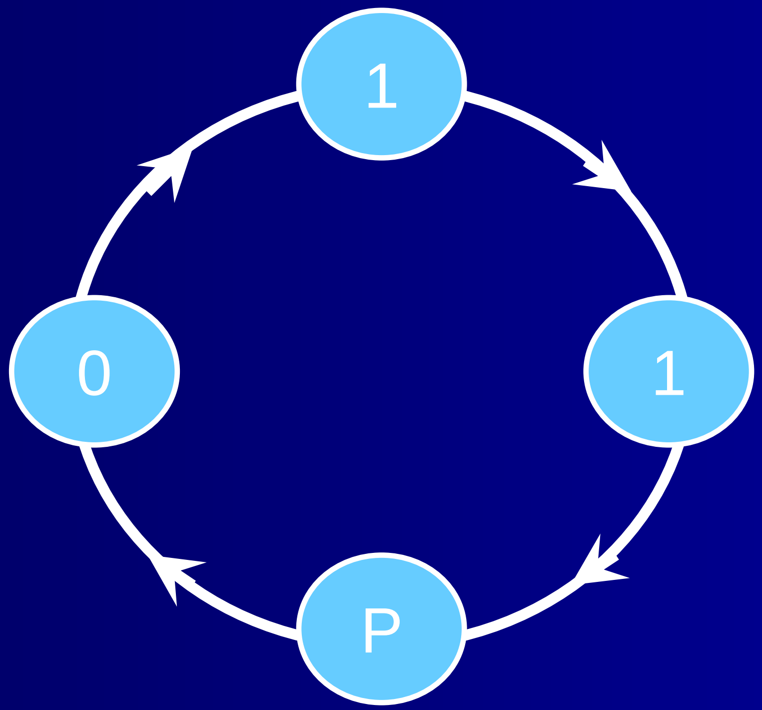


Probabilistic Parameterised Model Checking

- If any process receives counter of value N-1 then he becomes leader, else active processes choose again

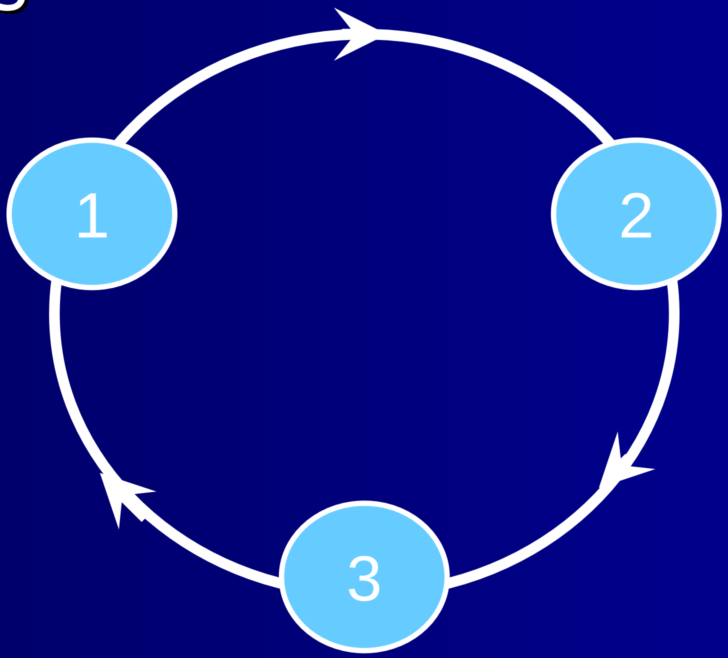


Probabilistic Parameterised Model Checking

- Itai Rodeh is partially degenerative
  - When process becomes passive it only passes on messages…
  - …but it can increment counter, whose max value is dependent on N
- Modelled in PRISM as an MDP [Kwiatkowska et al., Fokkink et al.]
- Our model is variation of these using buffers of size N

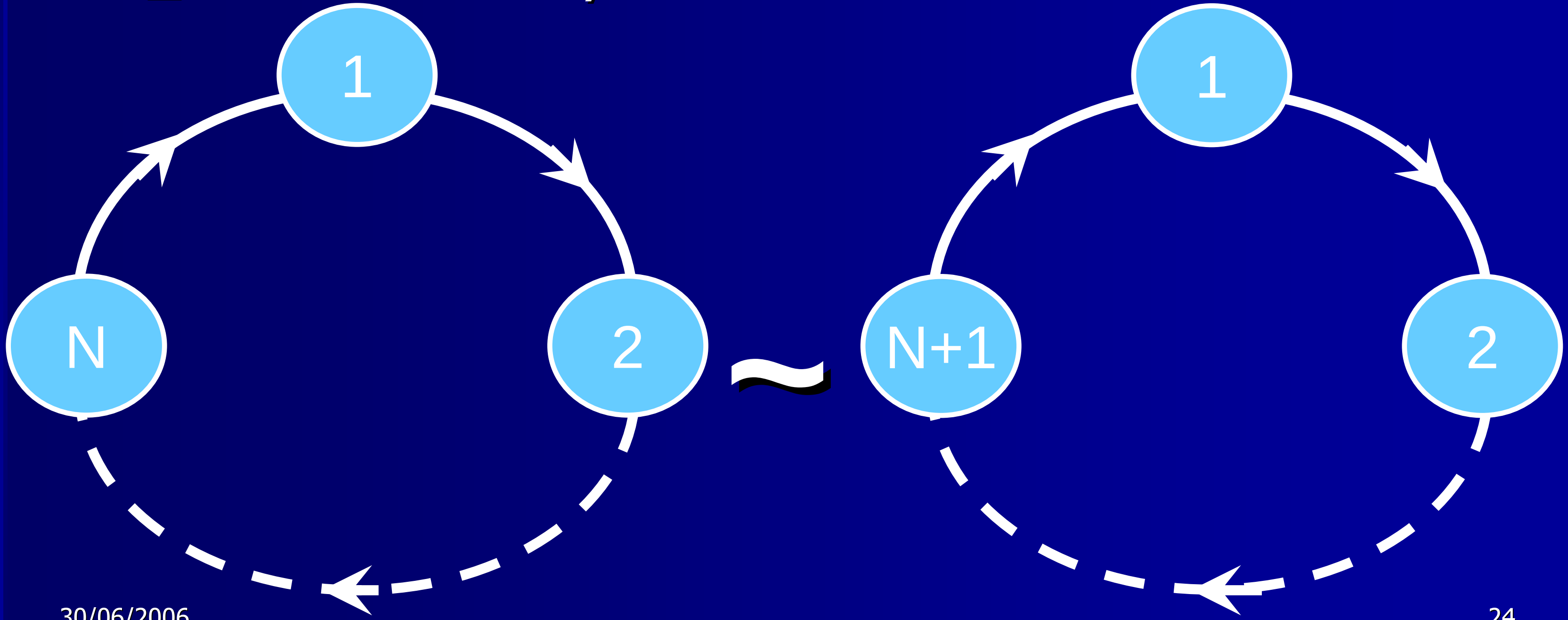
Probabilistic Parameterised Model Checking

- Apply same approach as for Firewire:
  - Base system is ring of size 3, say (could be anything that we can model check)



Probabilistic Parameterised Model Checking

- For N>2 show that *M_N ~ M_N+1* where:
  - ~ is some relationship between executions of MDPs
  - *M_N* is model of system of size N



Probabilistic Parameterised Model Checking

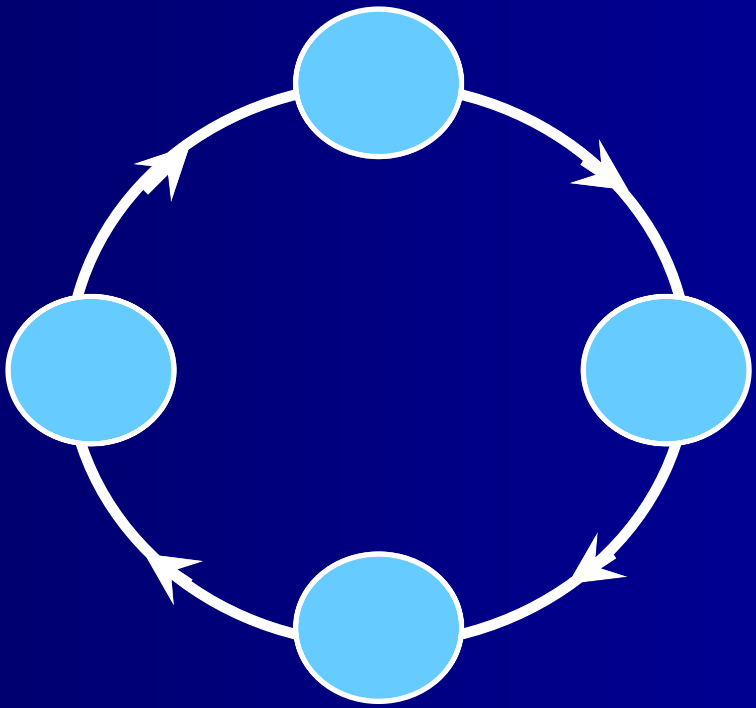
- Introduce series of "intermediate" models
- Define model Mc_N as for M_N but with buffer length N+1
- For system of size N, never more than N messages in buffers [Fokkink et al]
- Mc_N isomorphic to M_N

$$M\_N = Mc\_N$$

Probabilistic Parameterised Model Checking

- Define model *Mp_N*
- As for *M_N+1* except initial nondeterministic choice over processes with one selected as passive
- Passive process does not increment counter

Probabilistic Parameterised Model Checking



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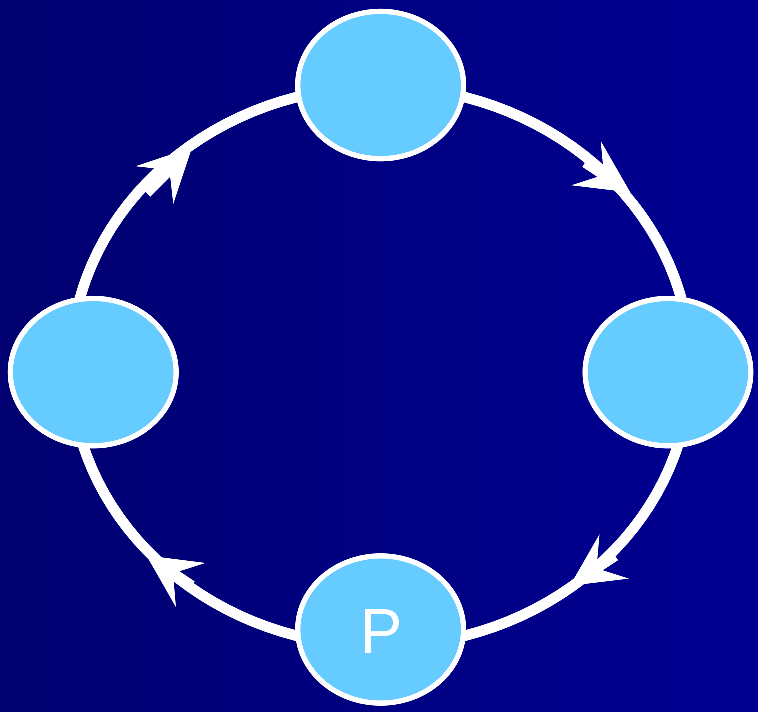

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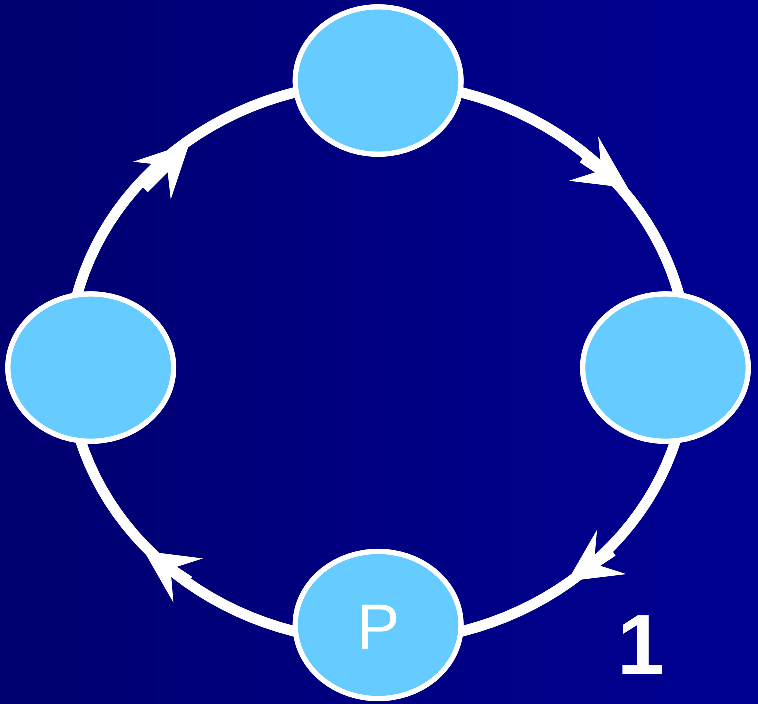
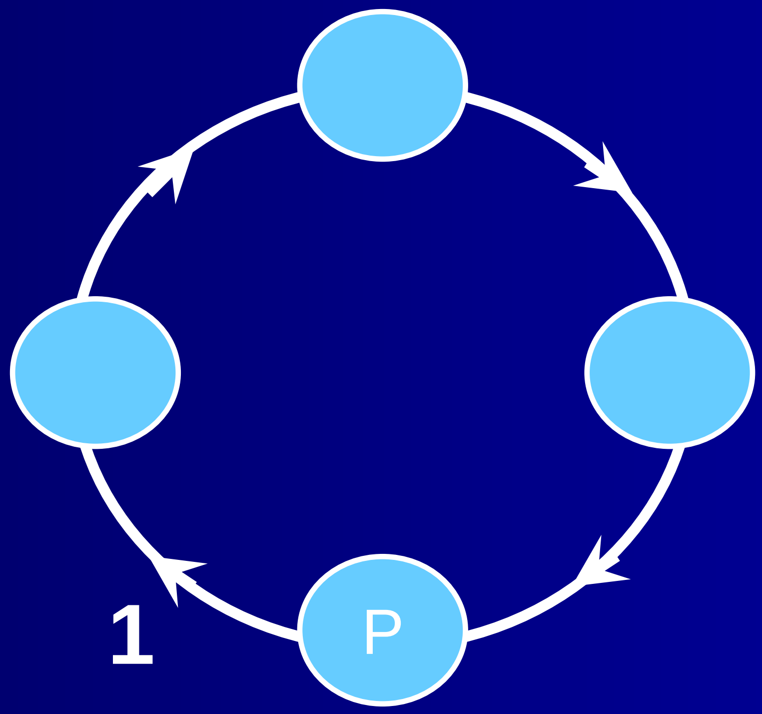


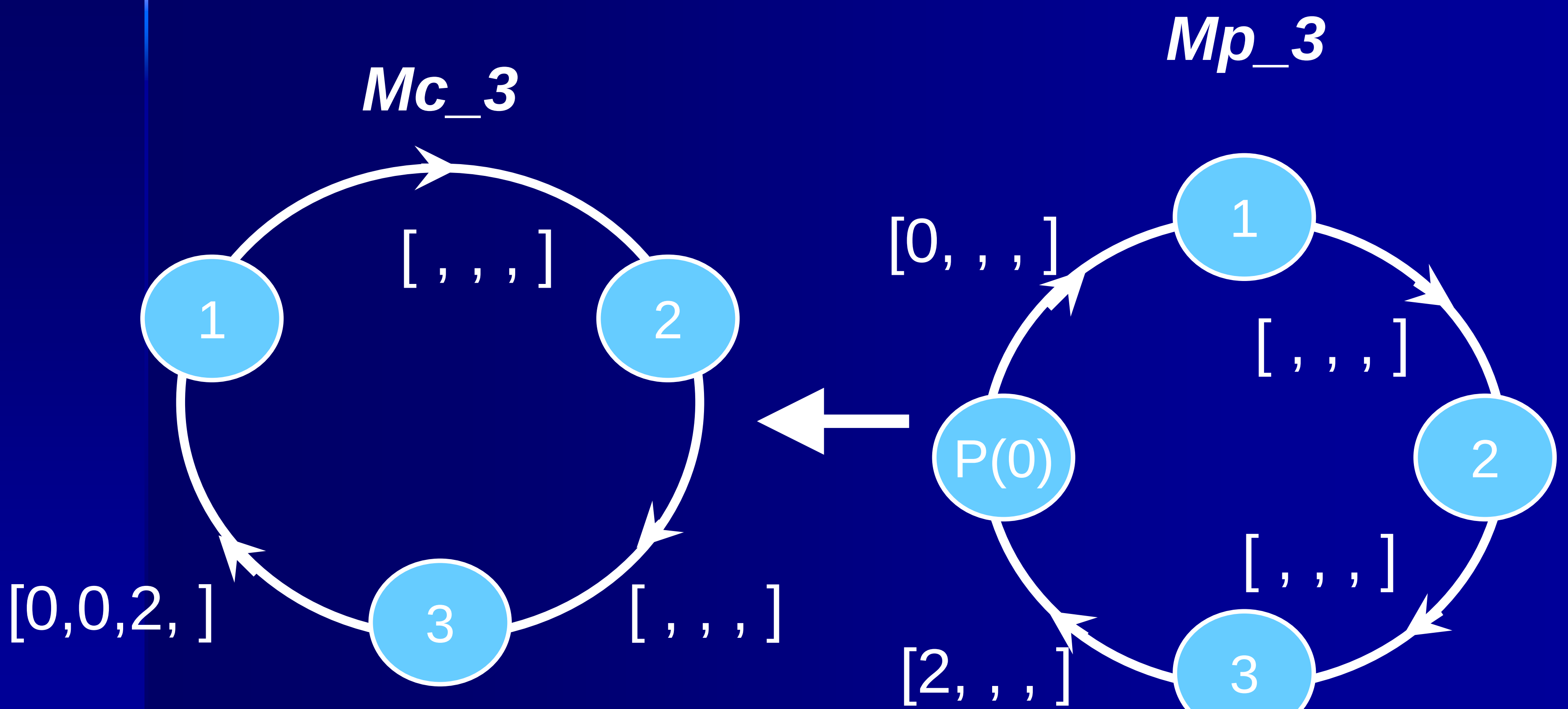
Probabilistic Parameterised Model Checking



Probabilistic Parameterised Model Checking

- In *Mp_N* buffers never contain>N messages
- If *p* initial passive, number of messages between *p-1* and *p+1* never > N
  – NB count *p* as a "buffer"
- Assume process *N+1* initial passive
- For *Mp_N* relate buffers between *N* and 1 to buffer between *N* and 1 in *Mc_N*

Probabilistic Parameterised Model Checking



Probabilistic Parameterised Model Checking

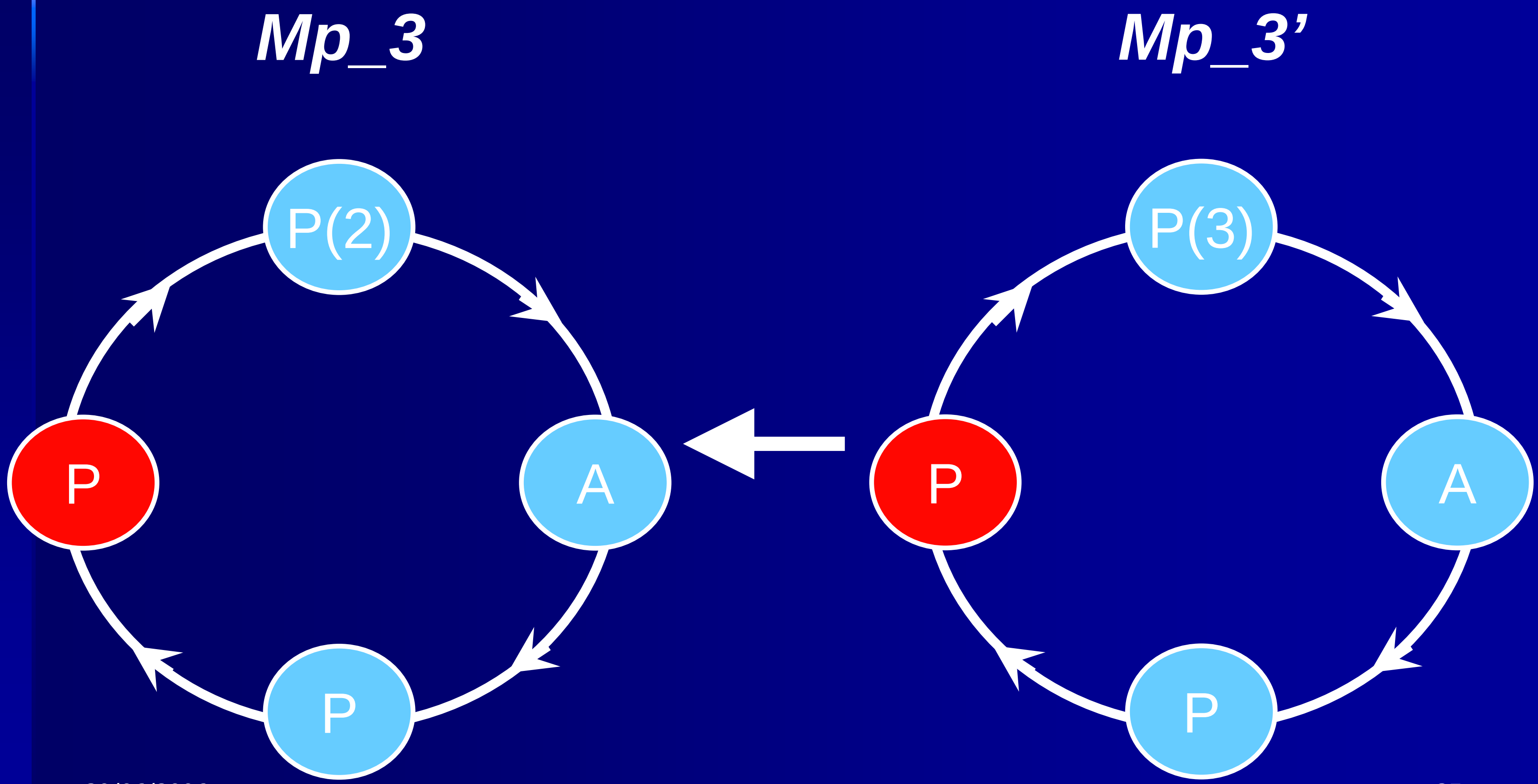
- For each execution of Mp_N there exists execution of Mc_N that is weakly bisimilar (under relation) and vice versa

$$M\_N = Mc\_N \approx Mp\_N$$

Probabilistic Parameterised Model Checking

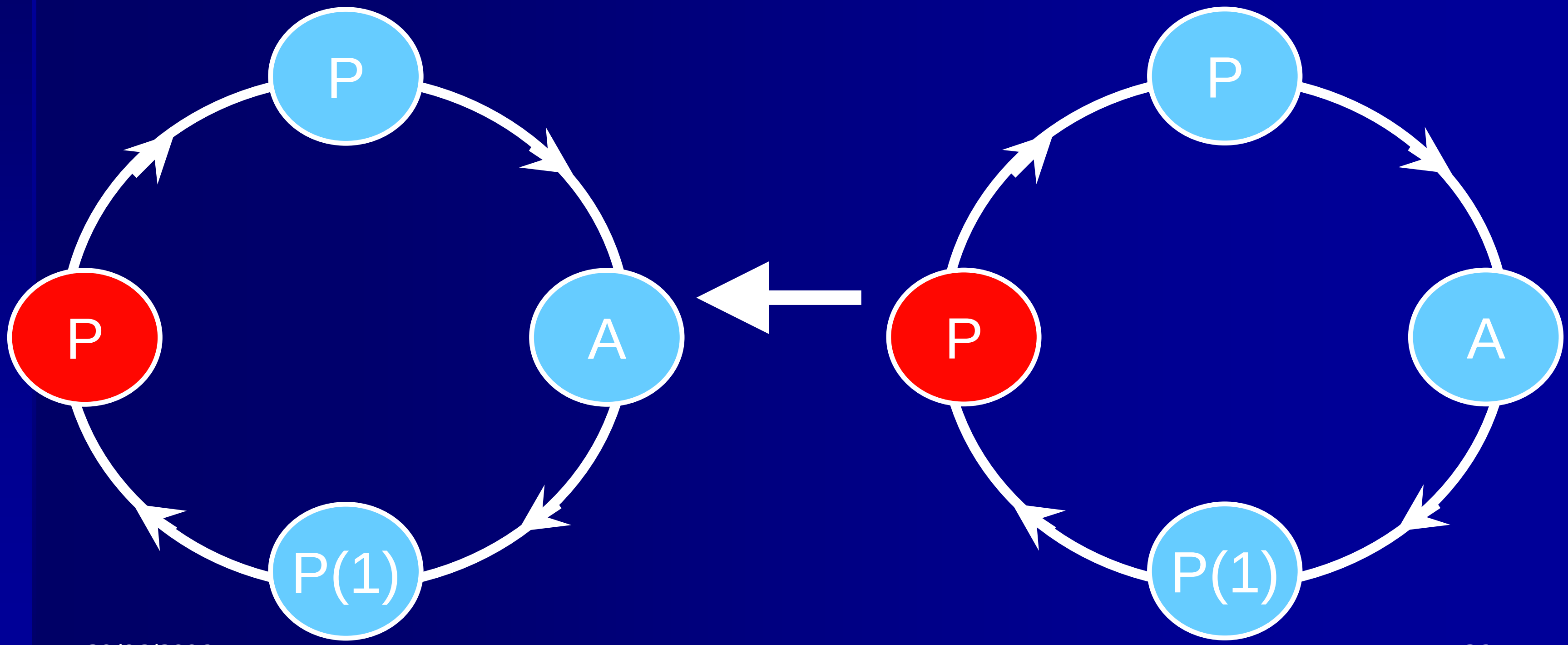
- Define *Mp_N′* as for *Mp_N* but initial passive increments counter
- Assume process *p* initial passive
- If counter has passed through *p* then relate state in *Mp_N′* to state in *Mp_N* with counter-1

Probabilistic Parameterised Model Checking



Probabilistic Parameterised Model Checking

Mp_3 ***Mp_3'***



Probabilistic Parameterised Model Checking

- Again relation gives weak bisimulation between executions of models

$$M\_N = Mc\_N \approx Mp\_N \approx Mp\_N'$$

Probabilistic Parameterised Model Checking

- Finally want to show that *Mp_N′* and *M_N+1* are related
- But choice of inital passive probabilistic in *M_N+1* and nondeterministic in *Mp_N′*
- Definition of relation between states is more complex and remains to be resolved

Probabilistic Parameterised Model Checking

- Hence we have:

  $M\_N = Mc\_N \approx Mp\_N \approx Mp\_N'$?≈? M_N+1
- So assuming

  $Mp\_N' \approx M\_N+1$

  then by induction,

  $M\_3 \models \Phi \implies$ for all N, $M\_N \models \Phi$

  where Φ is a PCTL property that
  - does not index any process id
  - does not contain next time or time bounded until operators
- *E.g.* "with probability 1, a leader is elected"

Further Work

- Complete proof for Itai Rodeh leader election
- Apply to other degenerative systems
  - Randomised consensus weak shared coin protocol (Aspnes & Herlihy)