

An attempt to give a clear semantics of the extension of PEPA for massively parallel processes

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Outline

Motivations

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Extensions of PEPA, massively parallel processes and mass actions

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Semantics of this double extension

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2. To have a semantics that can be explained by the **standard operational semantics of PEPA**.

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Massively parallel processes

Defined in [Calter & al 2004, Benoit & al 2005, Hillston 2005].

- A syntactic shortcut to **represent massive parallelism** :

$$P[n] = \underbrace{P \parallel \dots \parallel P}_n$$

$$P[a, b][n] = \underbrace{P \bowtie_{a,b} \dots \bowtie_{a,b} P}_n$$

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- A method to derive a **small set of ODEs** that describes the **evolution of the number of components** in their different states.

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$$P \overset{H}{\boxtimes}_L Q$$

► $H \Rightarrow$ multiplication of mass actions.

$$R = r_1 \times r_2 \quad r_\alpha(P \overset{\alpha}{\boxtimes}_\beta Q) = r_\alpha(P) \times r_\alpha(Q)$$

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$$R = r_1 \times r_2 \quad r_\alpha(P \underset{\beta}{\overset{\alpha}{\boxtimes}} Q) = r_\alpha(P) \times r_\alpha(Q)$$

- $\textcolor{red}{L} \Rightarrow$ standard operator, bounded capacity.

$$R = \frac{r_1}{r_\beta(P)} \times \frac{r_2}{r_\beta(Q)} \times \min(r_\beta(P), r_\beta(Q))$$

$$r_\beta(P \underset{\beta}{\overset{\alpha}{\boxtimes}} Q) = \min(r_\beta(P), r_\beta(Q))$$

PEPA models of this double extension

1. a set of declarations, each noted :

$$C \stackrel{\text{def}}{=} S$$

with S a PEPA term designing a **sequential process**

$$S := (\alpha, r).S \mid S + S \mid C$$

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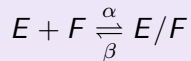
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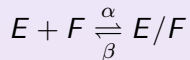
2. a root term :

$$P := P \overset{H}{\underset{L}{\boxtimes}} P \mid P/L \mid (S_1[n_1] \parallel \dots \parallel S_k[n_k])$$

Example : a simple kinetic reaction



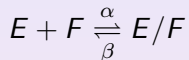
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PEPA model, set of
declarations :

$$\begin{aligned} E &\stackrel{def}{=} (\alpha, r_\alpha).E/F \\ F &\stackrel{def}{=} (\alpha, 1).F/E \\ E/F &\stackrel{def}{=} (\beta, r_\beta).E \\ F/E &\stackrel{def}{=} (\beta, r_\beta).F \end{aligned}$$

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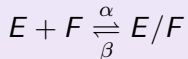
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$$(E[m_1] \parallel E/F[m_3]) \xrightleftharpoons[\beta]{\alpha} (F[m_2] \parallel F/E[m_3])$$

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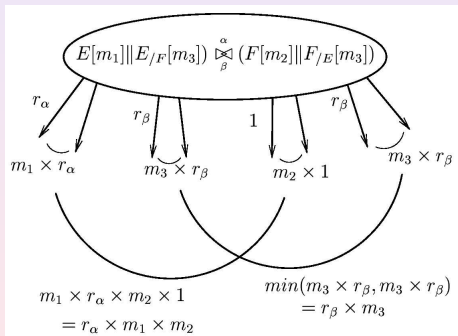
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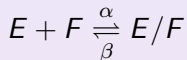
$$F_{/E} \stackrel{\text{def}}{=} (\beta, r_\beta).F$$

Root term :

$$(E[m_1] \parallel E_{/F}[m_3]) \xrightarrow[\beta]{\alpha} (F[m_2] \parallel F_{/E}[m_3])$$



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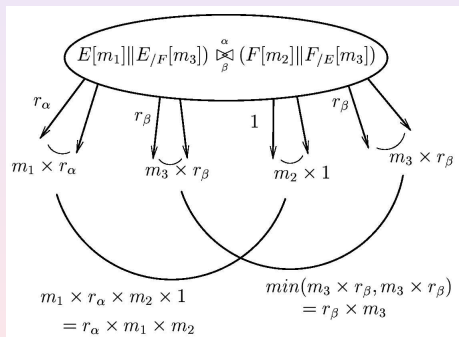
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- Decomposition of a process into a **vector of sequential processes** :

$$P = C[m_1] \overset{H}{\boxtimes} \underset{L}{C'[m_2]}$$

$\Downarrow$

$$X_t = (X^1, \dots, X^n) \quad \text{where } n = m_1 + m_2$$

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- Let $B_t^{i,s}$ be a **binary random process** :

$$\forall i \in \{1, \dots, n\}, \forall s \in \mathbf{S} \quad B_t^{i,s} = \begin{cases} 1 & \text{if } X_t^i = s \\ 0 & \text{otherwise} \end{cases}$$

Semantics of this double extension

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- ▶ The semantics is defined as the expectation of N_t^s :

$$\forall s \in \mathbf{S} \quad \mathbb{E}[N_t^s]$$

1st example : independent processes

Let P be the parallel product of **identical independent processes** with k states $\{s_1, \dots, s_k\}$:

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$$\frac{d\mathbf{p}(t)}{dt} = \mathbf{p}(t) \cdot \mathbf{Q} \quad \mathbf{p}(0) = \pi$$

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- Evolution of the **expectation of the number of processes** :

$$\frac{d(\mathbb{E}[N_t^{s_1}], \dots, \mathbb{E}[N_t^{s_k}])}{dt} = (\mathbb{E}[N_t^{s_1}], \dots, \mathbb{E}[N_t^{s_k}]) \cdot \mathbf{Q}$$

$$(\mathbb{E}[N_t^{s_1}], \dots, \mathbb{E}[N_t^{s_k}]) = (m_1, \dots, m_k)$$

2nd example : dependent processes

Let P be the PEPA model of
two processes, X^1 and X^2 :

$$X^1 : \quad \begin{array}{l} A \stackrel{\text{def}}{=} (\alpha, r_\alpha).A' \\ A' \stackrel{\text{def}}{=} (\tau, r_A).A \end{array}$$

$$X^2 : \quad \begin{array}{l} B \stackrel{\text{def}}{=} (\alpha, r_\alpha).B' \\ B' \stackrel{\text{def}}{=} (\tau, r_B).B \end{array}$$

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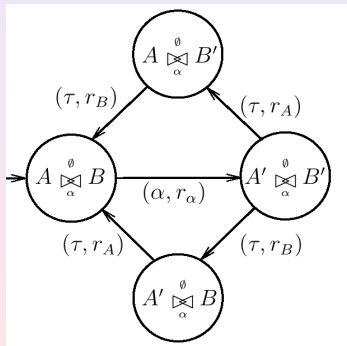
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- **Expectations** of the number of components in each state :

$$\begin{array}{ll} \mathbb{E}[N_t^A] &= \mathbb{P}(X_t^1 = A) & \mathbb{E}[N_t^B] &= \mathbb{P}(X_t^2 = B) \\ \mathbb{E}[N_t^{A'}] &= \mathbb{P}(X_t^1 = A') & \mathbb{E}[N_t^{B'}] &= \mathbb{P}(X_t^2 = B') \end{array}$$

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- The following **ODEs** describe these expectations :

$$\begin{aligned} \frac{d\mathbb{E}[N_t^A]}{dt} &= -r_\alpha \times \mathbb{P}(X_t = A \boxtimes_\alpha^\emptyset B) + r_A \times \mathbb{E}[N_t^{A'}] \\ \frac{d\mathbb{E}[N_t^{A'}]}{dt} &= r_\alpha \times \mathbb{P}(X_t = A \boxtimes_\alpha^\emptyset B) - r_A \times \mathbb{E}[N_t^{A'}] \\ \frac{d\mathbb{E}[N_t^B]}{dt} &= -r_\alpha \times \mathbb{P}(X_t = A \boxtimes_\alpha^\emptyset B) + r_B \times \mathbb{E}[N_t^{B'}] \\ \frac{d\mathbb{E}[N_t^{B'}]}{dt} &= r_\alpha \times \mathbb{P}(X_t = A \boxtimes_\alpha^\emptyset B) - r_B \times \mathbb{E}[N_t^{B'}] \end{aligned}$$

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- We cannot **get rid of** $\mathbb{P}(X_t = A \boxtimes_\alpha^\emptyset B)$.

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- ▶ Work to do :
 1. to clarify **when** this semantics **can be reduced** as a small set of ODEs, **when it cannot**.
 2. to see in which cases this semantics can be **approximated** by a **small set of ODEs**. Maybe when the number of processes grows the previous semantics becomes a good approximation of this one?