Ants and Agents: Global Complexity from Local Simplicity

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Credit

Based on an original publication:

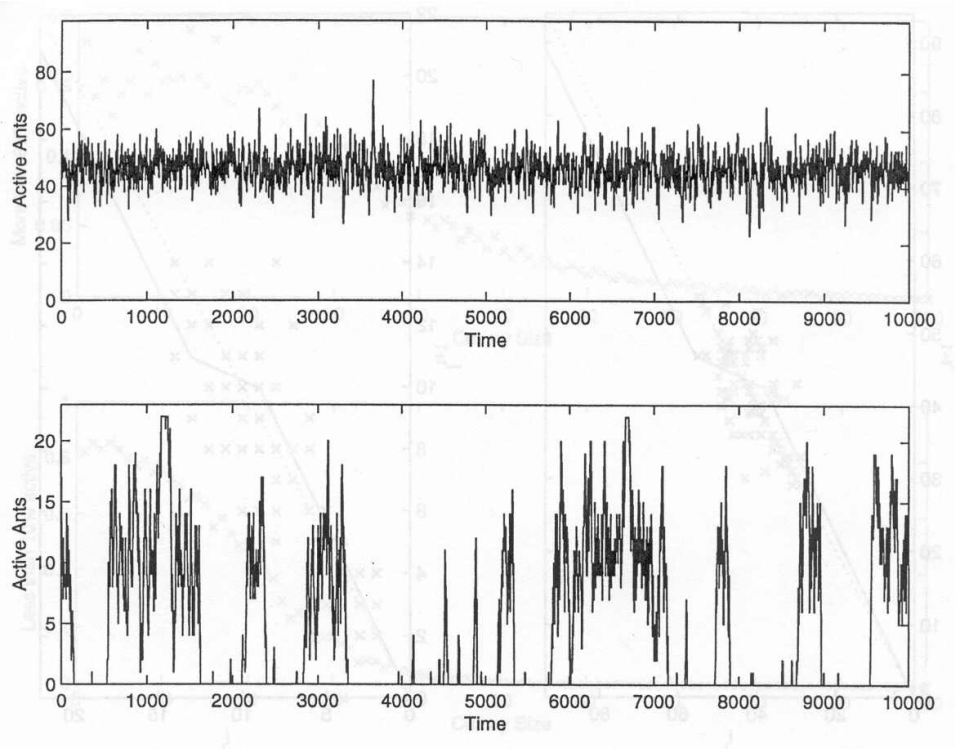
Ants and Agents: a Process Algebra Approach to Modelling Ant Colony Behaviour, D.J.T. Sumpter, G.B. Blanchard and D.S. Broomhead, Bulletin of Mathematical Biology, **63**(2001).

Introduction

- What's the issue?
- Some models of ants
- A fancy probability distribution or two
- Mean difference equations as an approximation
- An answer?

What's going on?

Plot of activity in: (Top) large colonies and (bottom) small colonies of ants.



The Issues

- Do not doubt the force of the state-space explosion!

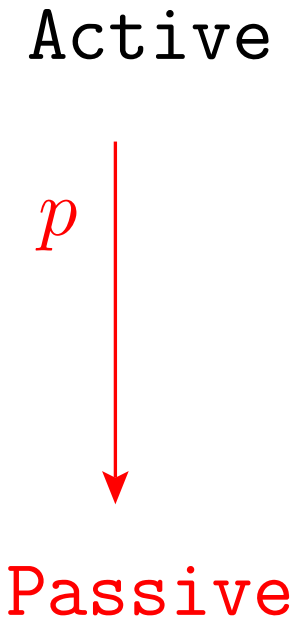
  $\Rightarrow$ complex global dynamics can be generated from very simple local dynamics — if there are sufficient interacting agents

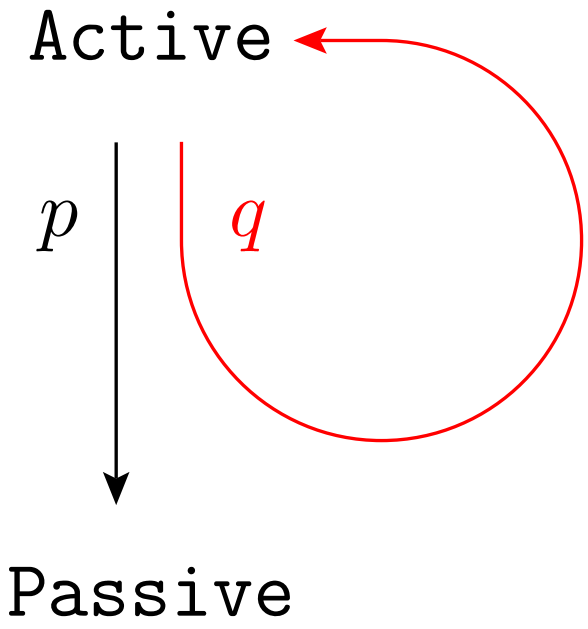
- On a technical level — paper uses probabilistic process algebra, WSCCS, for:

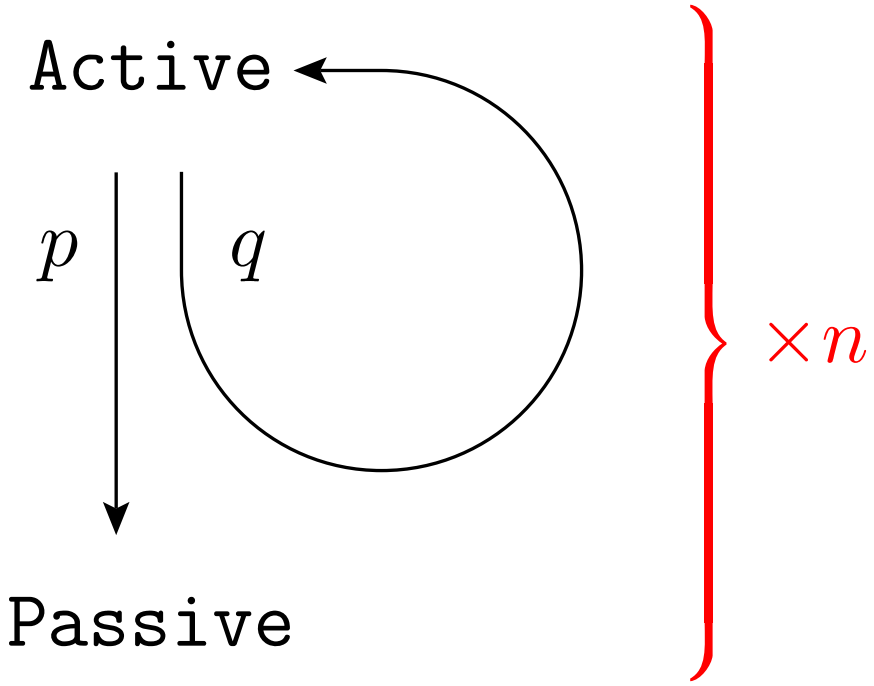
  1. specification for simulations
  2. Markov chain analysis
  3. mean difference equation techniques

Active and Passive Ants

- n agents
- Active agent can:
  1. go passive with probability p
  2. stay active with probability q
- Passive agents remain passive


Active
p
Passive


Active
p
q
Passive

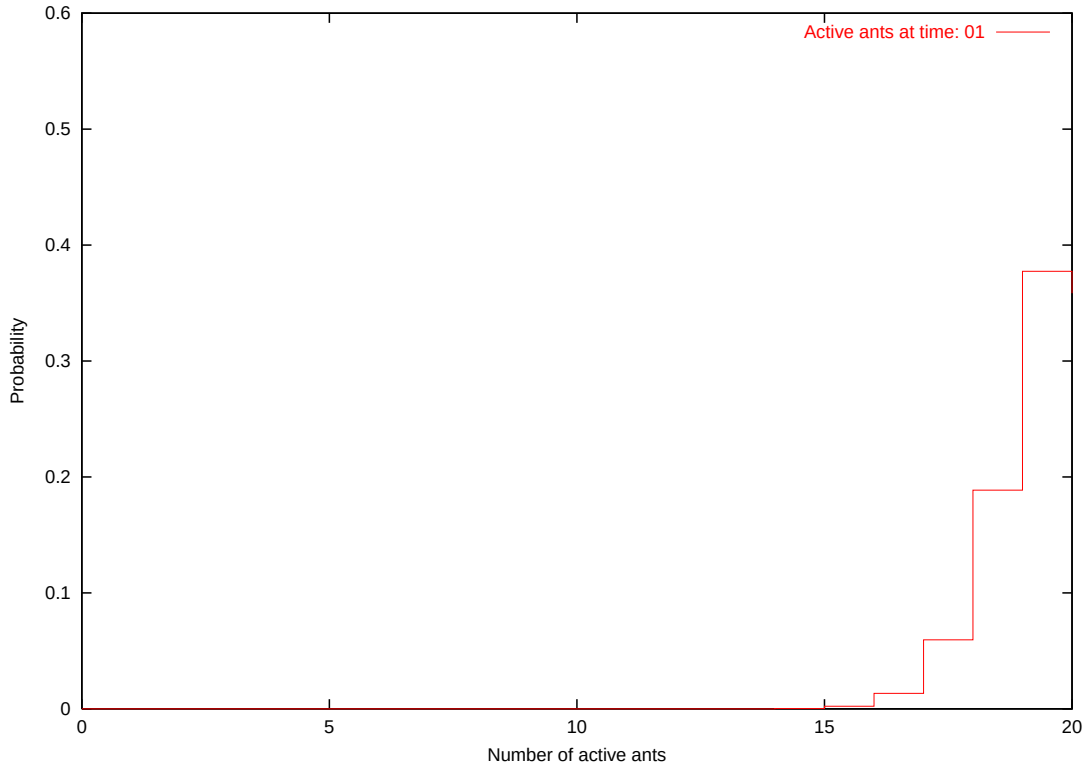

Active
p
q
Passive
$\times n$

WSCCS Description

$$\begin{aligned} \texttt{Active} &\equiv p : \surd.\texttt{Passive} + q : \surd.\texttt{Active} \\ \texttt{Passive} &\equiv 1 : \surd.\texttt{Passive} \\ \\ \texttt{Colony}_n(i) &\equiv \prod^{i} \texttt{Active} \times \prod^{n-i} \texttt{Passive} \end{aligned}$$

Probability Mass Function of Activity

System 1: $p = 0.05$, $q = 0.95$, $n = 20$, $1 \leq t \leq 71$.



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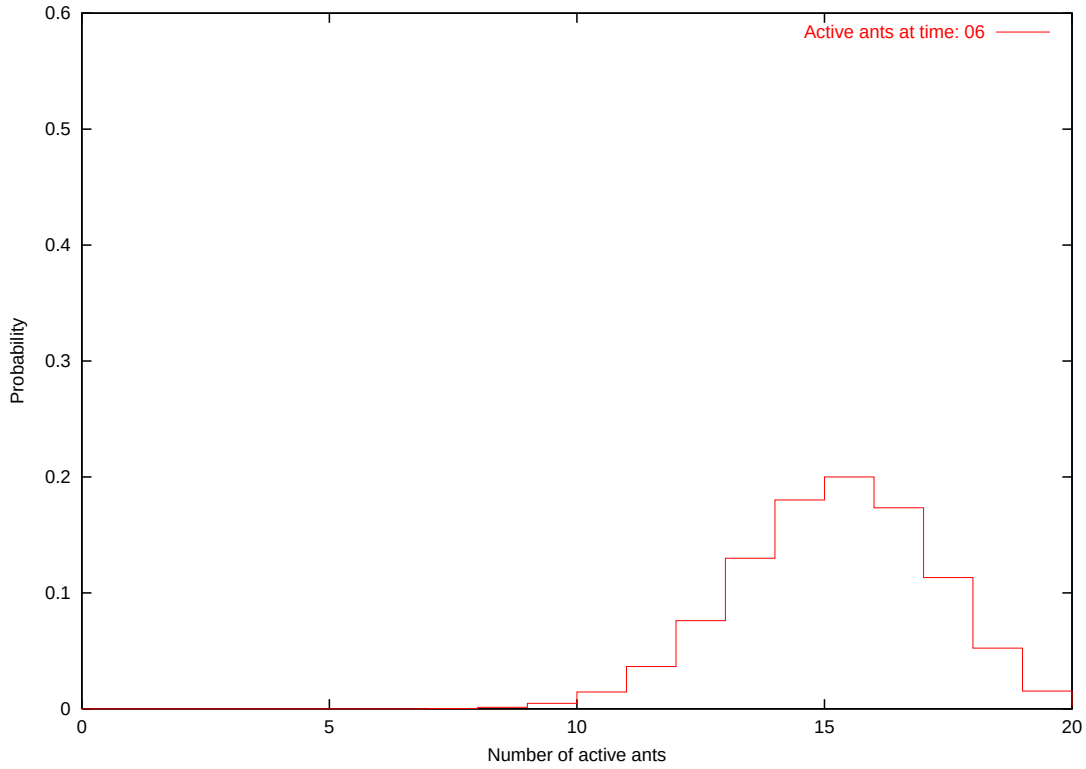

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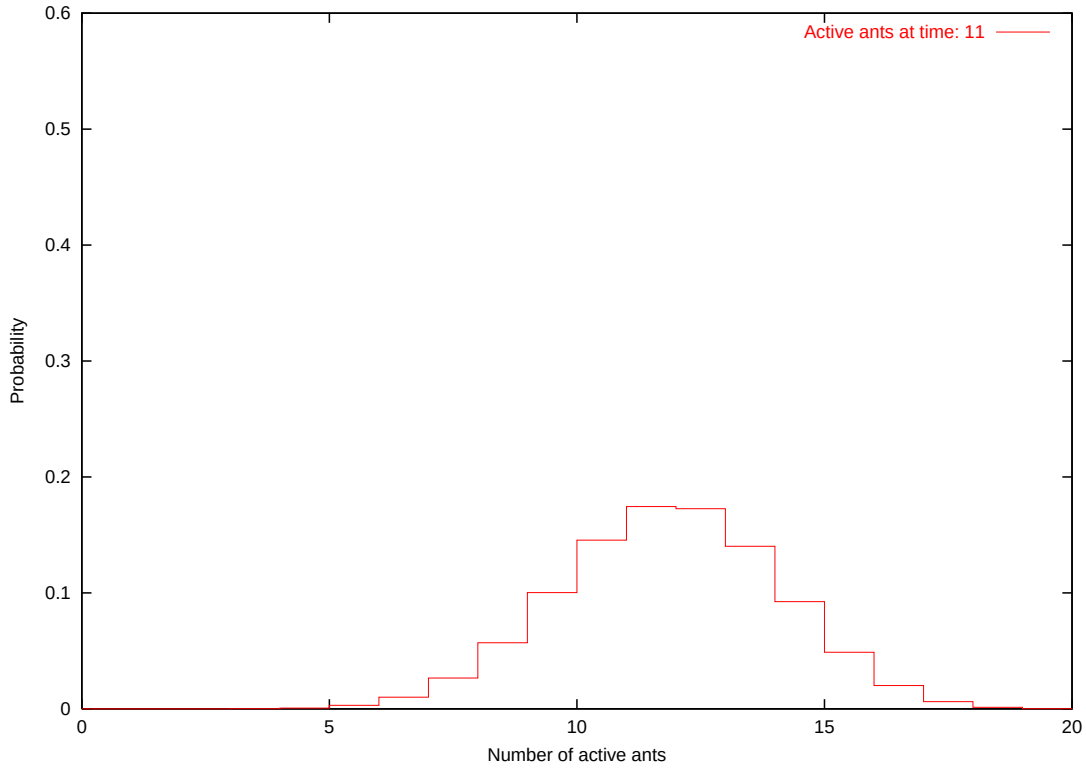

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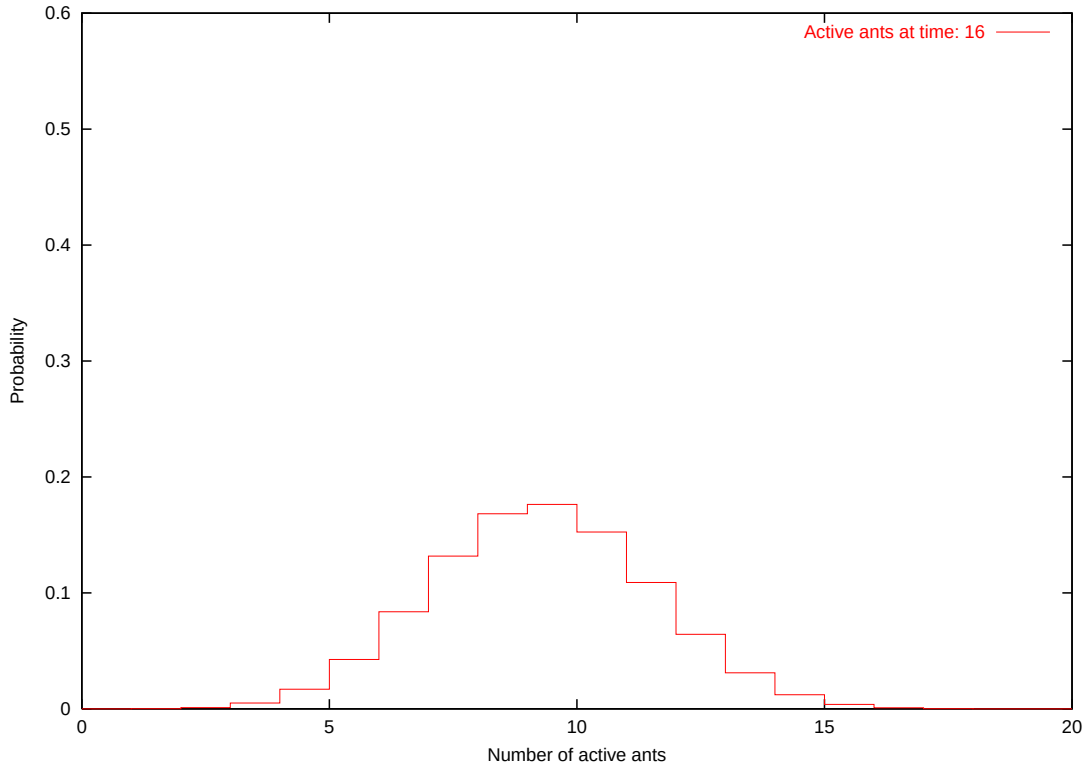

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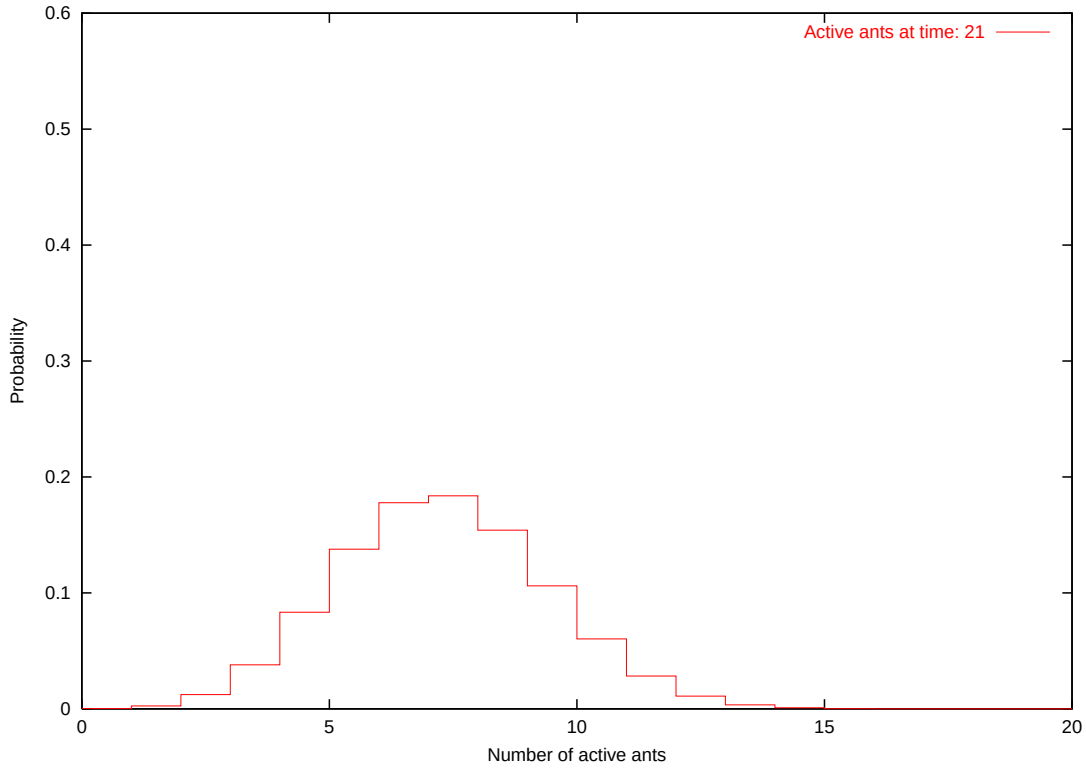

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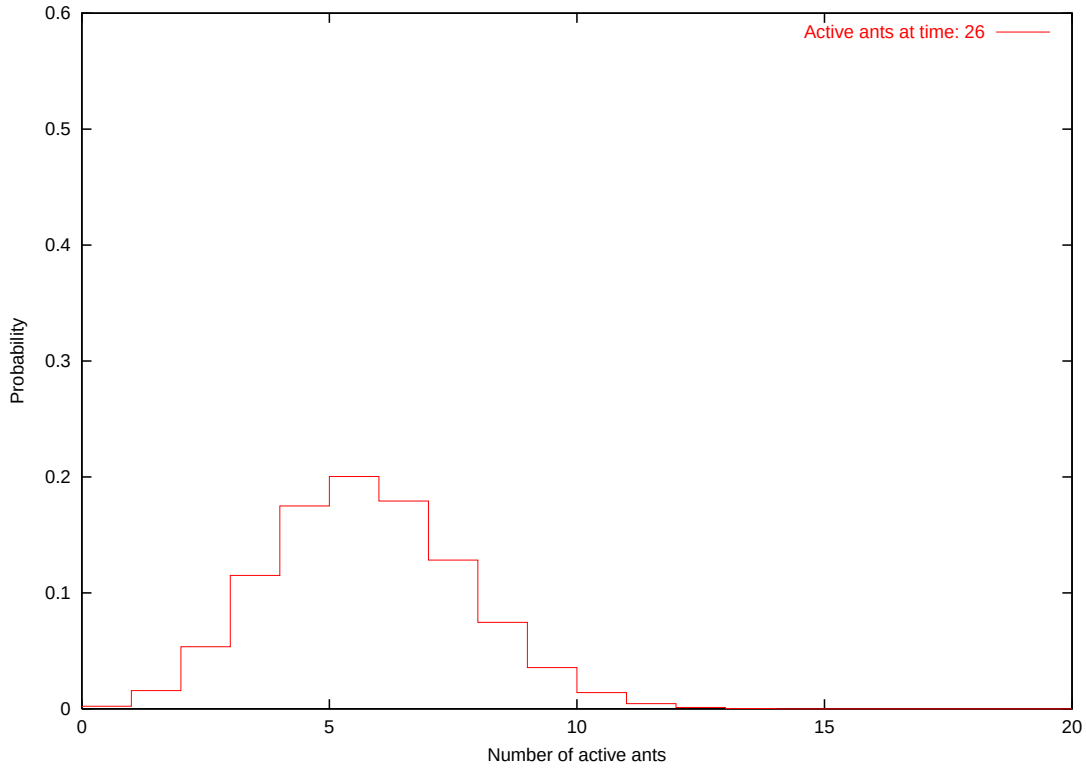

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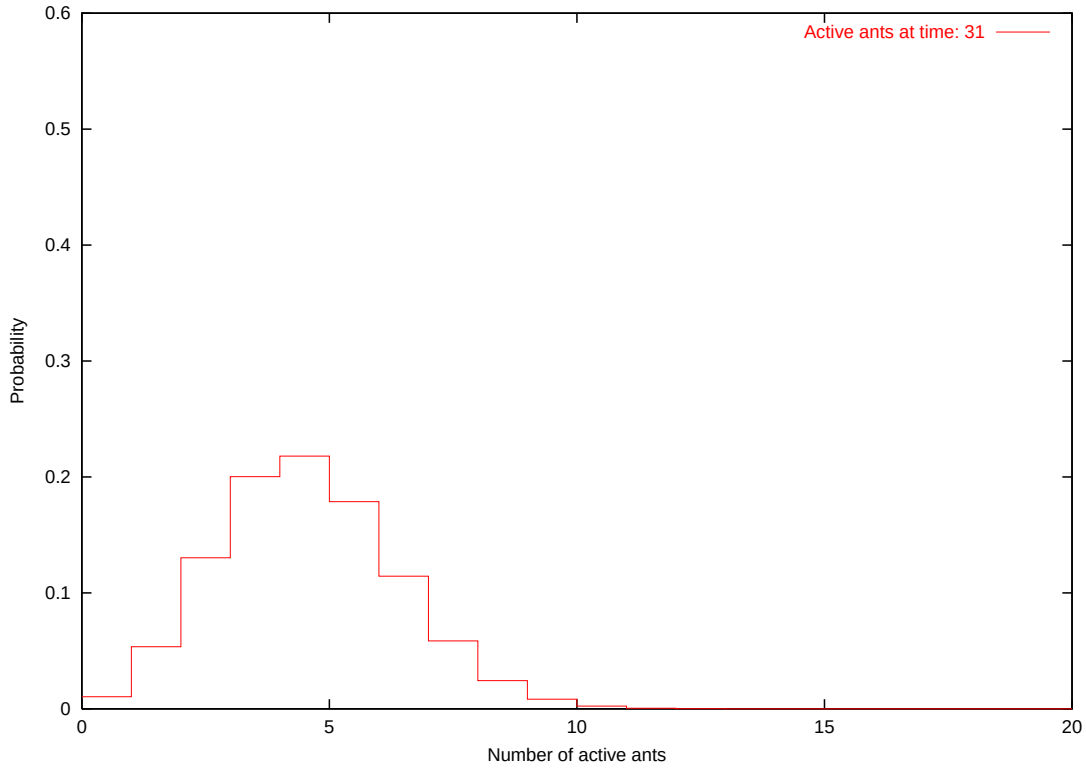

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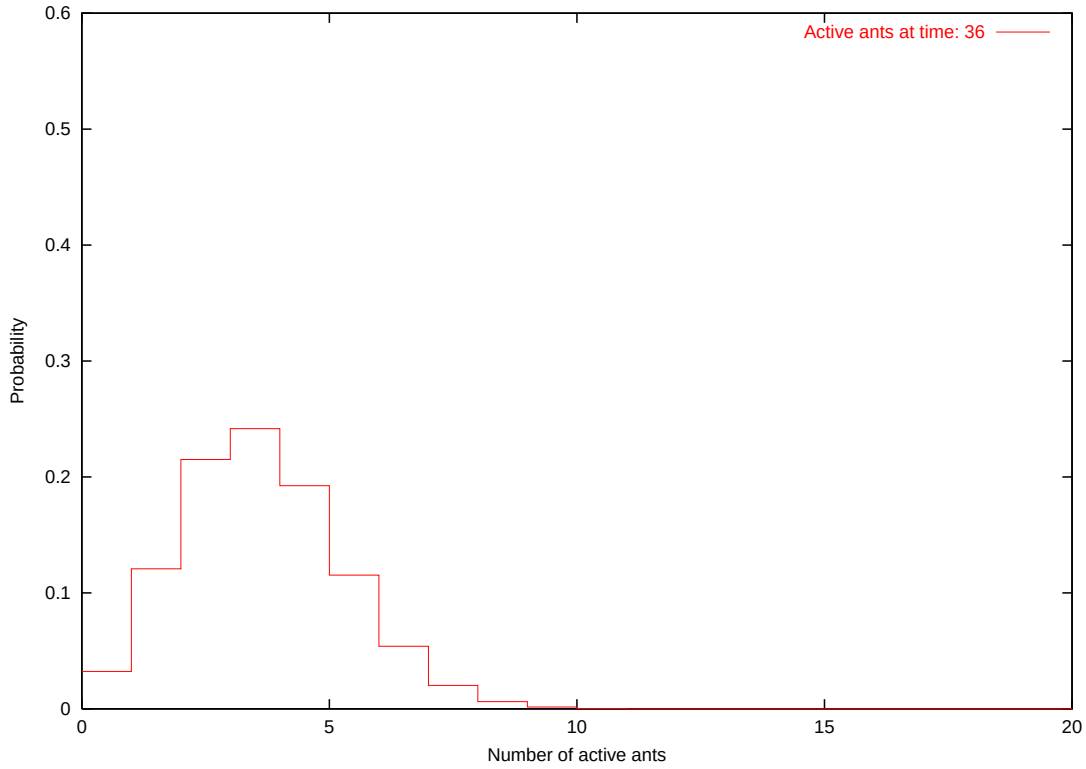

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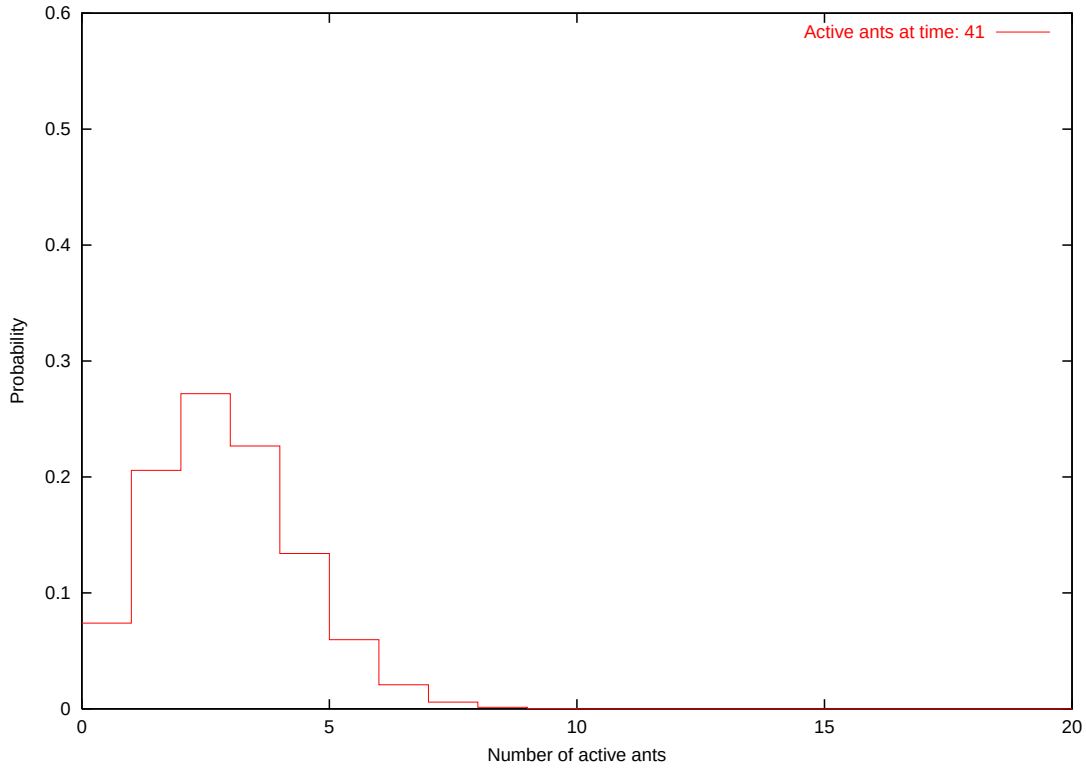

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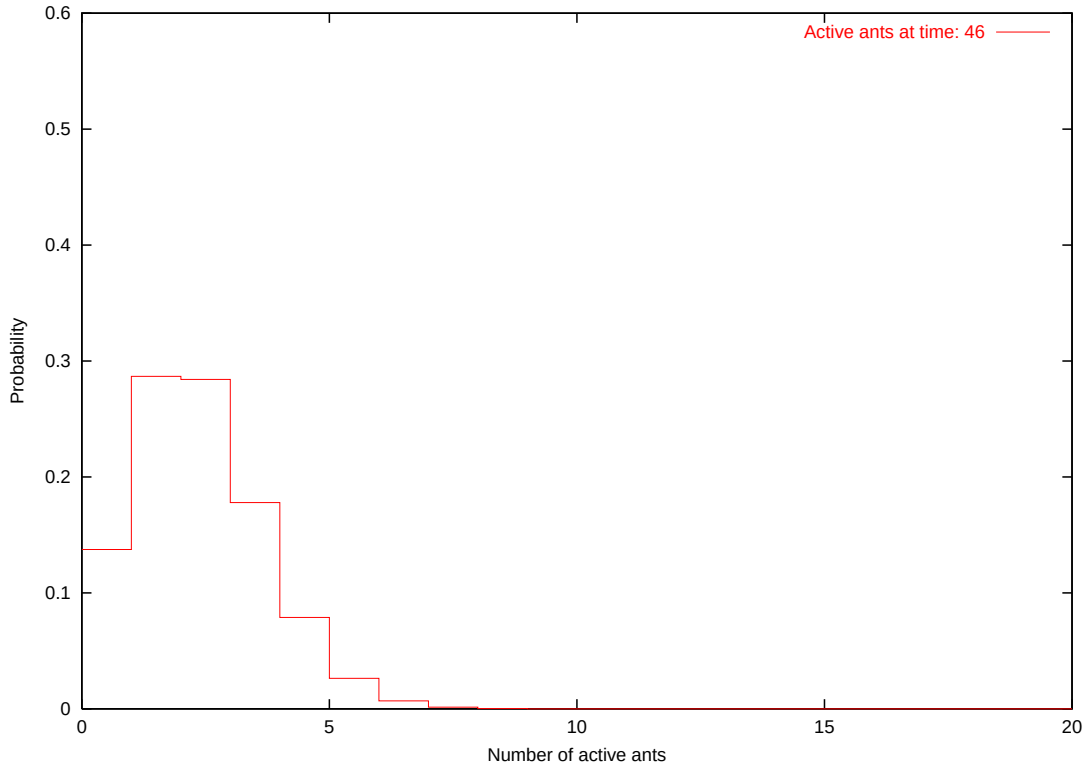

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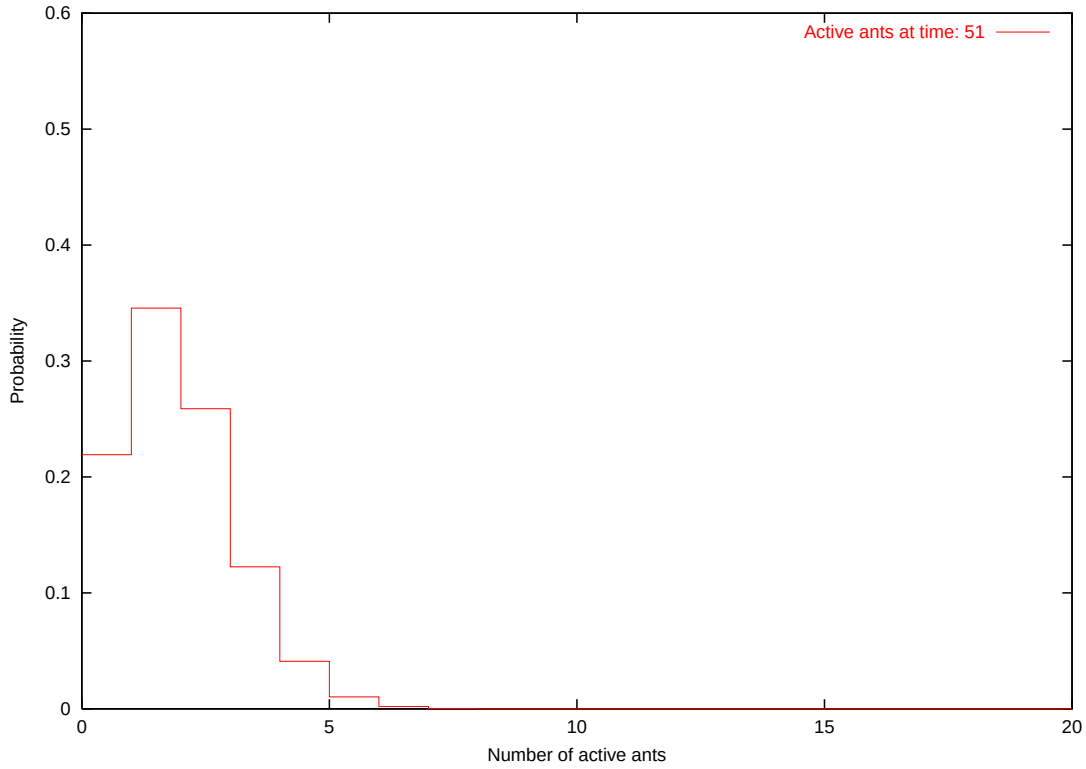

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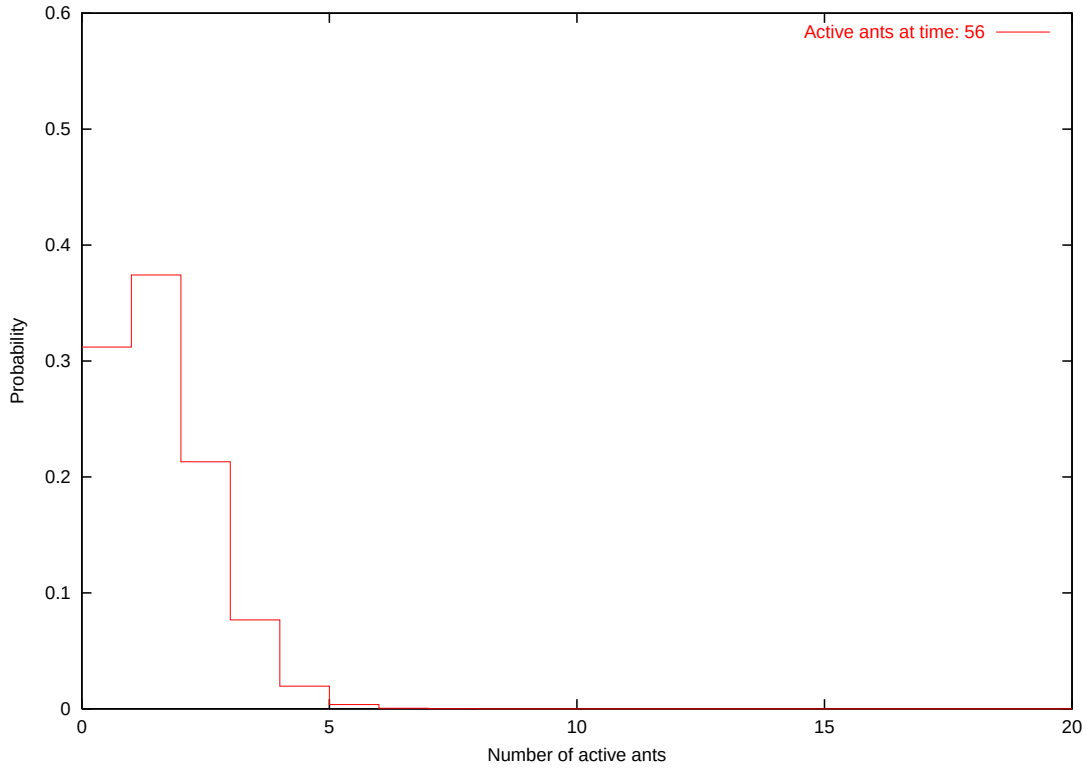

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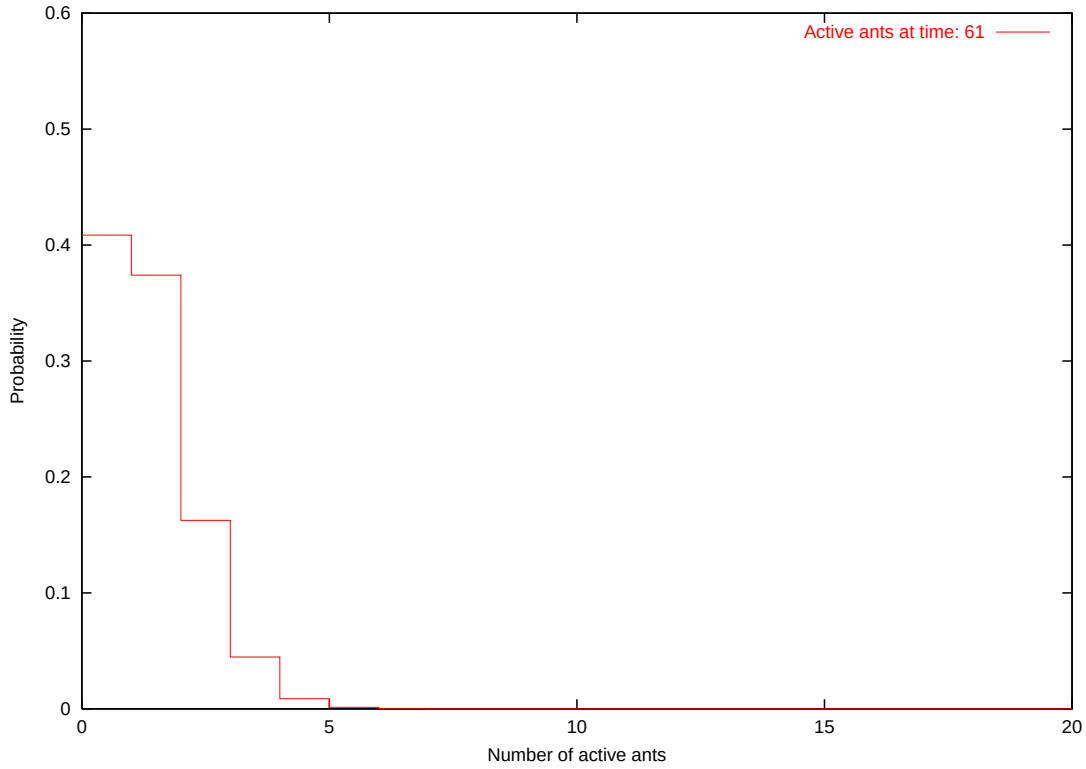

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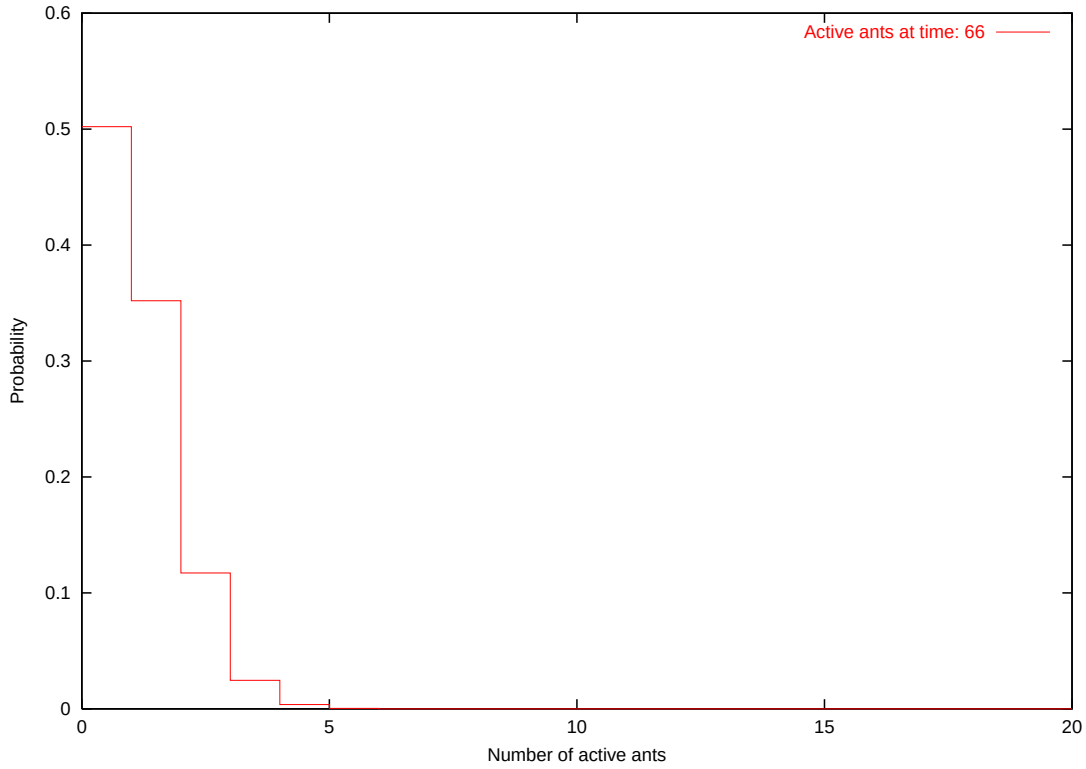

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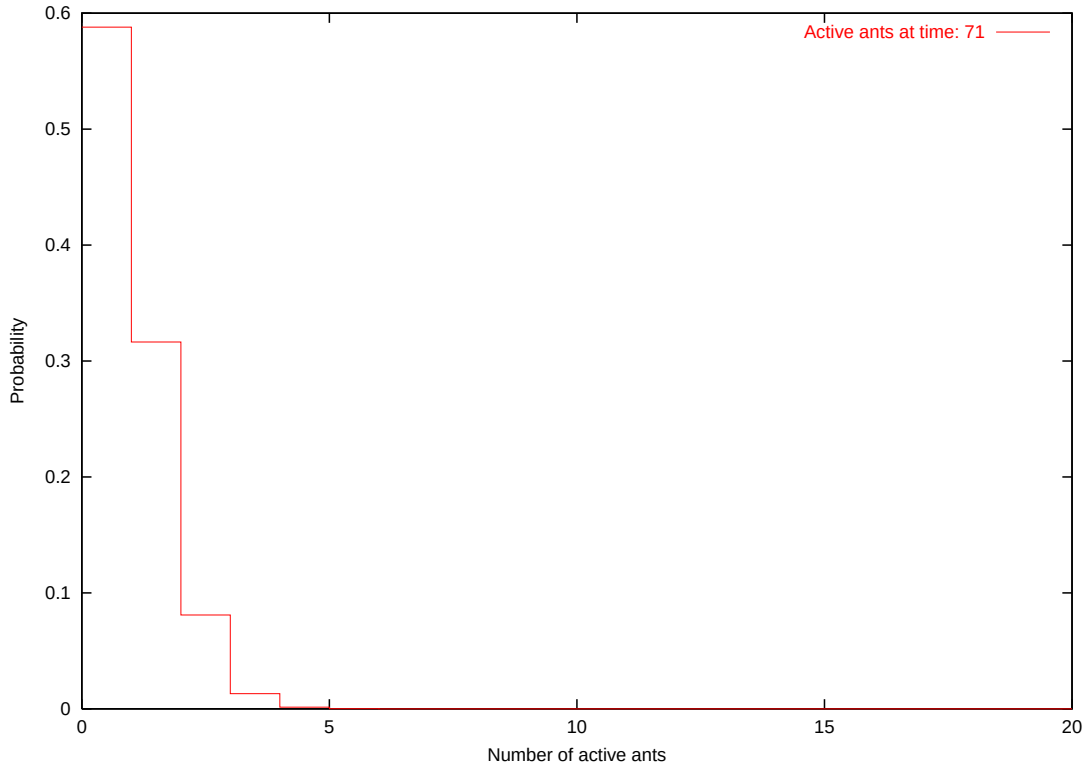


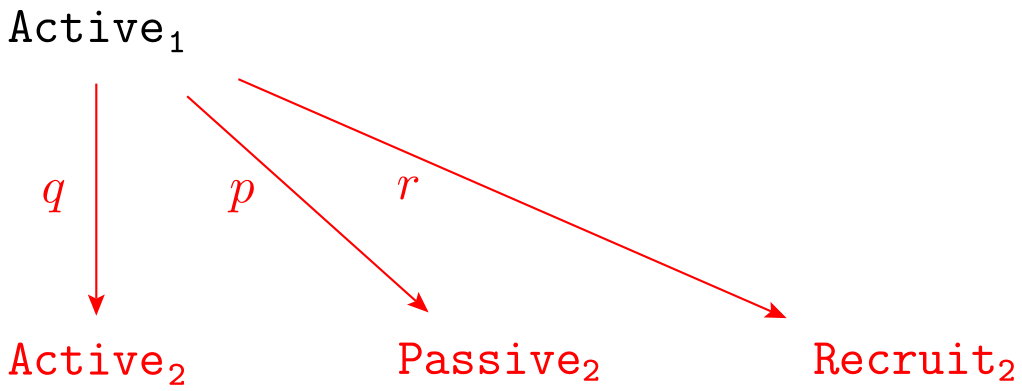
Ant Agent Model (I)

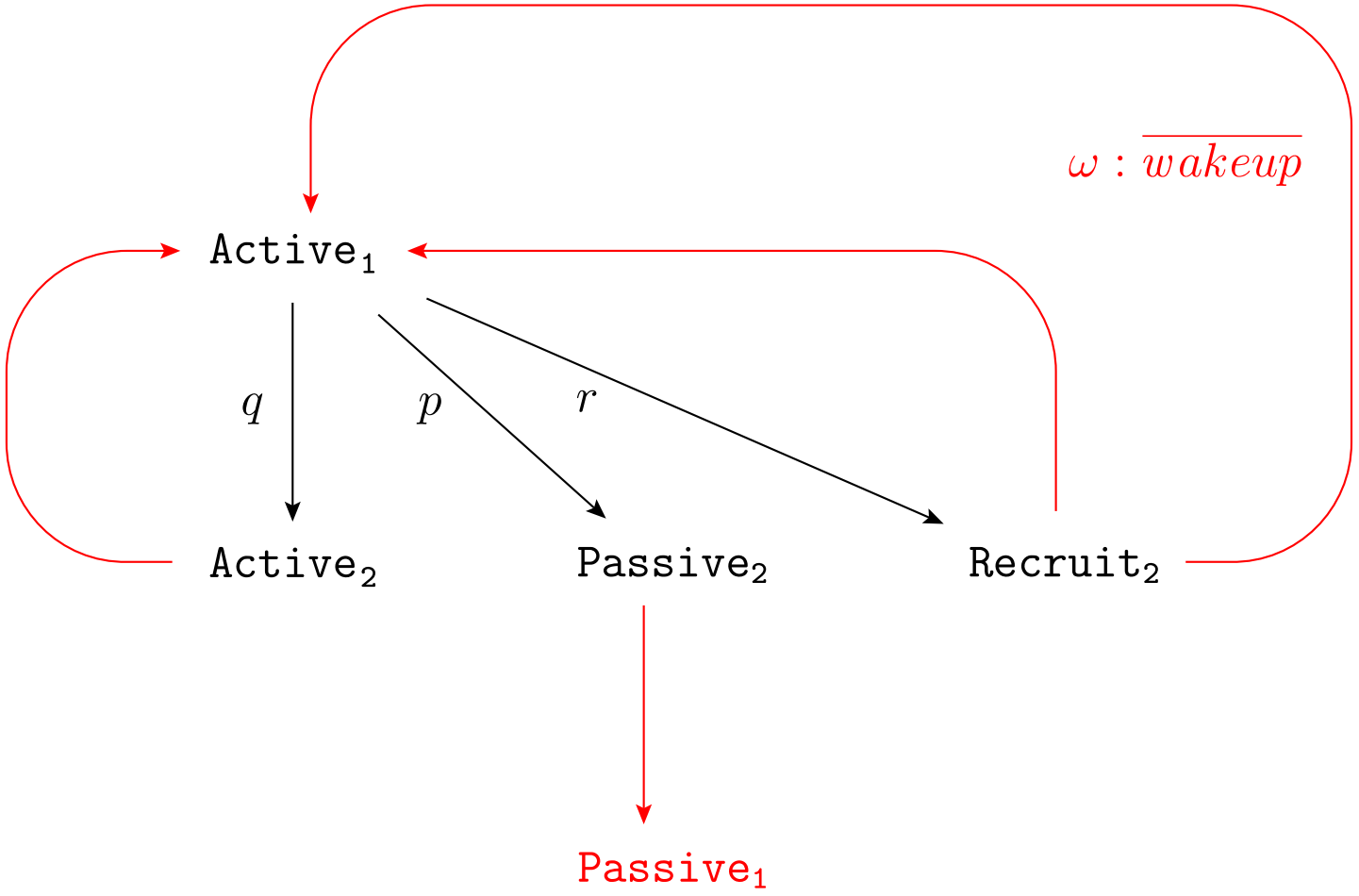
- Communication between ants introduced
- Active ants can now:
  1. remain active
  2. go passive
  3. become a recruiting ant
- Now passive ants can be woken up by recruiting ants

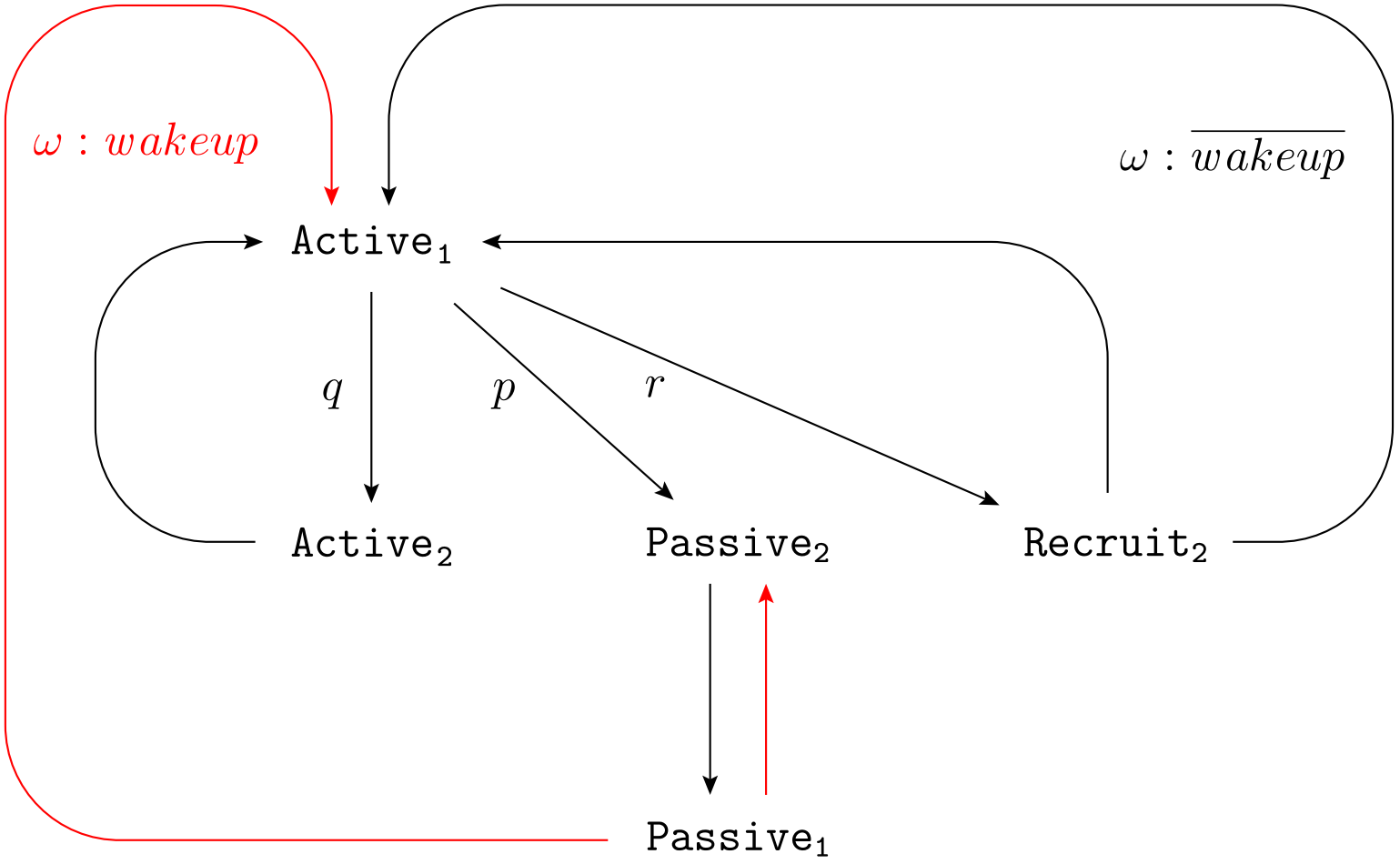
$\Rightarrow$ population does not just decline — it can fluctuate

WSCCS Description

$$
\begin{aligned}
\mathtt{Active}_1 &\equiv p : \surd.\mathtt{Passive}_2 + q : \surd.\mathtt{Active}_2 + r : \surd.\mathtt{Recruit}_2 \\
\mathtt{Active}_2 &\equiv 1 : \surd.\mathtt{Active}_1 \\
\mathtt{Recruit}_2 &\equiv \omega : \overline{\mathtt{wakeup}}.\mathtt{Active}_1 + 1 : \mathtt{Active}_1 \\
\\
\mathtt{Passive}_1 &\equiv 1 : \surd.\mathtt{Passive}_2 \\
\mathtt{Passive}_2 &\equiv \omega : \mathtt{wakeup}.\mathtt{Active}_1 + 1 : \surd.\mathtt{Passive}_1 \\
\\
\mathtt{Activity}_n(i) &\equiv \textstyle\prod^i \mathtt{Active}_1 \times \prod^{n-i} \mathtt{Passive}_1
\end{aligned}
$$


Active1
q
p
r
Active2
Passive2
Recruit2


$\omega : \overline{wakeup}$
Active$_1$
q
p
r
Active$_2$
Passive$_2$
Recruit$_2$
Passive$_1$

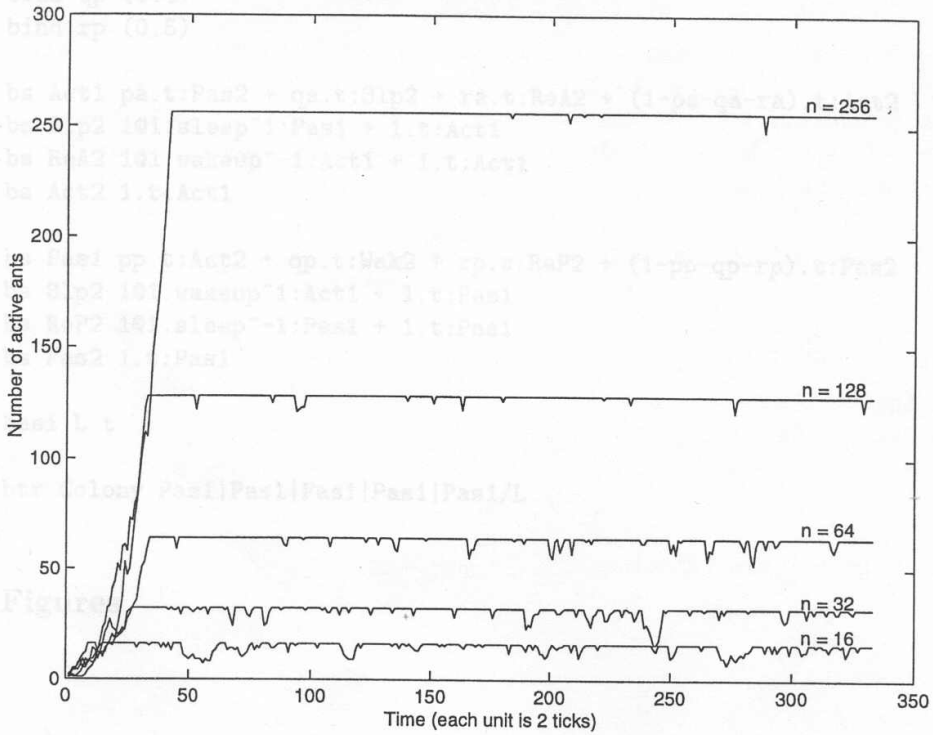

$\omega : wakeup$
$\omega : \overline{wakeup}$
$\mathtt{Active}_1$
q
p
r
$\mathtt{Active}_2$
$\mathtt{Passive}_2$
$\mathtt{Recruit}_2$
$\mathtt{Passive}_1$

Activity Extinction

- Activity extinction occurs when number of active ants reaches 0
- Traditional Markov chain analysis tells us system will become extinct eventually
- But an entire colony can be woken up from a single active ant: intruder alarm

Simulations: Number of Active Ants

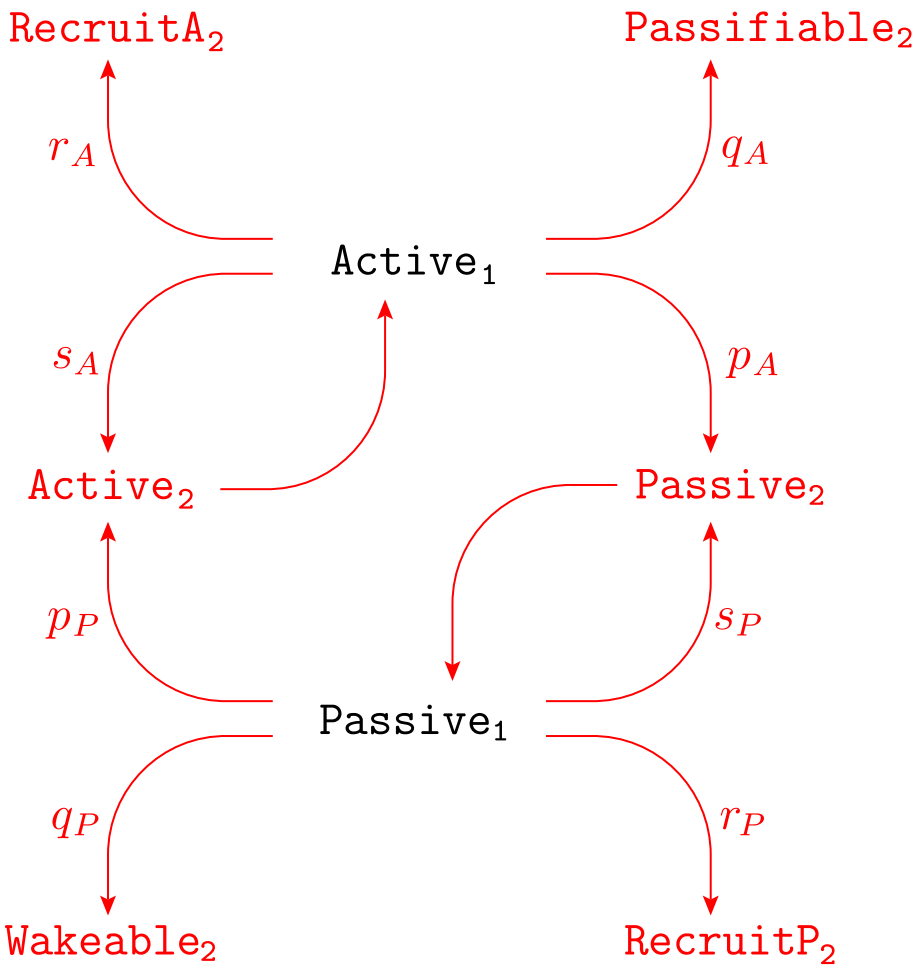
System 2: $p = 0.2$, $q = 0.3$, $r = 0.5$

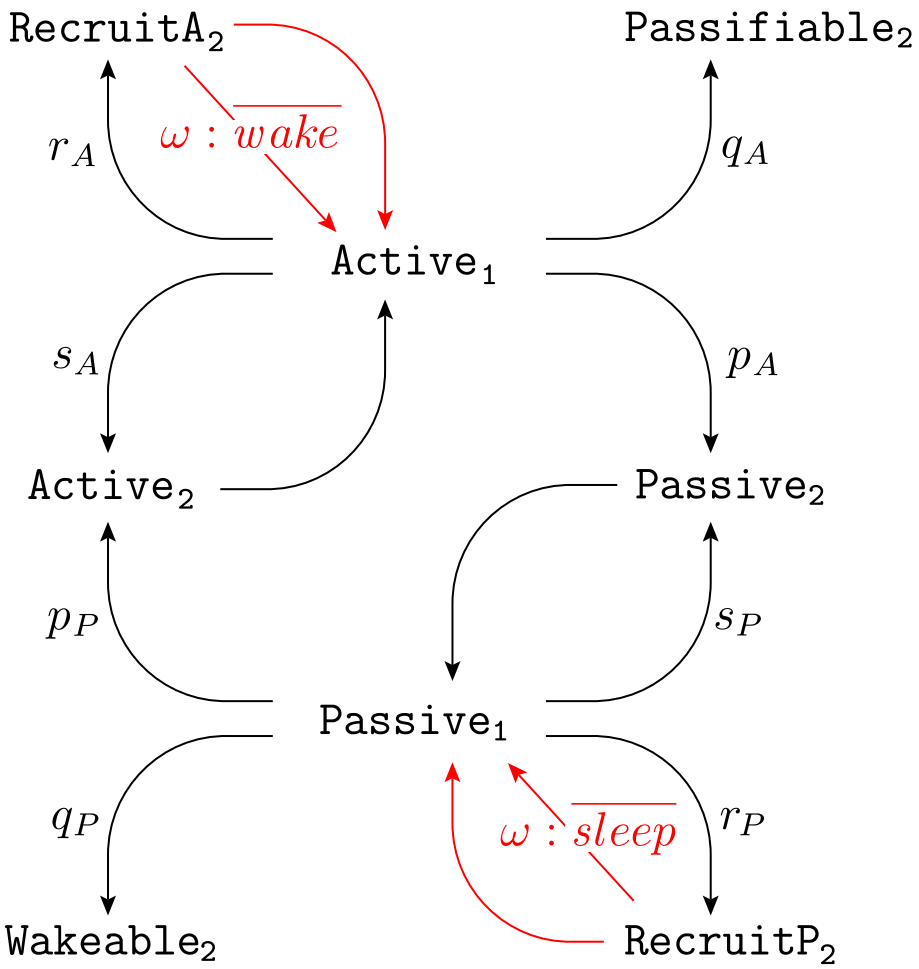


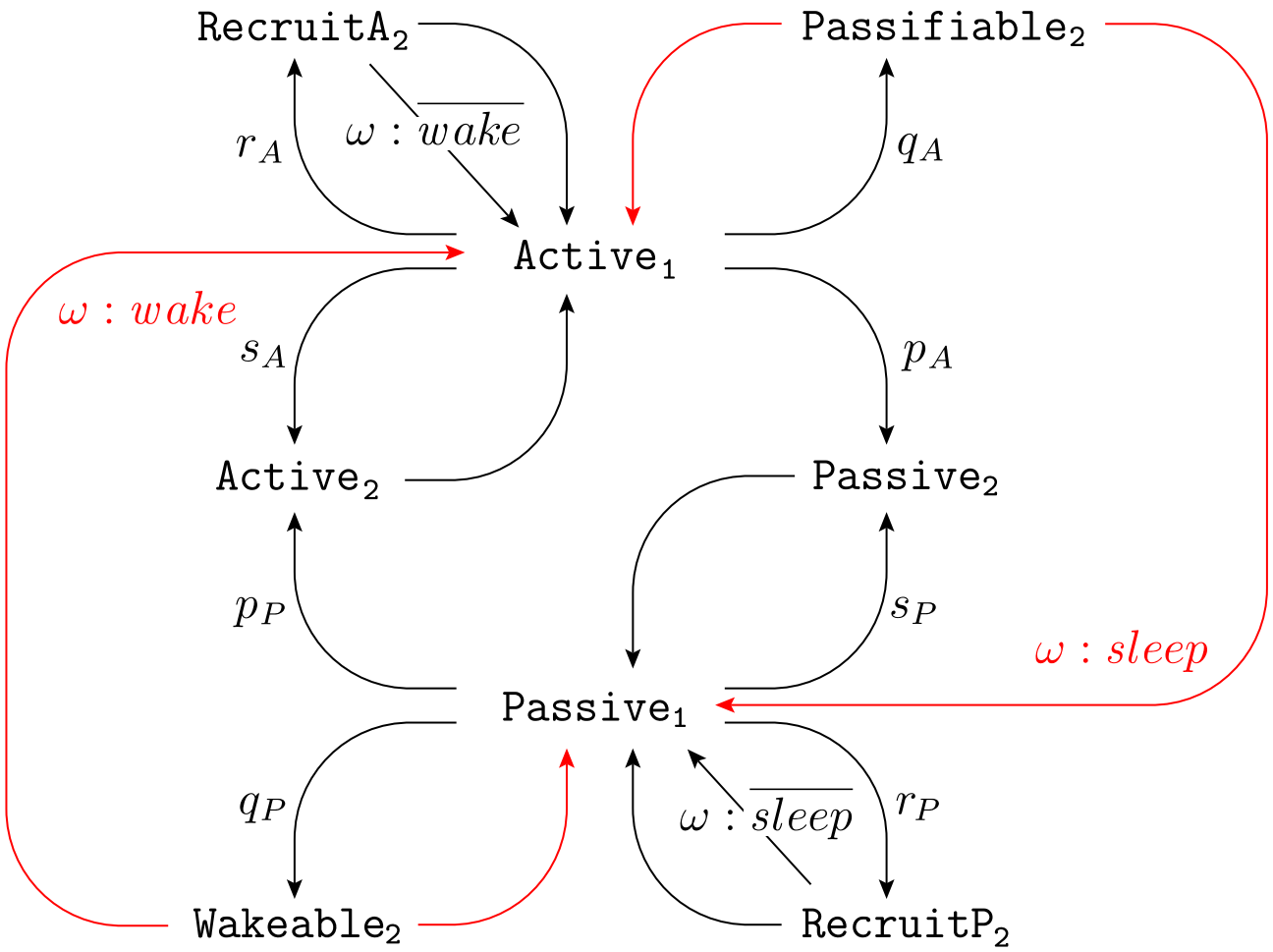
Ant Agent Model (II)

- As with previous model but with two types of recruiter ant:

  1. Allows passive ants to send active ants to sleep
  2. Allows active ants to wake passive ants up


RecruitA$_2$
Passifiable$_2$
r_A
q_A
Active$_1$
s_A
p_A
Active$_2$
Passive$_2$
p_P
s_P
Passive$_1$
q_P
r_P
Wakeable$_2$
RecruitP$_2$


RecruitA$_2$
Passifiable$_2$
r_A
$\omega : \overline{wake}$
q_A
Active$_1$
s_A
p_A
Active$_2$
Passive$_2$
p_P
s_P
Passive$_1$
q_P
$\omega : \overline{sleep}$
r_P
Wakeable$_2$
RecruitP$_2$


RecruitA$_2$
Passifiable$_2$
r_A
$\omega : \overline{wake}$
q_A
Active$_1$
$\omega : wake$
s_A
p_A
Active$_2$
Passive$_2$
p_P
s_P
$\omega : sleep$
Passive$_1$
q_P
$\omega : \overline{sleep}$
r_P
Wakeable$_2$
RecruitP$_2$

Introduction to Mean Difference Analysis

- Constructs function:

$$f(i) = \mathbb{E}(A_{t+1}|A_t = i)$$

- i.e. average activity at time $(t+1)$ given that activity at time t is i

- Iterate equation to obtain an approximation of the activity at time t:

$$i_{t+1} = f(i_t)$$

Back to System 1

$$\begin{aligned}
\texttt{Active} &\equiv p : \surd.\texttt{Passive} + q : \surd.\texttt{Active} \\
\texttt{Passive} &\equiv 1 : \surd.\texttt{Passive} \\
\\
\texttt{Colony}_n(i) &\equiv \prod^{i} \texttt{Active} \times \prod^{n-i} \texttt{Passive}
\end{aligned}$$

Brief Analysis

- Let X_k represent number of active ants that the kth active ant contributes to the next time step — $\mathbb{P}(X_k = 0) = p$, and $\mathbb{P}(X_k = 1) = q$

$$A_{t+1} = X_1 + X_2 + \cdots + X_{A_t}$$

- So A_t is a binomial distribution (c.f. tossing a coin)

$$\mathbb{P}(A_{t+1} = j | A_t = i) = \left(\begin{array}{c} i \\ j \end{array} \right) q^j p^{i-j}$$

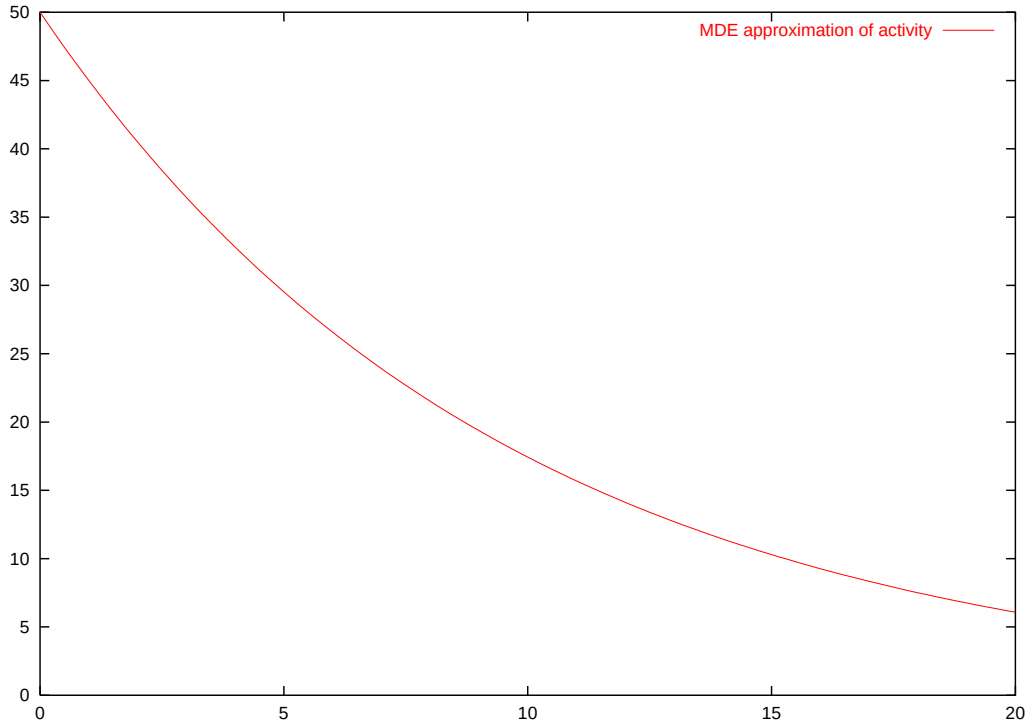
- Construct Mean Difference Equation:

$$\begin{aligned} f(i) &= \mathbb{E}(A_{t+1}|A_t = i) = qi \\ i_{t+1} &= qi_t \\ i_t &= q^t n \end{aligned}$$

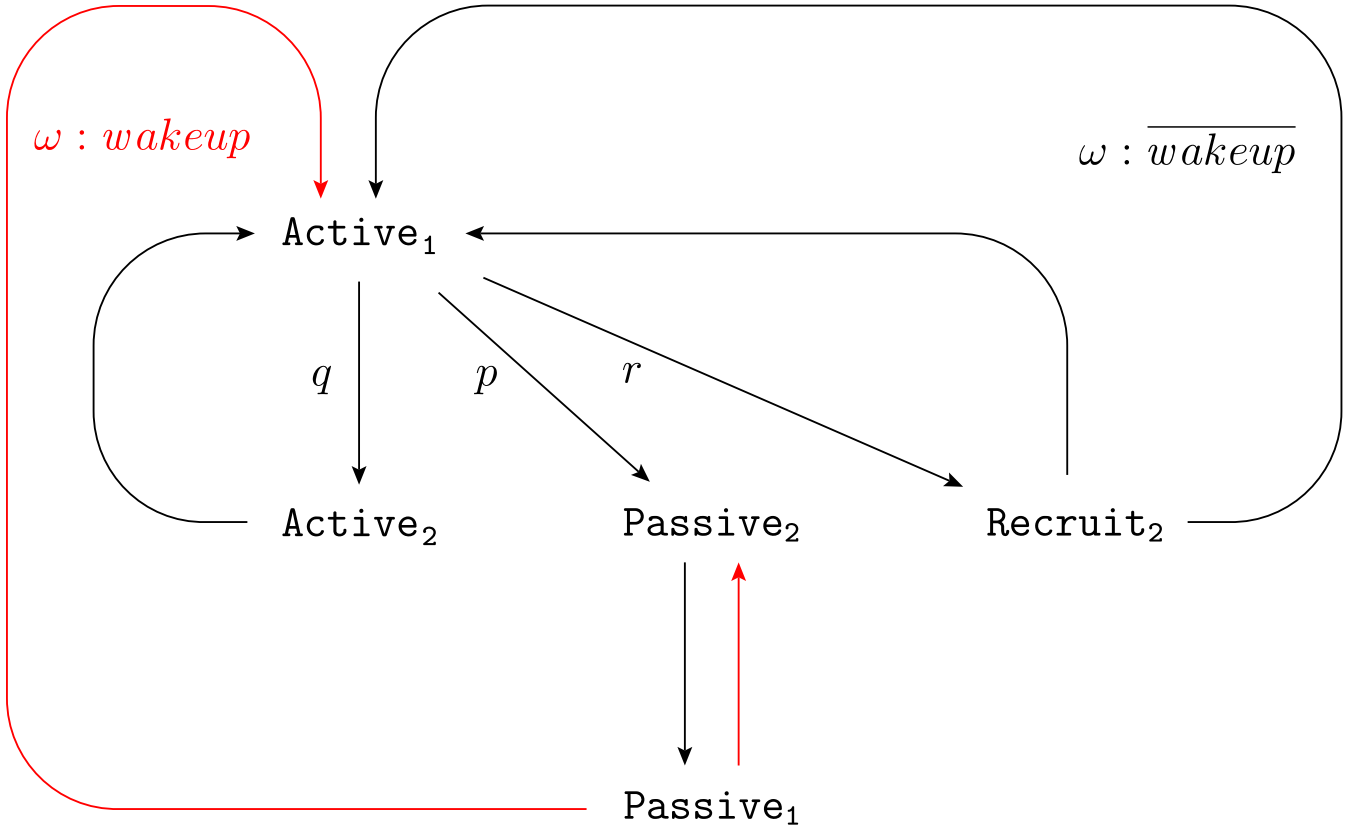
- So average activity at time t is $q^t n$ — which looks like. . .

Plot of Mean Difference Equation

System 1: $p = 0.1$, $q = 0.9$, $n = 50$. Activity against time.

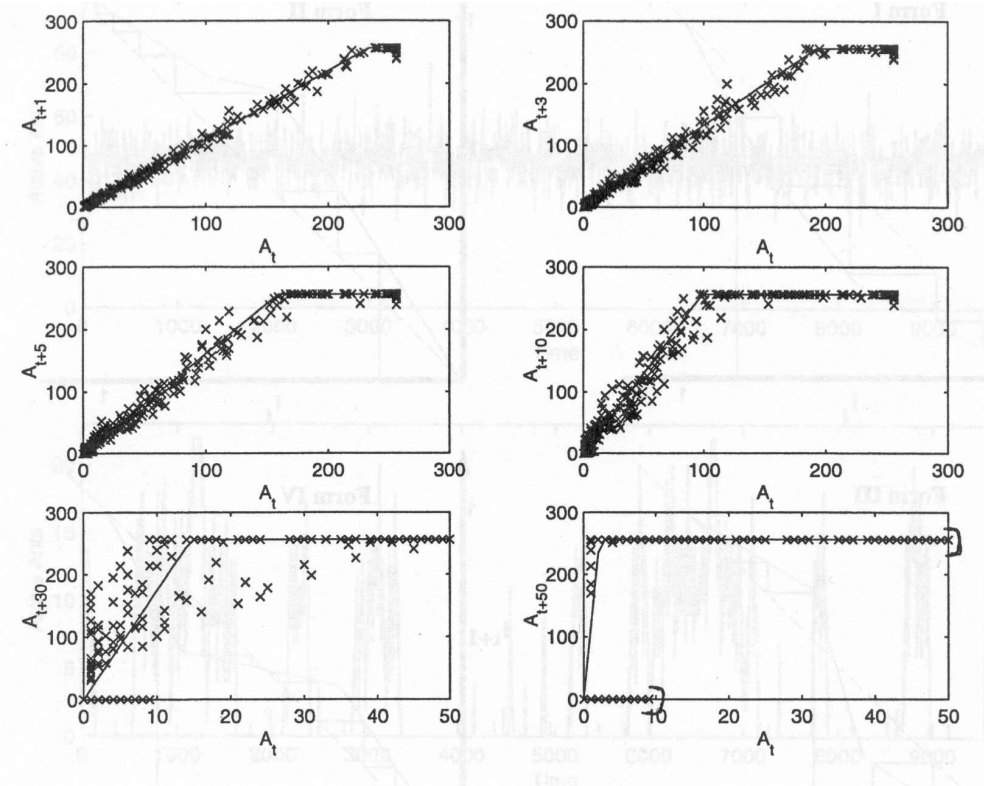


Back to System 2

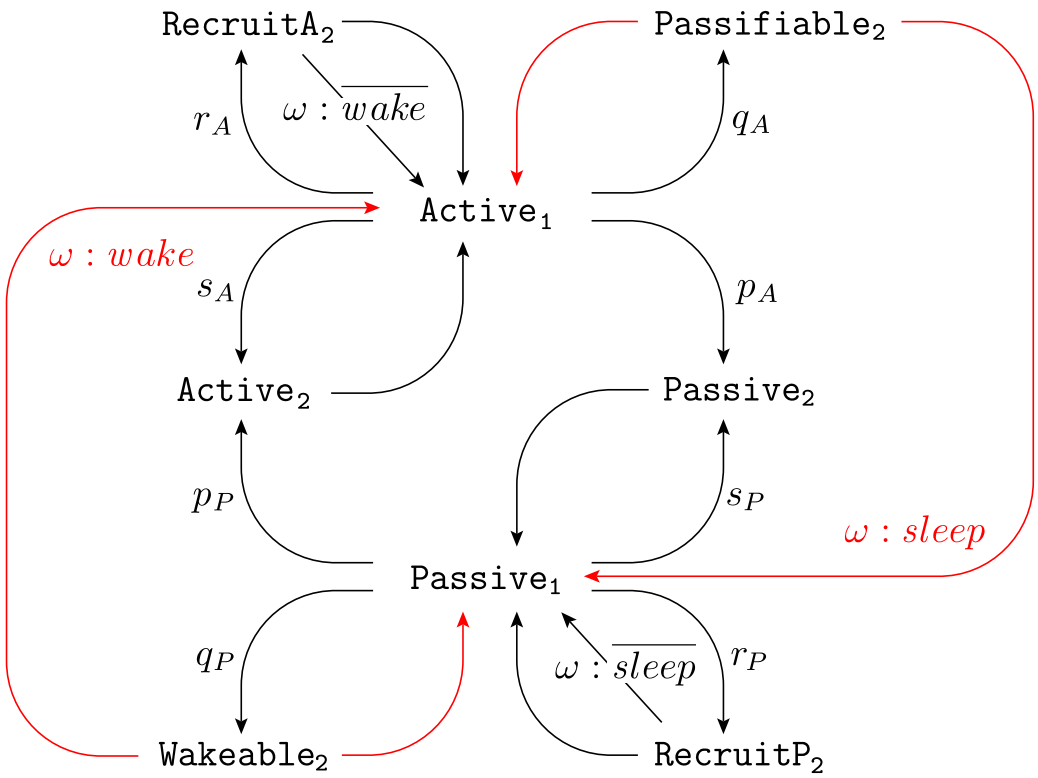


Accuracy of Mean Difference Equation

System 2: $p = 0.2$, $q = 0.3$, $r = 0.5$. A_t against A_{t+s} for $s \in \{1, 3, 5, 10, 30, 50\}$.

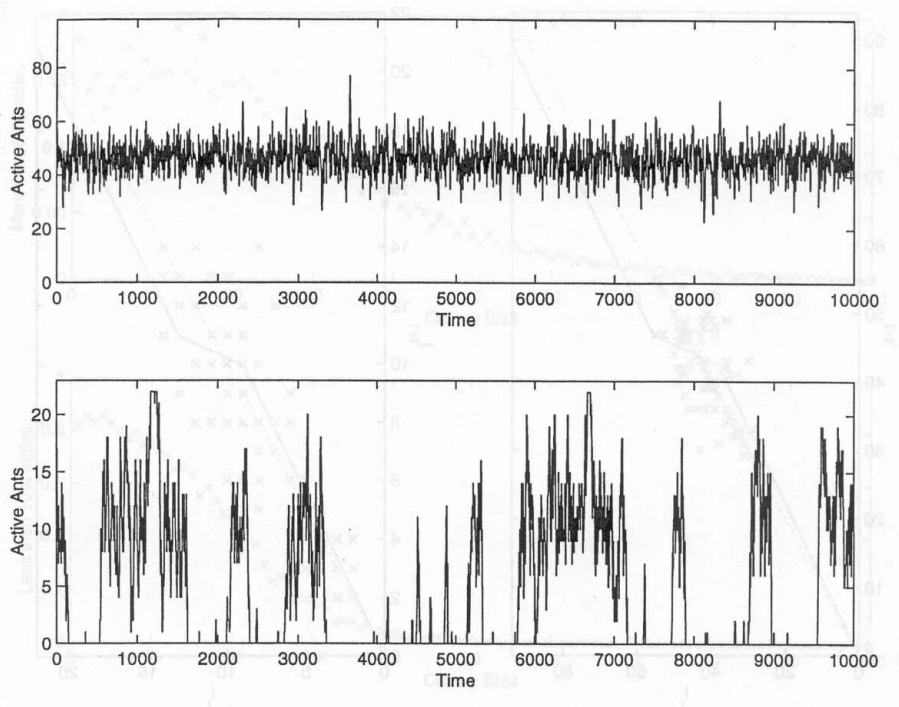


Back to System 3



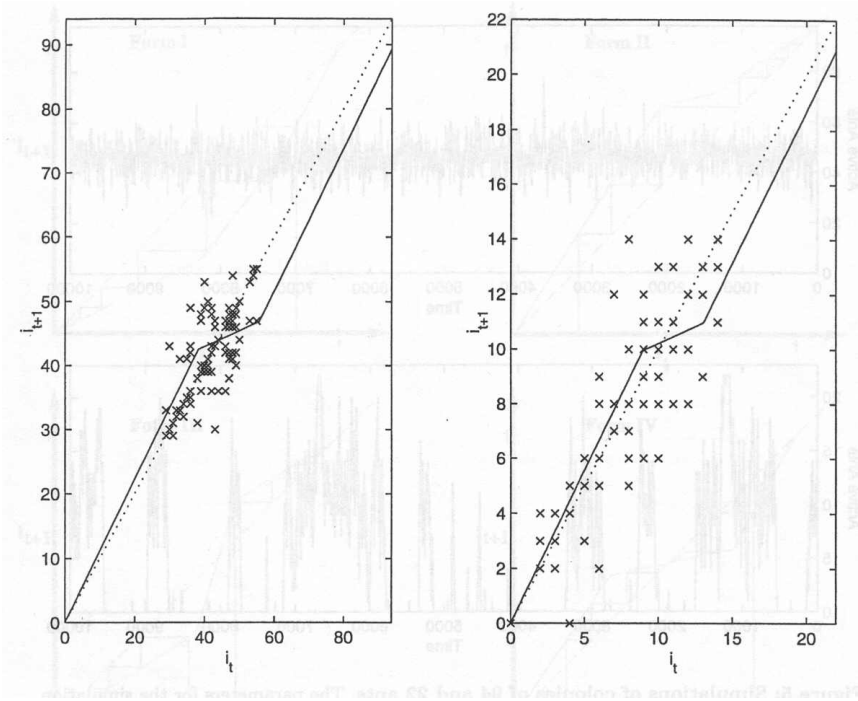
Simulations of Activity

System 3: $p_P = 0.001$, $p_A = 0.005$, $q_p = 0.36$, $q_A = 0.36$, $r_P = 0.53$, $r_A = 0.53$. Top: $n = 94$; Bottom: $n = 22$.



Activity Maps: Simulation vs MDE

System 3: $p_P = 0.001$, $p_A = 0.005$, $q_p = 0.36$, $q_A = 0.36$, $r_P = 0.53$, $r_A = 0.53$. Left: $n = 94$; Right: $n = 22$.



Conclusion

- Simple agent dynamics can produce complex non-linear global behaviour
- Mean difference equations are a good way of looking at global time-varying behaviour without looking at the global state space
- Uses process algebra descriptions as they were always meant to be used!
- Plenty of applications in computer science as well as biology