ipc :: PEPA $\longrightarrow$ DNAmaca

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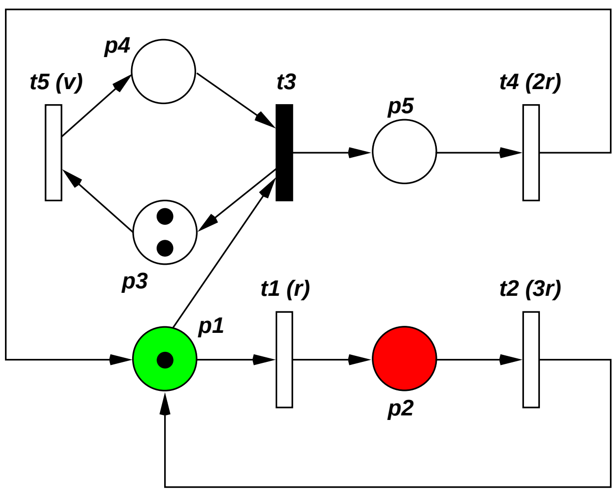
What is PEPA?

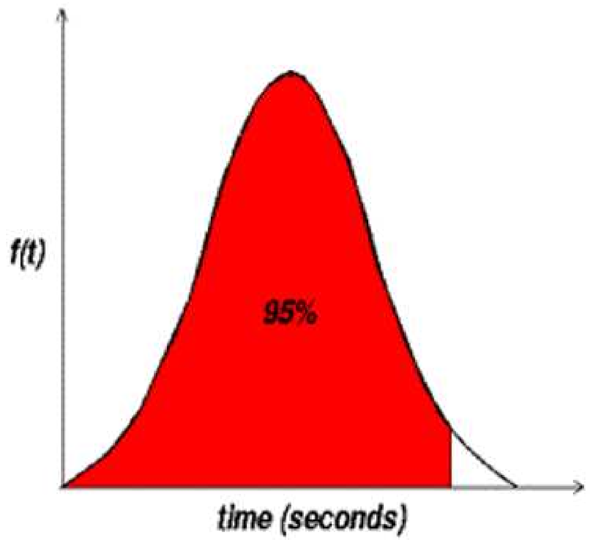
- A stochastic process algebra
- Markovian or exponential distributions
- Traditionally analysed with steady-state measures

What is DNAmaca/HYDRA?

Extracting a passage-time density from a GSPN:



$\Rrightarrow$

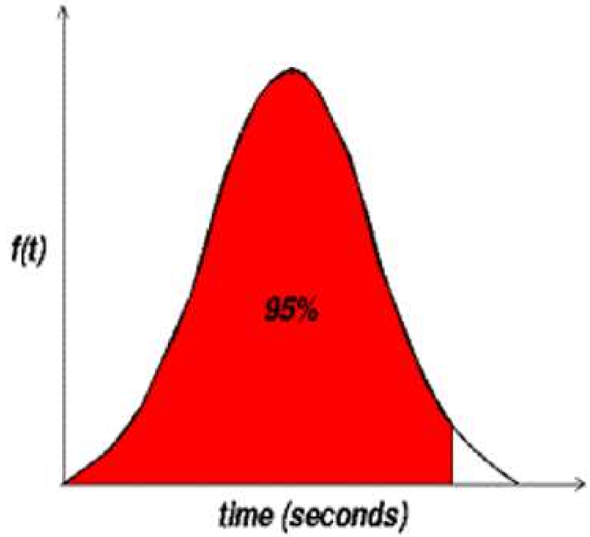


Motivation

Extract a passage-time density from a PEPA model:

$$
\begin{aligned}
A1 &\stackrel{\text{def}}{=} (\text{start}, r_1).A2 + (\text{pause}, r_2).A3 \\
A2 &\stackrel{\text{def}}{=} (\text{run}, r_3).A1 + (\text{fail}, r_4).A3 \\
A3 &\stackrel{\text{def}}{=} (\text{recover}, r_1).A1 \\
AA &\stackrel{\text{def}}{=} (\text{run}, \top).(\text{alert}, r_5).AA \\
Sys &\stackrel{\text{def}}{=} AA \underset{\{run\}}{\bowtie} A1
\end{aligned}
$$

$\Longrightarrow$



PEPA: Process Algebra

- Syntax:

$$P \ ::= \ (a,\lambda).P \ \Big| \ P + P \ \Big| \ P \underset{S}{\bowtie} P \ \Big| \ P/L \ \Big| \ A$$

- $(a,\lambda).P$: prefix operation $\quad$ $P+P$: competitive choice

- $P \underset{S}{\bowtie} P$: component cooperation $\quad$ P/L: action hiding

PEPA: Example

$$
\begin{aligned}
\mathsf{A1} &\stackrel{\text{def}}{=} (\mathsf{start}, r_1).\mathsf{A2} + (\mathsf{pause}, r_2).\mathsf{A3} \\
\mathsf{A2} &\stackrel{\text{def}}{=} (\mathsf{run}, r_3).\mathsf{A1} + (\mathsf{fail}, r_4).\mathsf{A3} \\
\mathsf{A3} &\stackrel{\text{def}}{=} (\mathsf{recover}, r_1).\mathsf{A1} \\
\mathsf{AA} &\stackrel{\text{def}}{=} (\mathsf{run}, \top).(\mathsf{alert}, r_5).\mathsf{AA}
\end{aligned}
$$

PEPA: Example

$$
\begin{aligned}
A1 &\stackrel{\text{def}}{=} (\mathit{start}, r_1).A2 + (\mathit{pause}, r_2).A3 \\
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\end{aligned}
$$

$$
\begin{aligned}
B1 &\stackrel{\text{def}}{=} (\mathit{start}, r_1).B2 + (\mathit{pause}, r_2).B1 \\
B2 &\stackrel{\text{def}}{=} (\mathit{run}, r_3).B1 \\
BB &\stackrel{\text{def}}{=} (\mathit{run}, \top).(\mathit{alert}, r_5).BB
\end{aligned}
$$

PEPA: Example

$$\mathsf{A1} \stackrel{\text{def}}{=} (\mathsf{start}, r_1).\mathsf{A2} + (\mathsf{pause}, r_2).\mathsf{A3}$$
$$\mathsf{A2} \stackrel{\text{def}}{=} (\mathsf{run}, r_3).\mathsf{A1} + (\mathsf{fail}, r_4).\mathsf{A3}$$
$$\mathsf{A3} \stackrel{\text{def}}{=} (\mathsf{recover}, r_1).\mathsf{A1}$$
$$\mathsf{AA} \stackrel{\text{def}}{=} (\mathsf{run}, \top).(\mathsf{alert}, r_5).\mathsf{AA}$$

$$\mathsf{B1} \stackrel{\text{def}}{=} (\mathsf{start}, r_1).\mathsf{B2} + (\mathsf{pause}, r_2).\mathsf{B1}$$
$$\mathsf{B2} \stackrel{\text{def}}{=} (\mathsf{run}, r_3).\mathsf{B1}$$
$$\mathsf{BB} \stackrel{\text{def}}{=} (\mathsf{run}, \top).(\mathsf{alert}, r_5).\mathsf{BB}$$

$$\mathsf{Sys} \stackrel{\text{def}}{=} (\mathsf{AA} \underset{\mathsf{run}}{\bowtie} \mathsf{A1}) \underset{\mathsf{alert}}{\bowtie} (\mathsf{BB} \underset{\mathsf{run}}{\bowtie} \mathsf{B1})$$

Example ipc Compilation

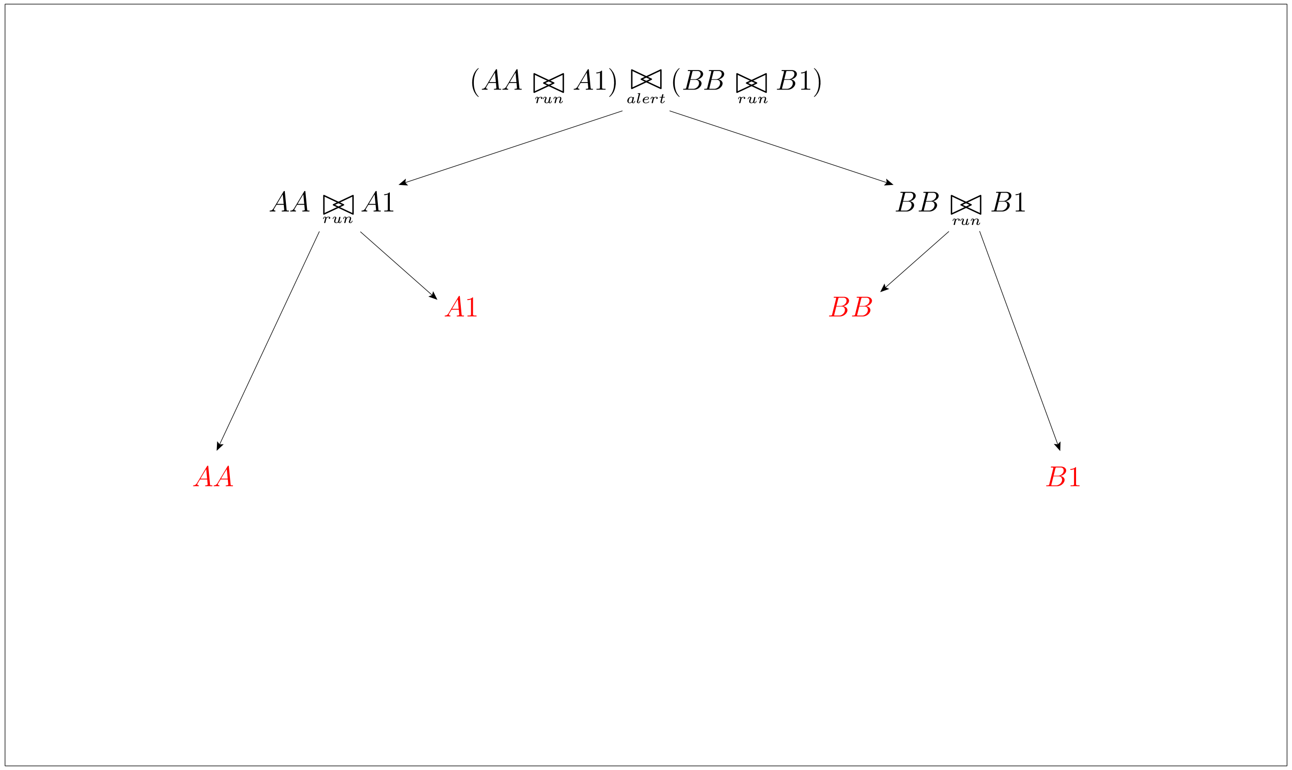
$$(AA \underset{run}{\bowtie} A1) \underset{alert}{\bowtie} (BB \underset{run}{\bowtie} B1)$$

Example ipc Compilation

$$(AA \underset{run}{\bowtie} A1) \underset{alert}{\bowtie} (BB \underset{run}{\bowtie} B1)$$

$$AA \underset{run}{\bowtie} A1 \qquad\qquad BB \underset{run}{\bowtie} B1$$

Example ipc Compilation



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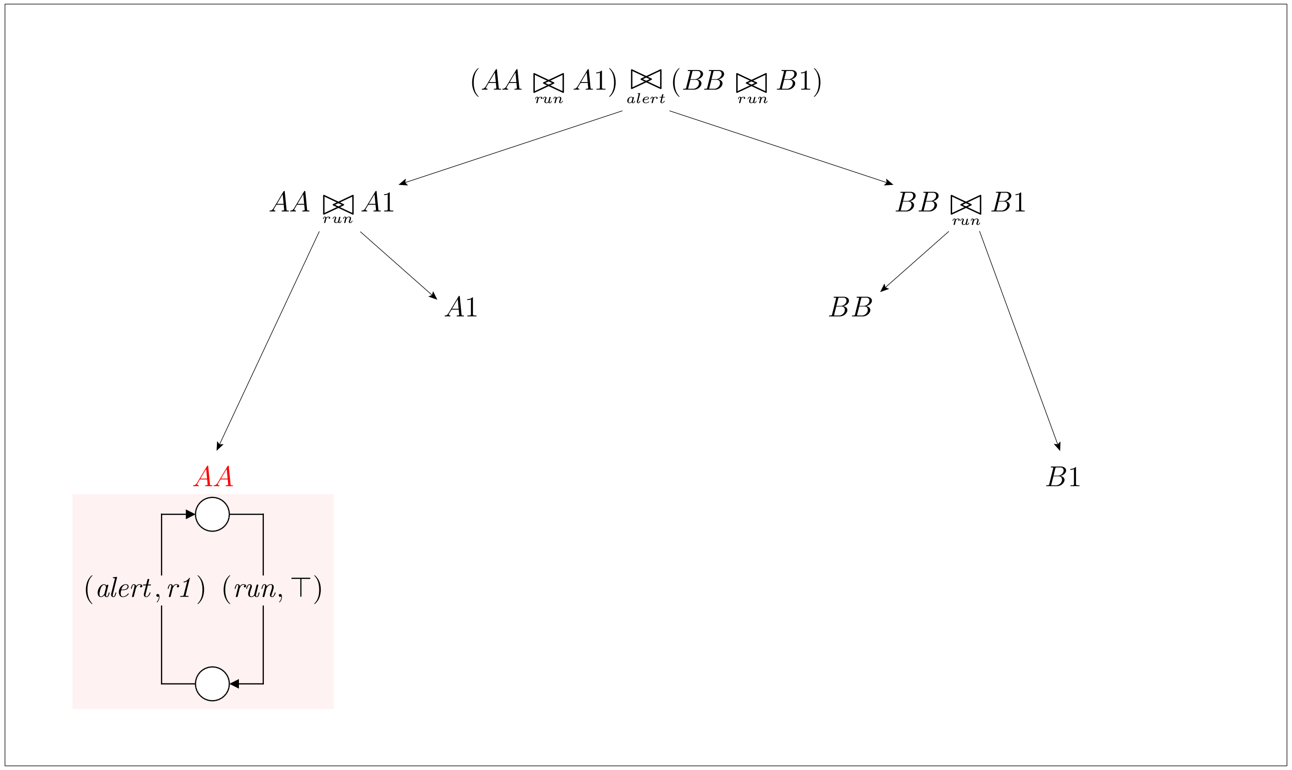

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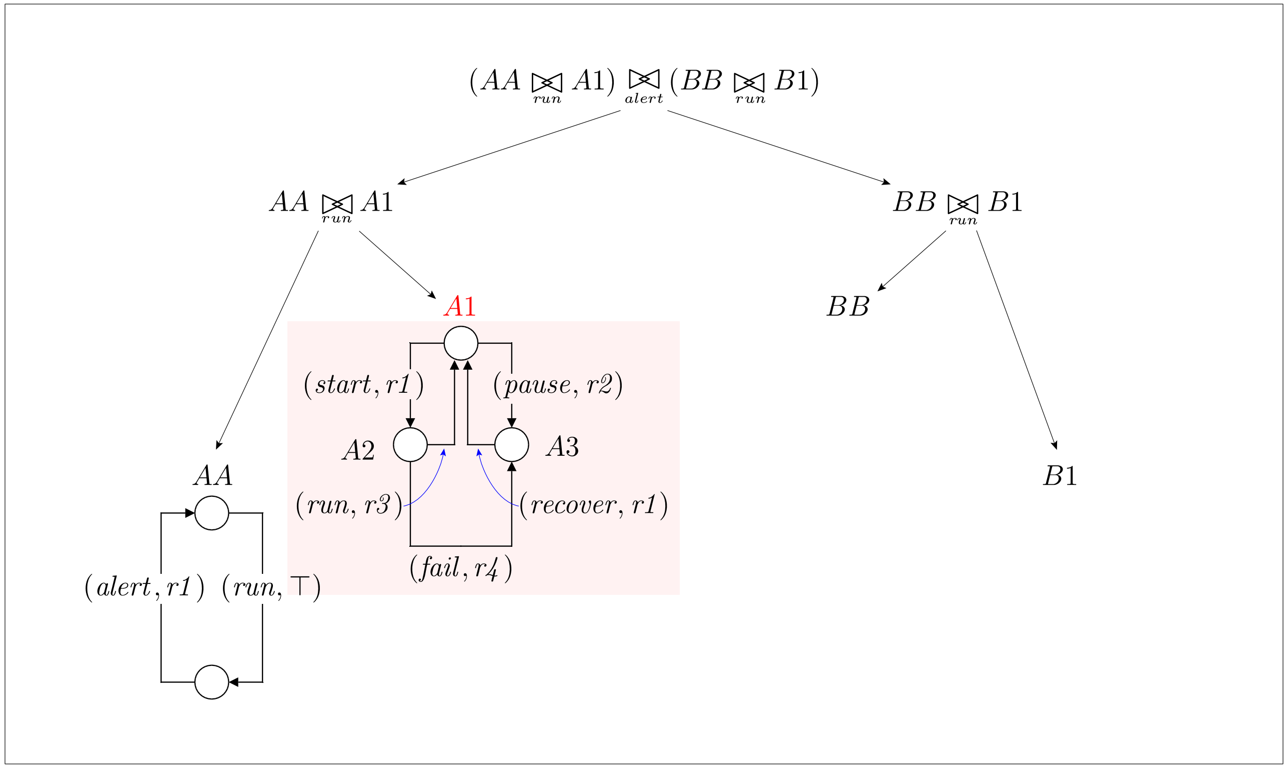

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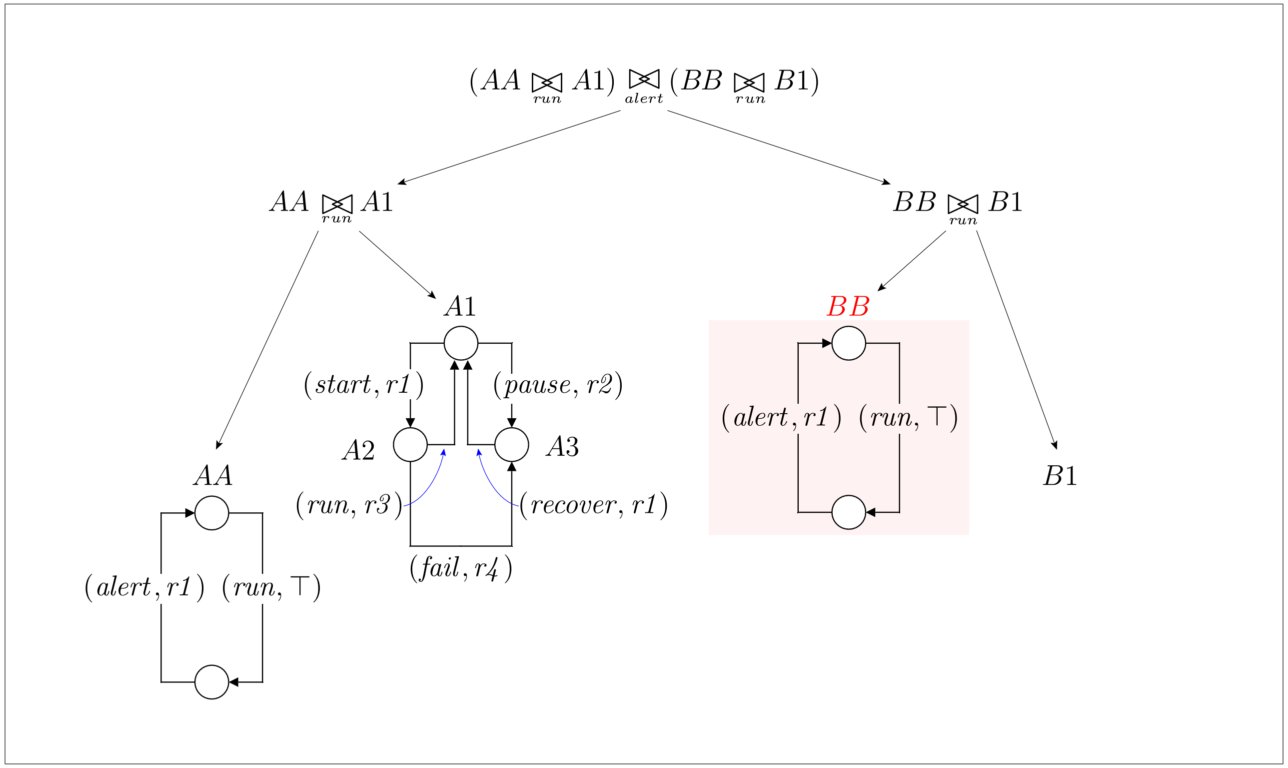

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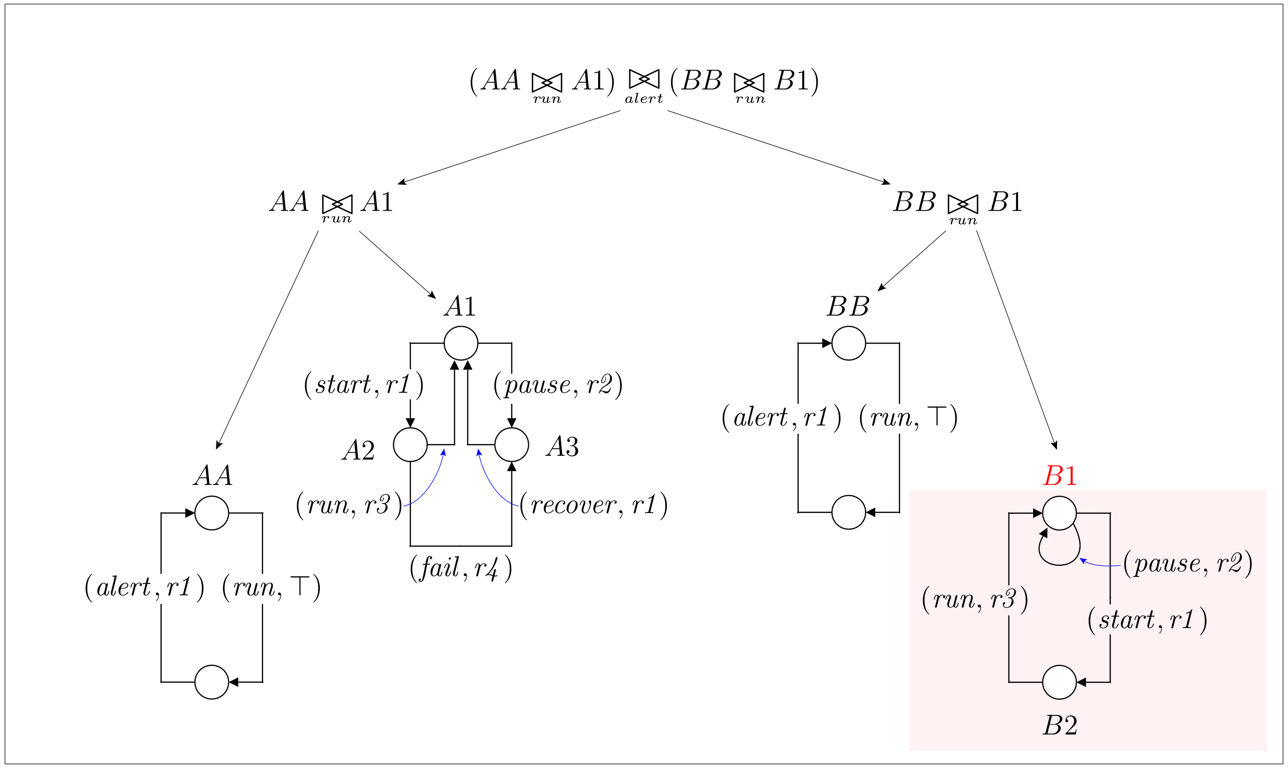

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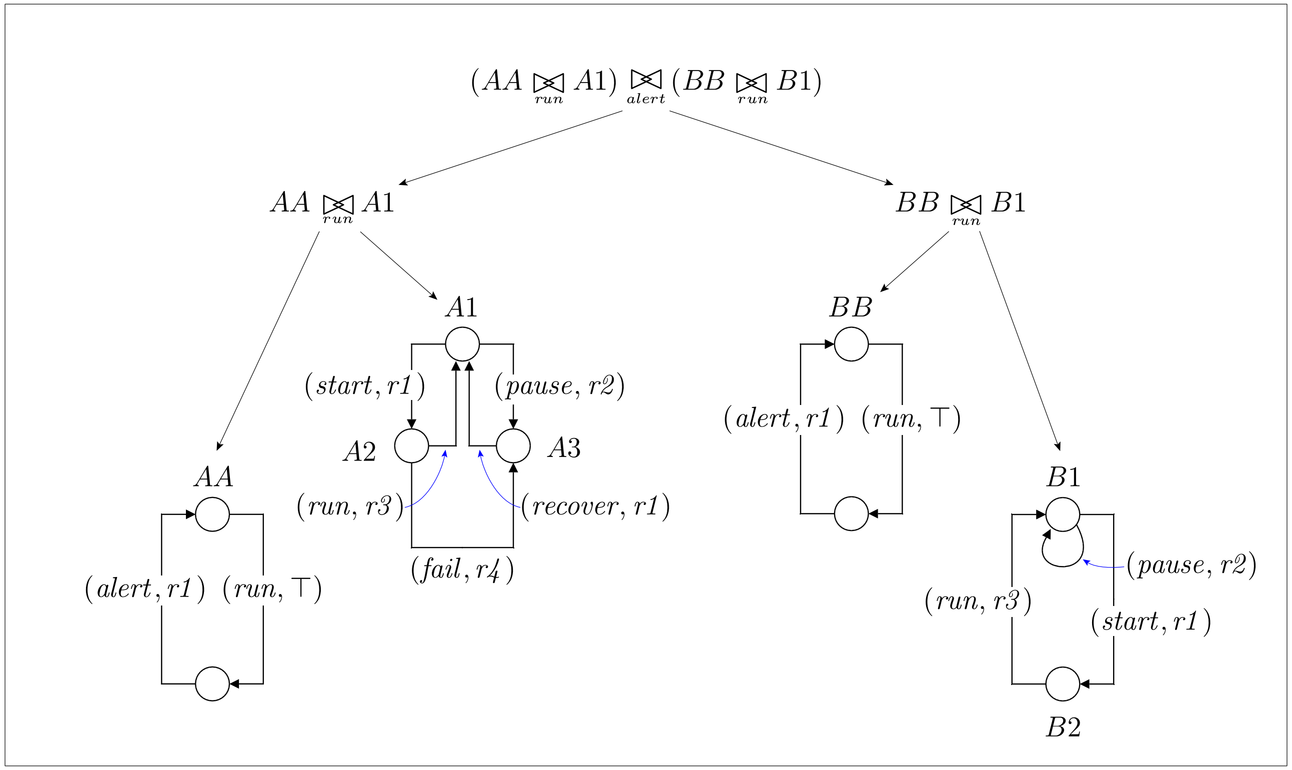

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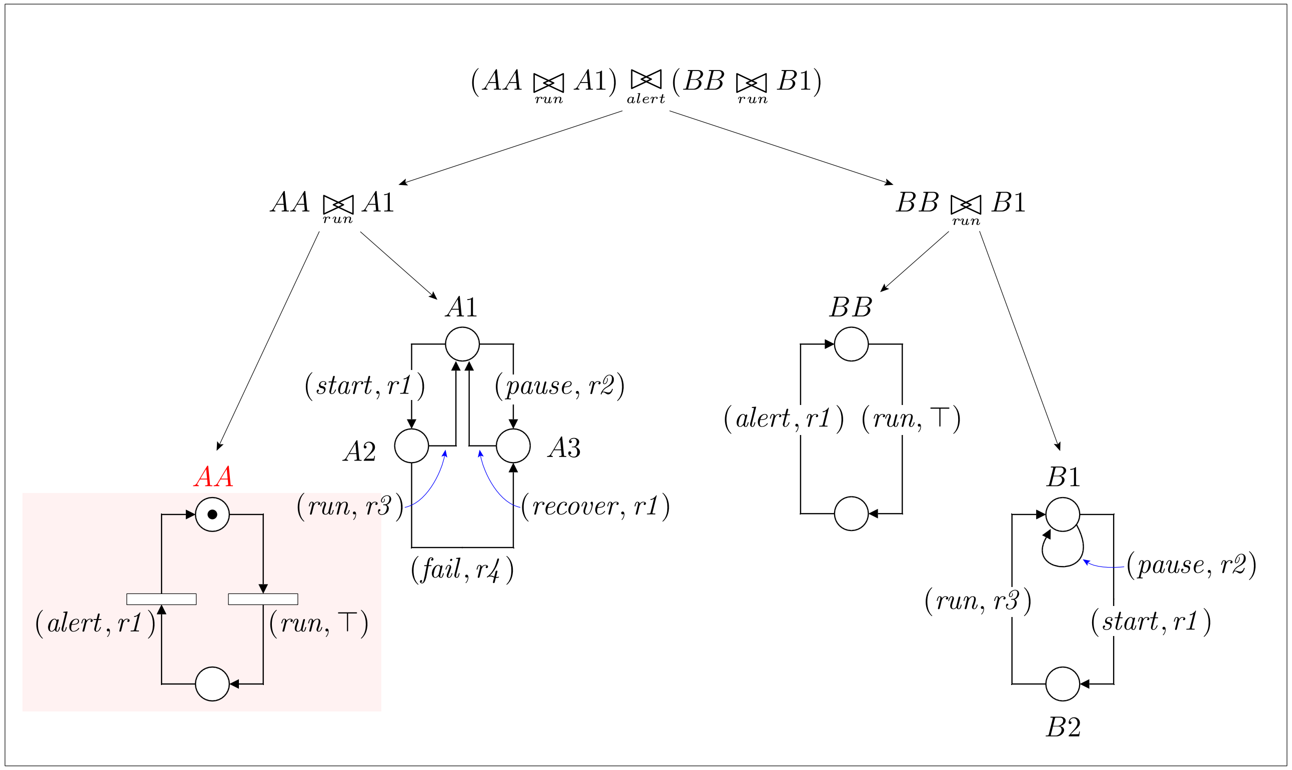

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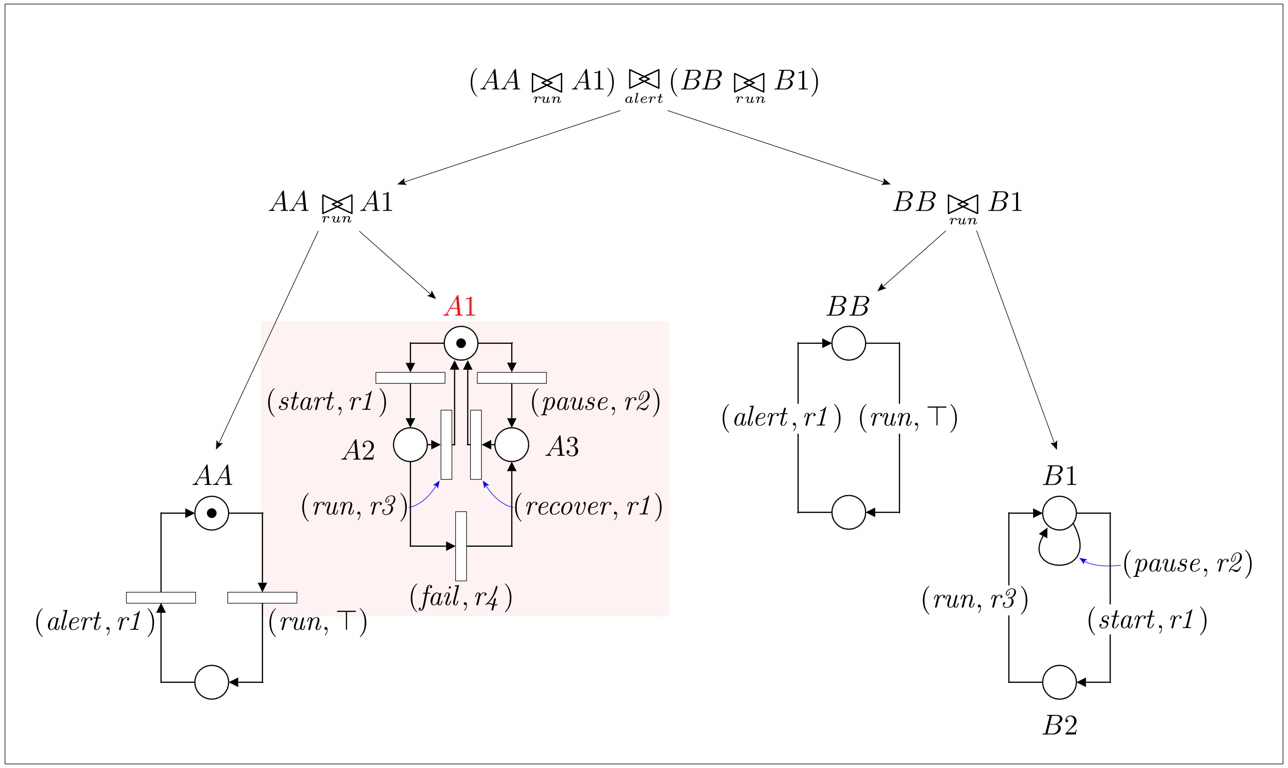

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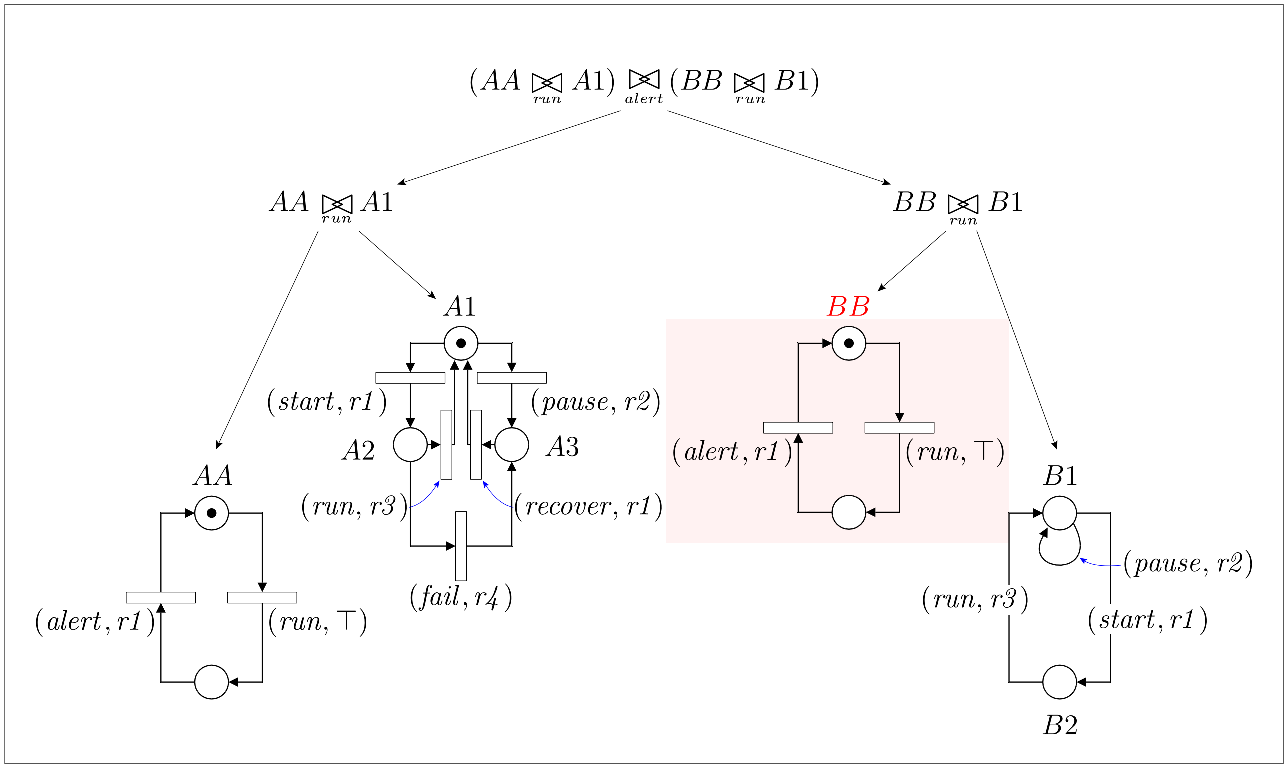

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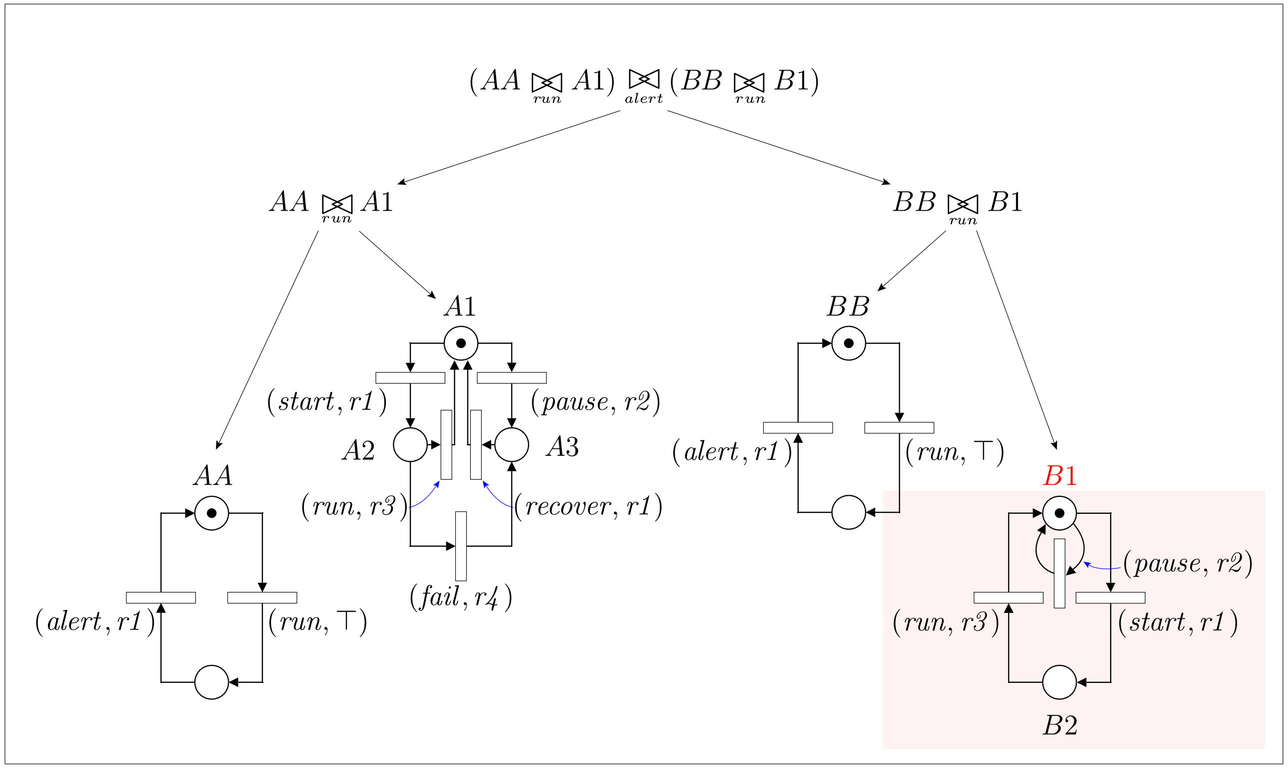

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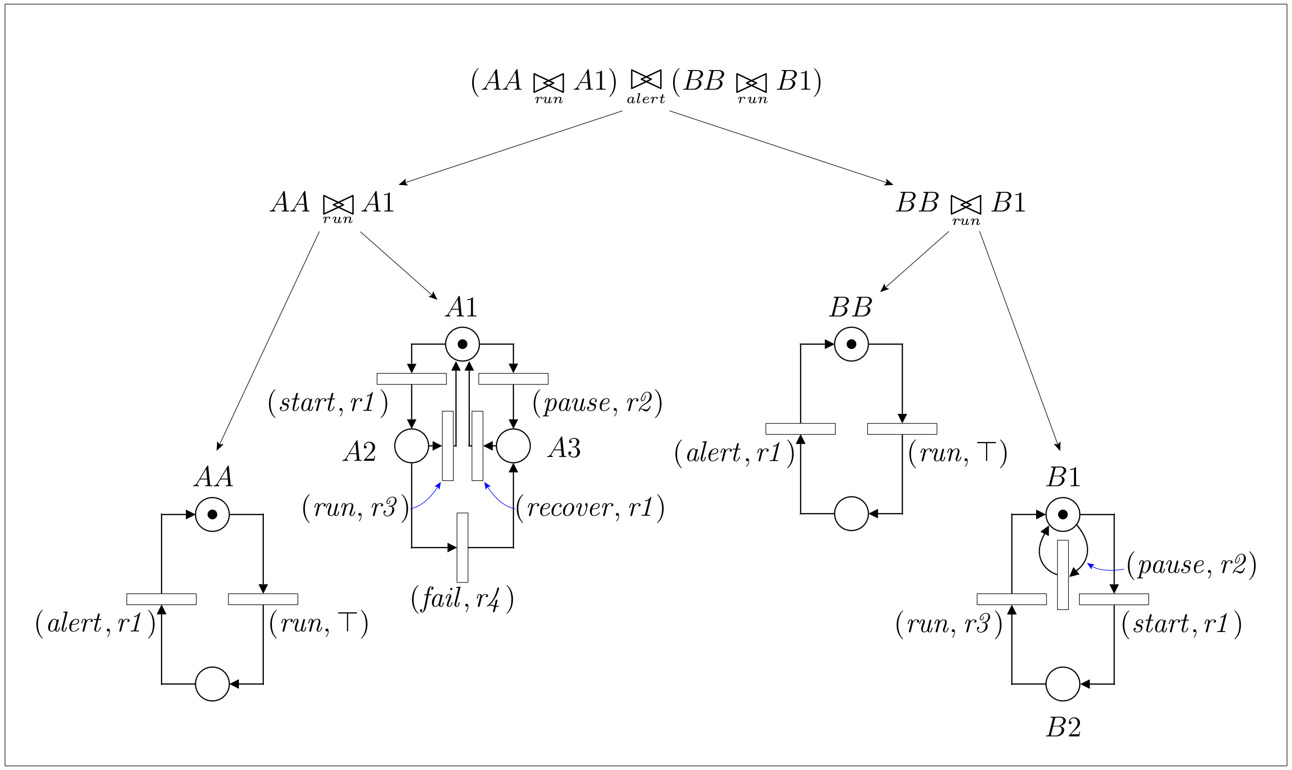

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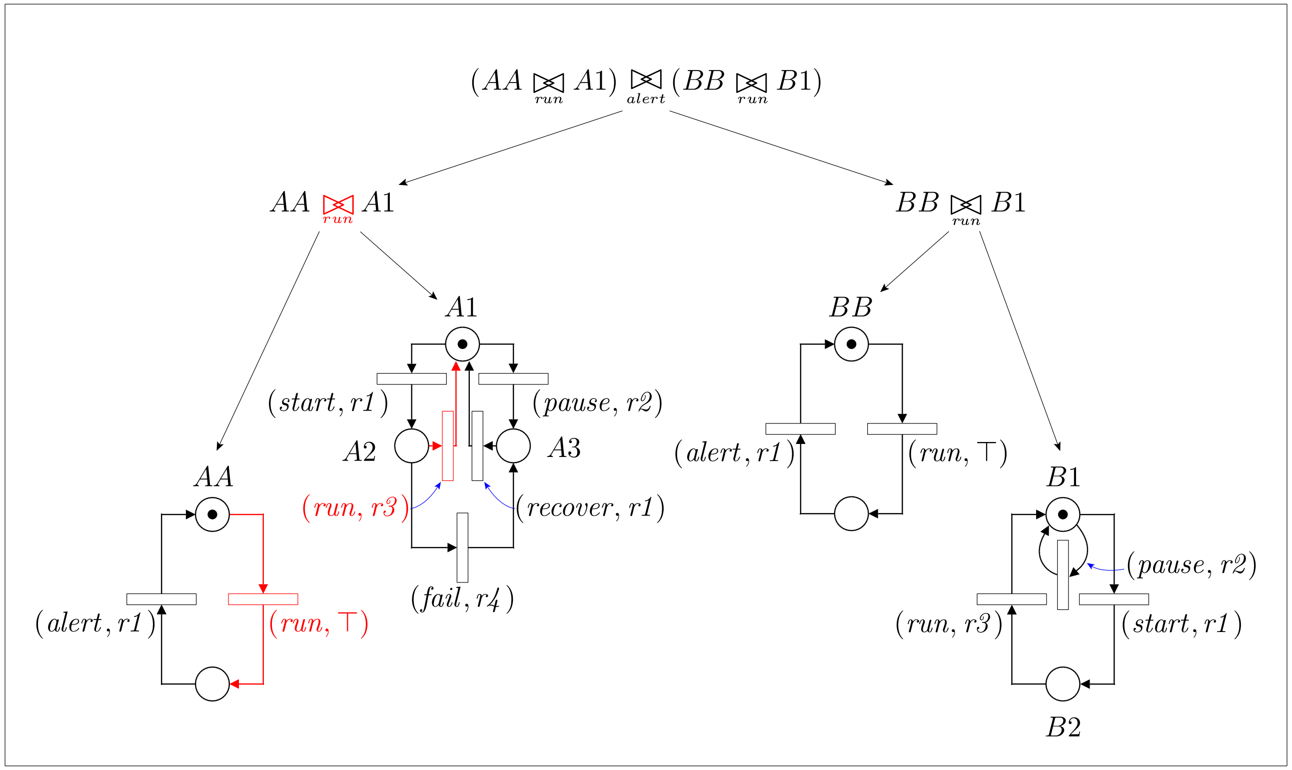

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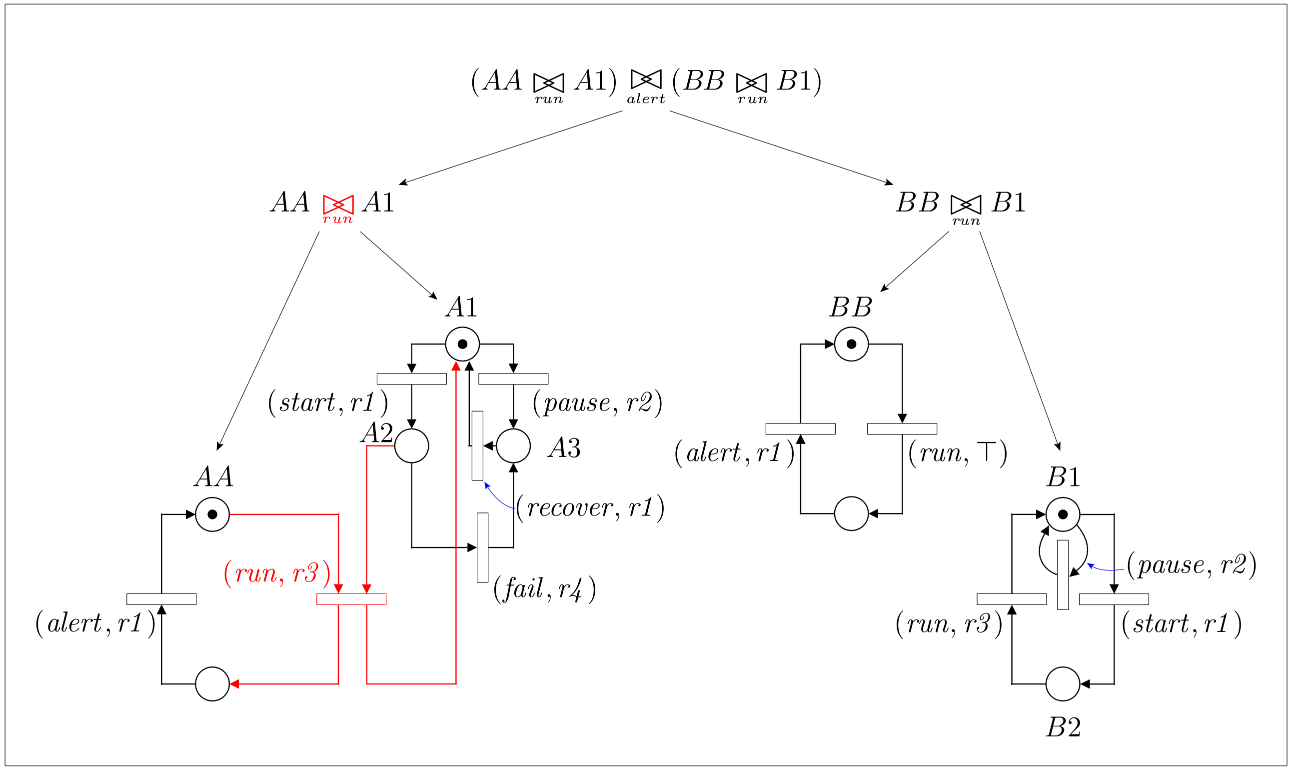

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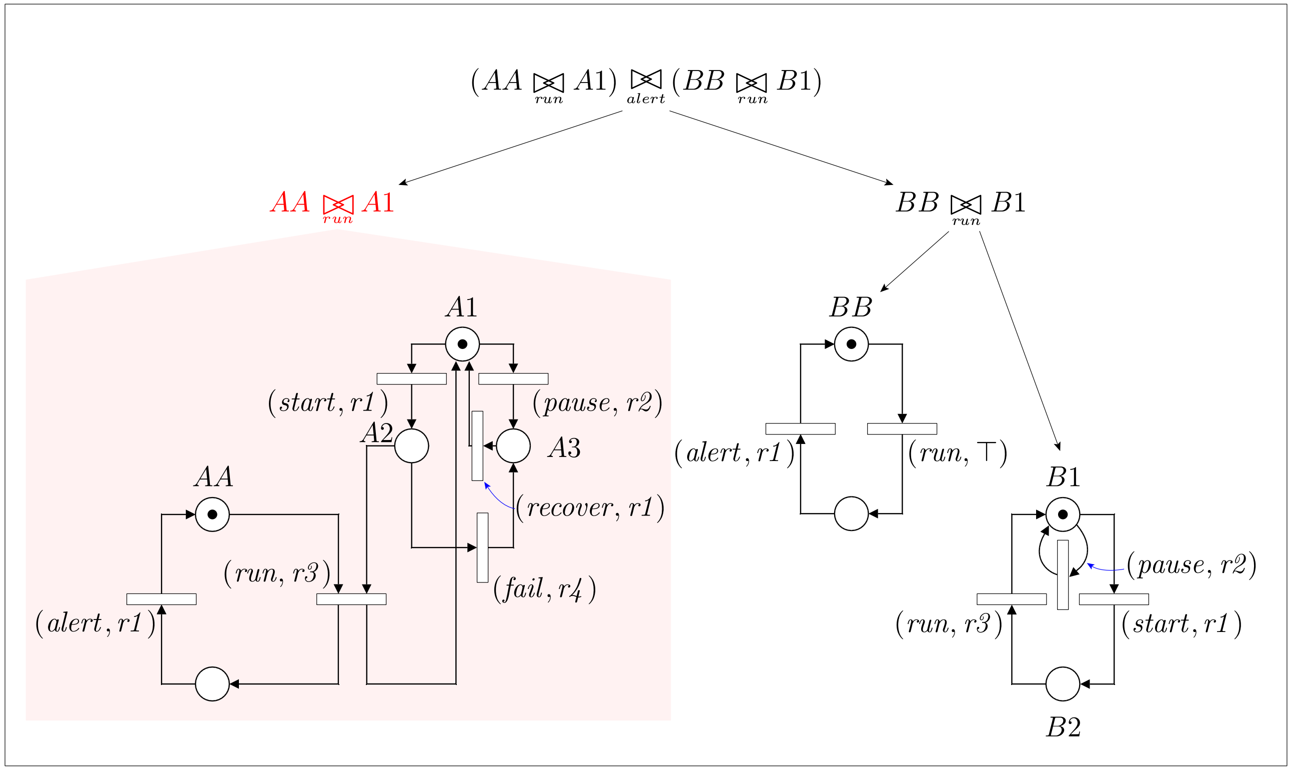

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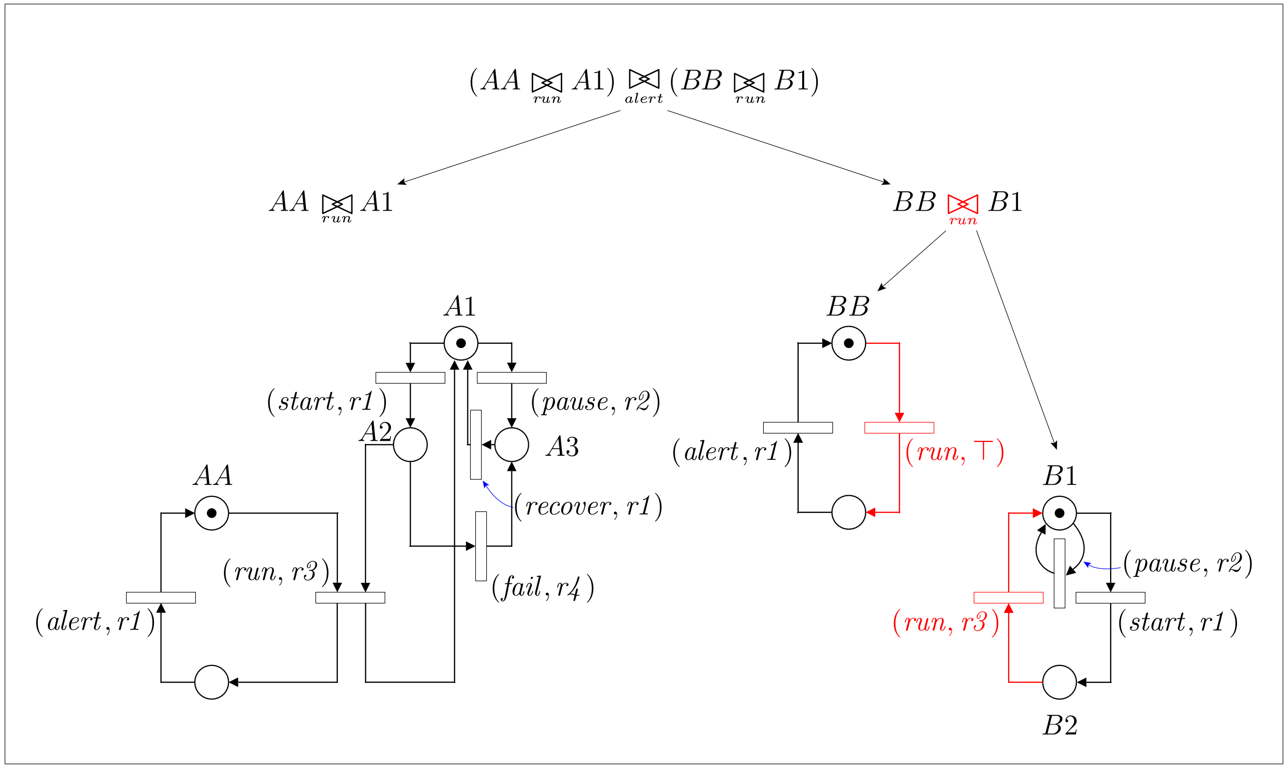

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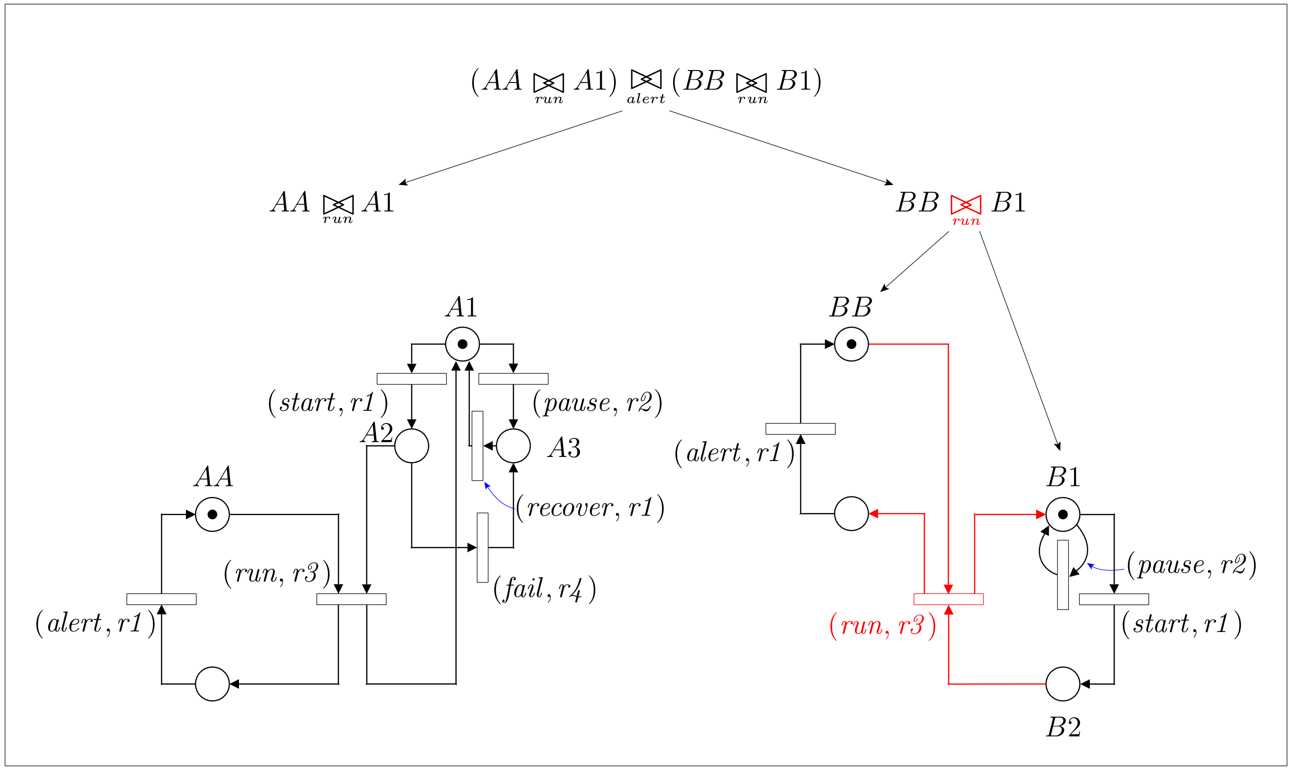

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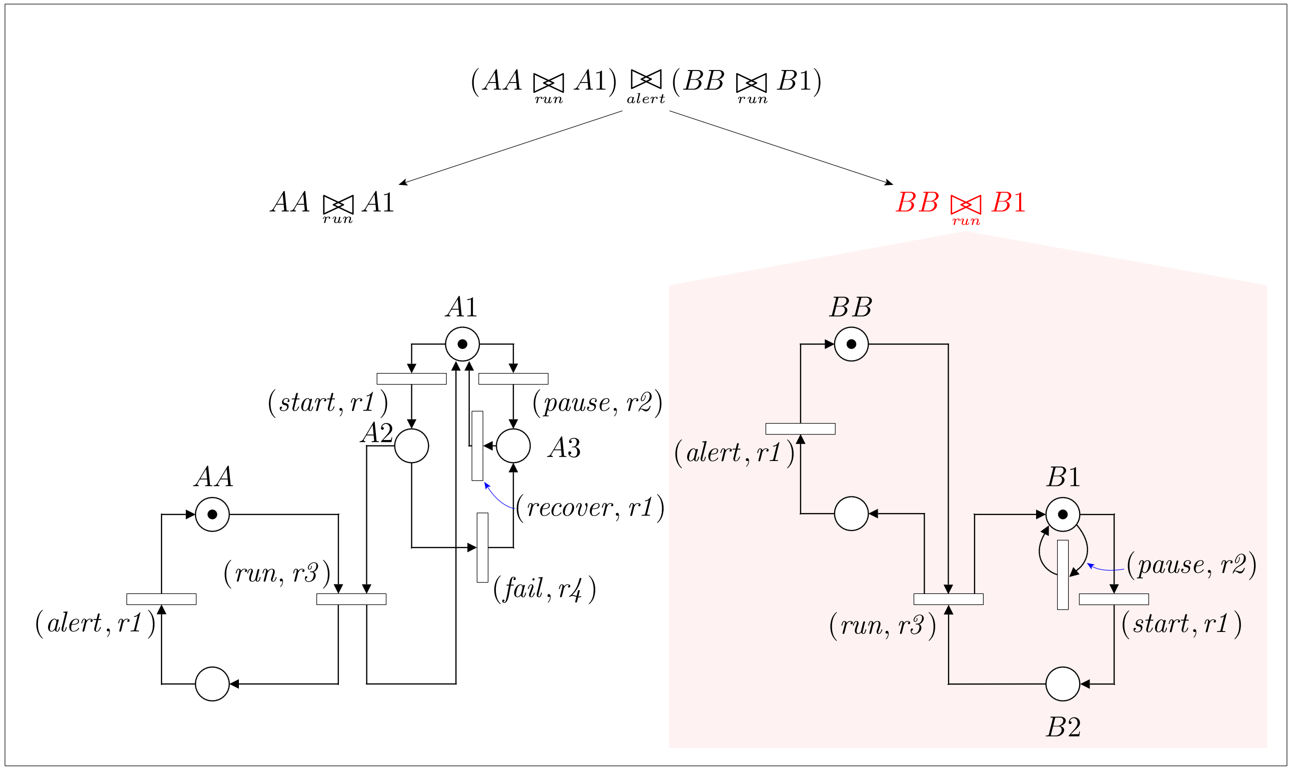

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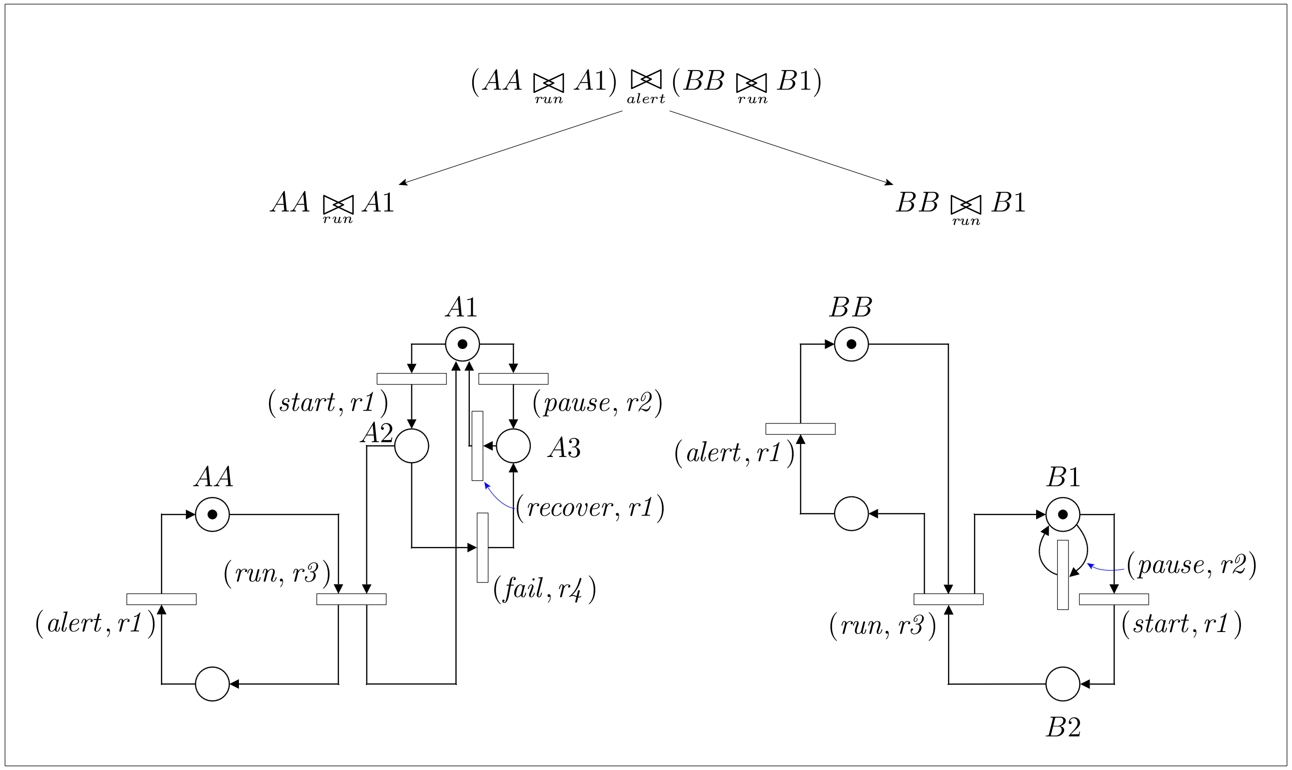

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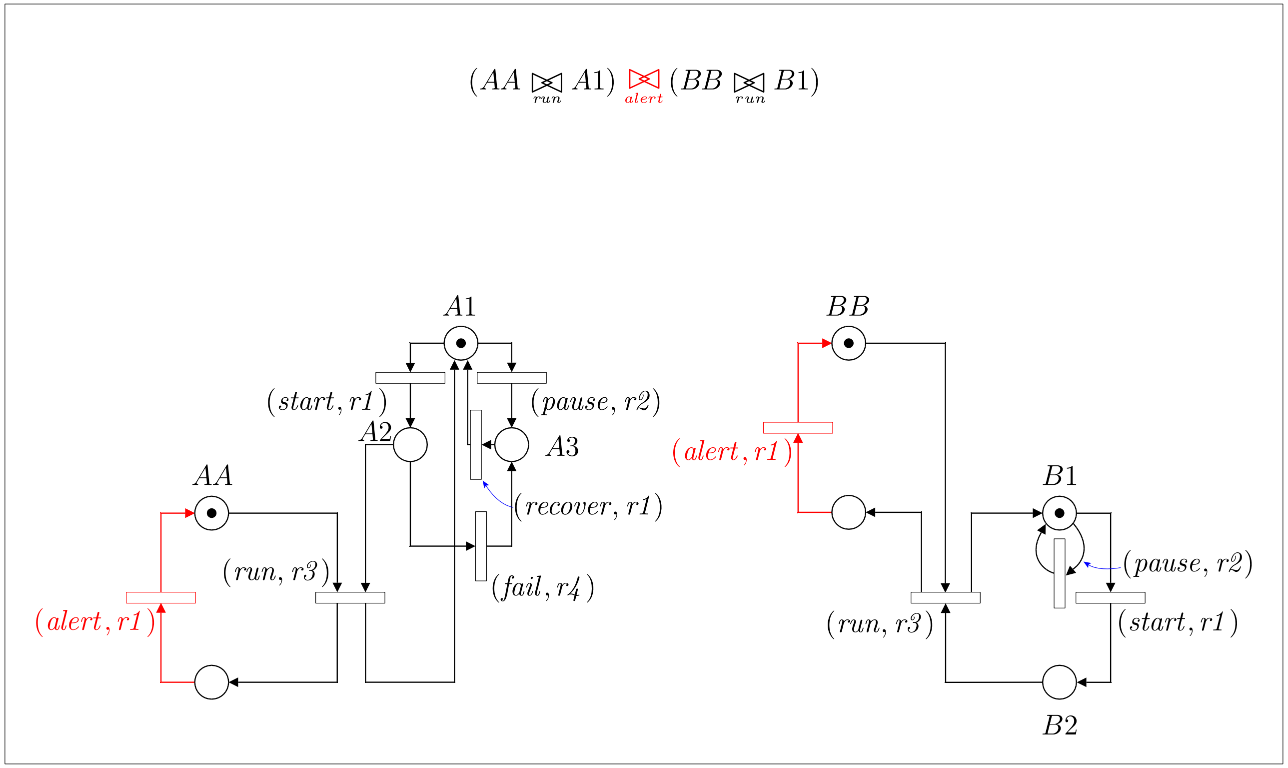

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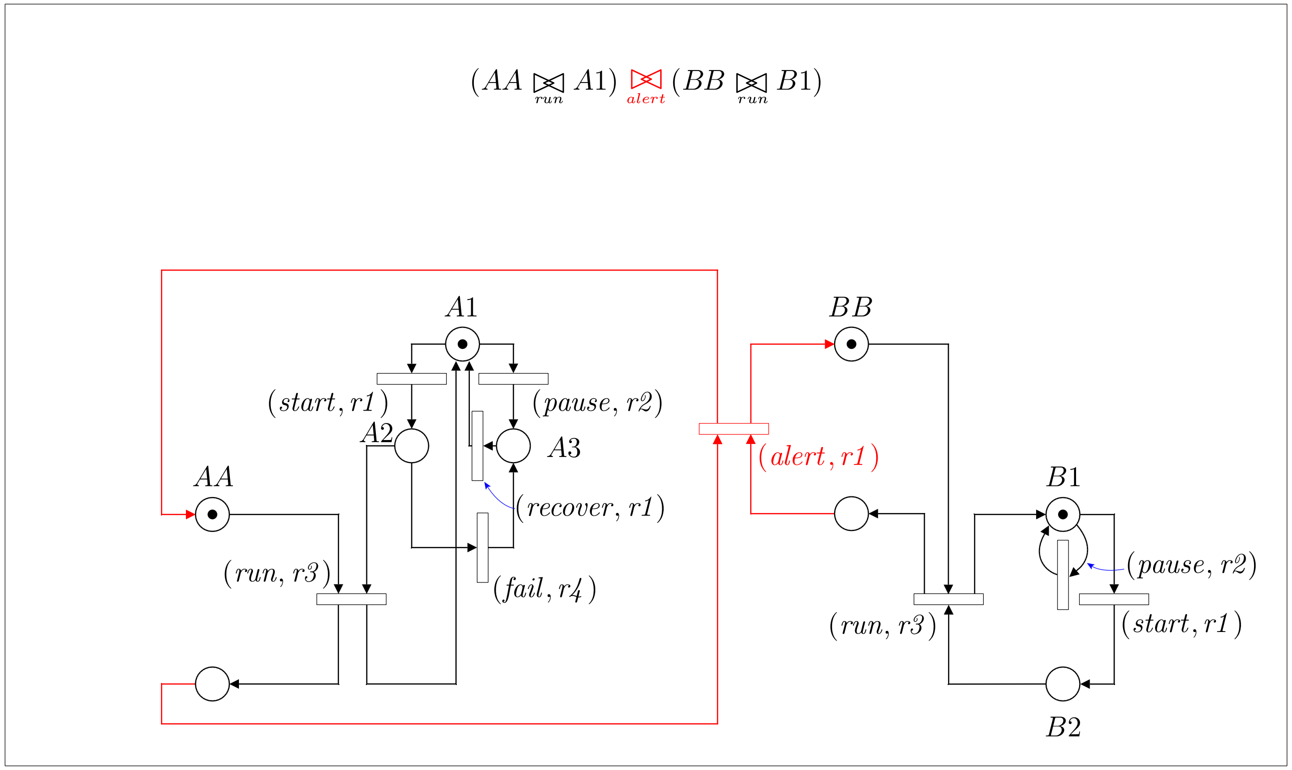

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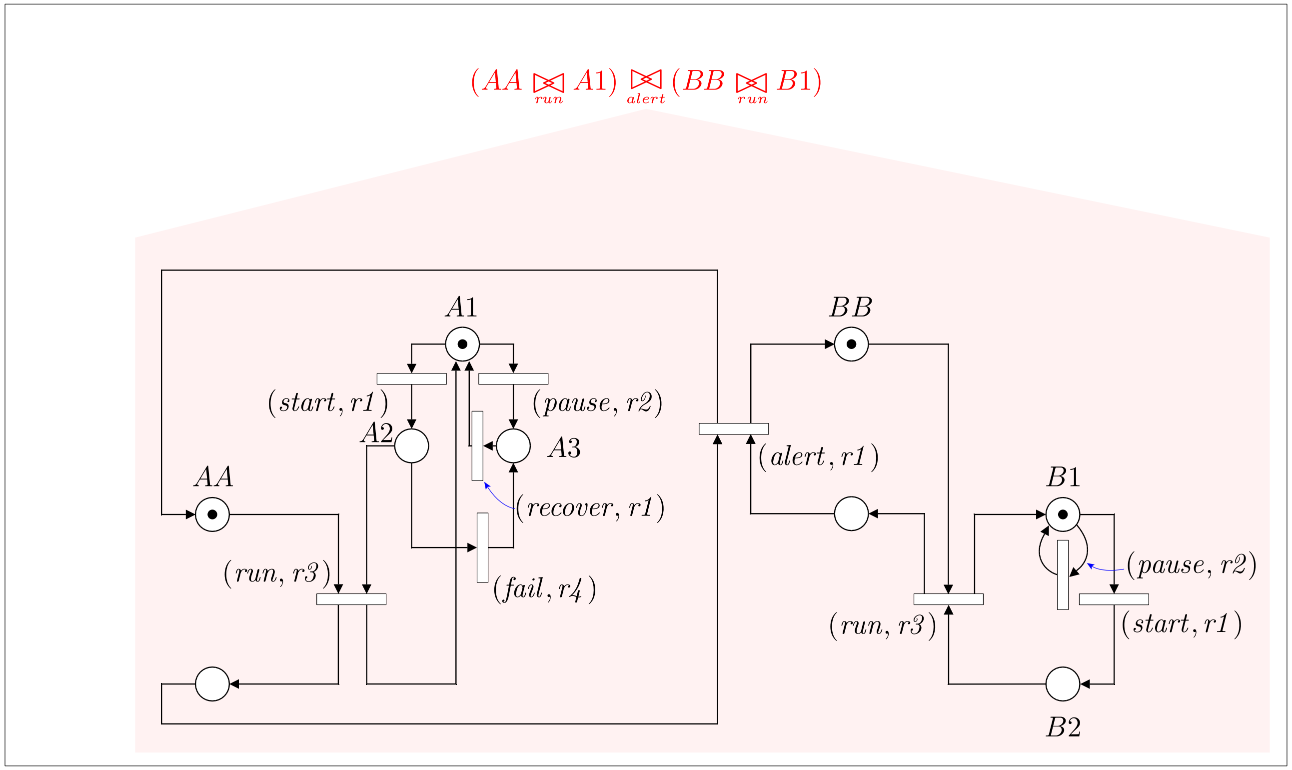

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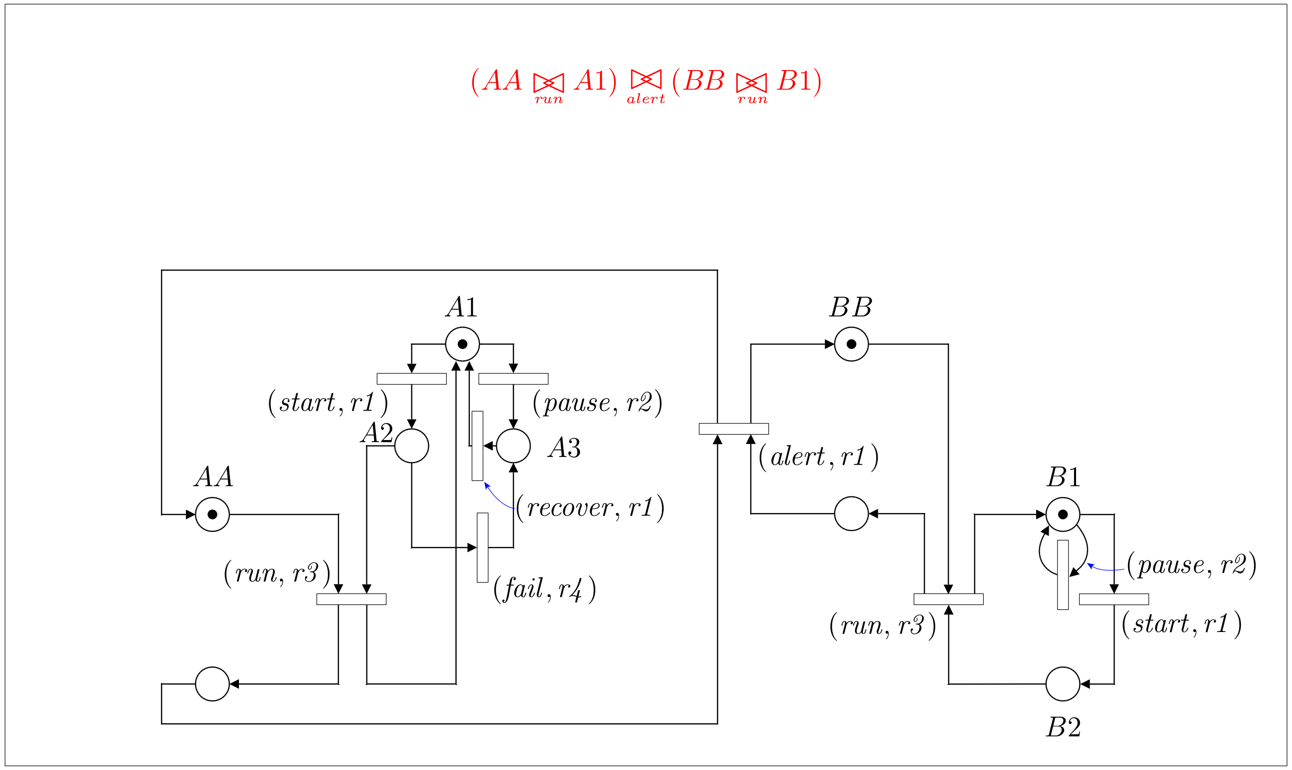


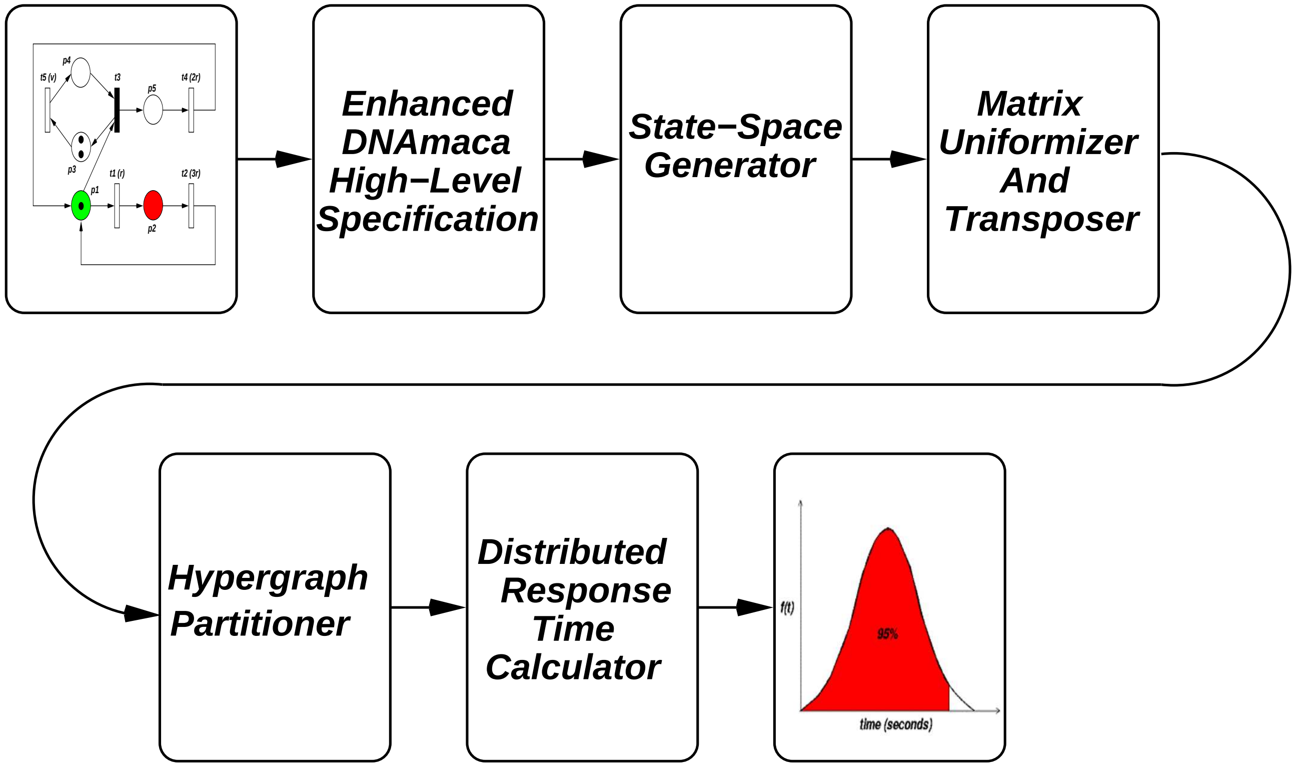
DNAmaca: Example

A transition definition in DNAmaca:

```
\transition{A2_fail} {
        %% PEPAaction type { fail }
        \condition{ A2 > 0 }
        \action {
                next -> A2 = A2 - 1;
                next -> A3 = A3 + 1;
        }
        \priority{1}
        \rate{ PEPA_r4 }
}
```

HYDRA :: DNAmaca $\rightarrow$ Passage

HYDRA Tool Pipeline



Uniformization (I)

- Used for transient and passage-time analysis of continuous-time Markov chains (CTMCs)

- A CTMC generator matrix , Q, is "uniformized" with:

$$P = Q/q + I$$

  where $q > \max_i |q_{ii}|$

- Transforms a CTMC into one in which all states have the same mean holding time $1/q$

Uniformization (II)

- Passage/transient calculation has two main components:
  - The time to complete n hops ($n = 1, 2, 3, \ldots$), which is an Erlang distribution with parameters n and q
  - The probability that the transition between source and target states occurs in exactly n hops
  - In equation form:

$$f_{\vec{i}\vec{j}}(t) = \sum_{n=1}^{\infty} \frac{q^n t^{n-1} e^{-qt}}{(n-1)!} \sum_{k \in \vec{j}} \tilde{\pi}_k^{(n)}$$

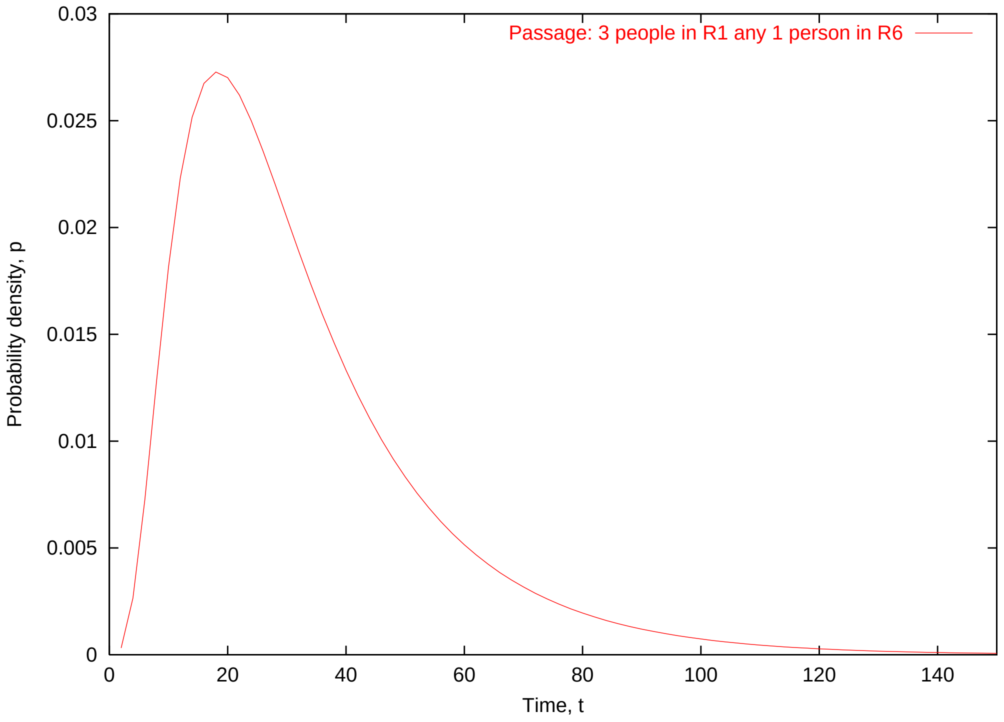
Active Badge Model

Example: 2 people with 2 rooms and sensors:

$$
\begin{aligned}
\mathrm{Person1} &\stackrel{\mathrm{def}}{=} (\mathrm{reg1}, r).\mathrm{Person1} + (\mathrm{move2}, m).\mathrm{Person2} \\
\mathrm{Person2} &\stackrel{\mathrm{def}}{=} (\mathrm{reg2}, r).\mathrm{Person2} + (\mathrm{move1}, m).\mathrm{Person1} \\
\mathrm{Sensor1} &\stackrel{\mathrm{def}}{=} (\mathrm{reg1}, \top).(\mathrm{rep1}, s).\mathrm{Sensor1} \\
\mathrm{Sensor2} &\stackrel{\mathrm{def}}{=} (\mathrm{reg2}, \top).(\mathrm{rep2}, s).\mathrm{Sensor2} \\
\mathrm{Dbase1} &\stackrel{\mathrm{def}}{=} (\mathrm{rep1}, \top).\mathrm{Dbase1} + (\mathrm{rep2}, \top).\mathrm{Dbase2} \\
\mathrm{Dbase2} &\stackrel{\mathrm{def}}{=} (\mathrm{rep1}, \top).\mathrm{Dbase1} + (\mathrm{rep2}, \top).\mathrm{Dbase2} \\
\mathrm{Sys} &\stackrel{\mathrm{def}}{=} (\mathrm{Person1} \parallel \mathrm{Person1}) \underset{\{\mathrm{reg1},\mathrm{reg2}\}}{\bowtie} ((\mathrm{Sensor1} \parallel \mathrm{Sensor2}) \underset{\{\mathrm{rep1},\mathrm{rep2}\}}{\bowtie} \mathrm{Dbase1})
\end{aligned}
$$

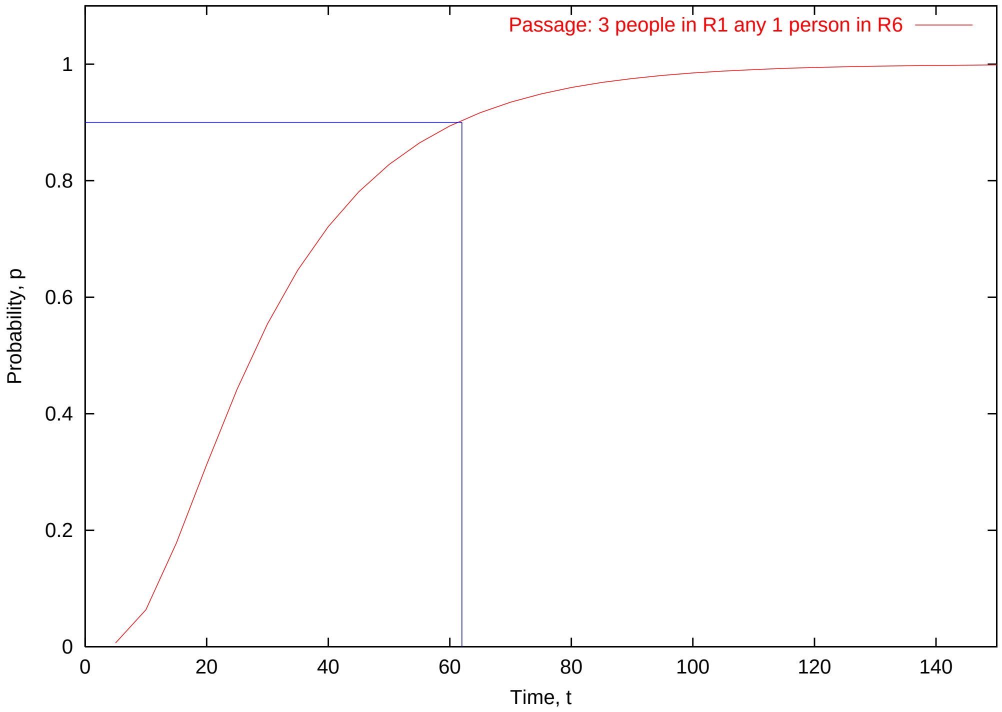
Output – passage time

[3 people/6 rooms]: 3 people from room 1 to any one in room 6



Output – passage time cdf

[3 people/6 rooms]: 3 people from room 1 to any one in room 6



Conclusions

- Purpose: to extract passage times from PEPA models
  - Introduced new tools ipc and DNAmaca/HYDRA
  - ipc: PEPA $\rightarrow$ DNAmaca
  - HYDRA: DNAmaca $\rightarrow$ passage time densities/quantiles