Performance Queries

on Semi-Markov SPNs with an Extended Continuous Stochastic Logic

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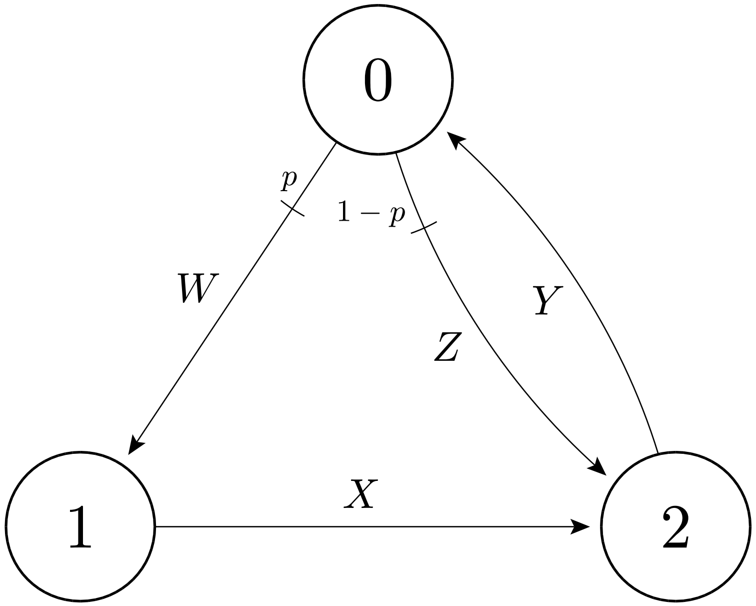
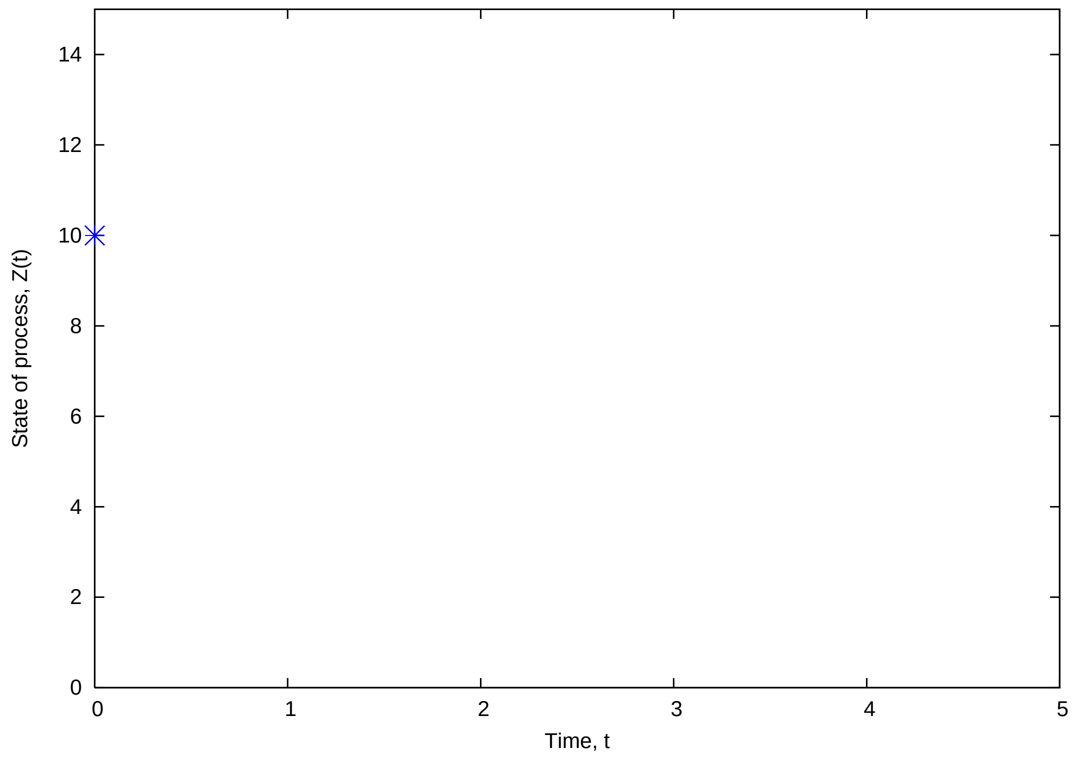
What is an SMP?



Distributions:
$W \sim \Gamma(\alpha, \lambda)$, $Z \sim \mathrm{Hyper}\left(\frac{2}{3}, \frac{1}{3}; \lambda, \mu\right)$, $X \sim \mathrm{U}\left[\frac{1}{\lambda}, \frac{1}{\mu}\right]$, $Y \sim \delta\left[\frac{1}{\mu}\right]$

Random Process Evolution



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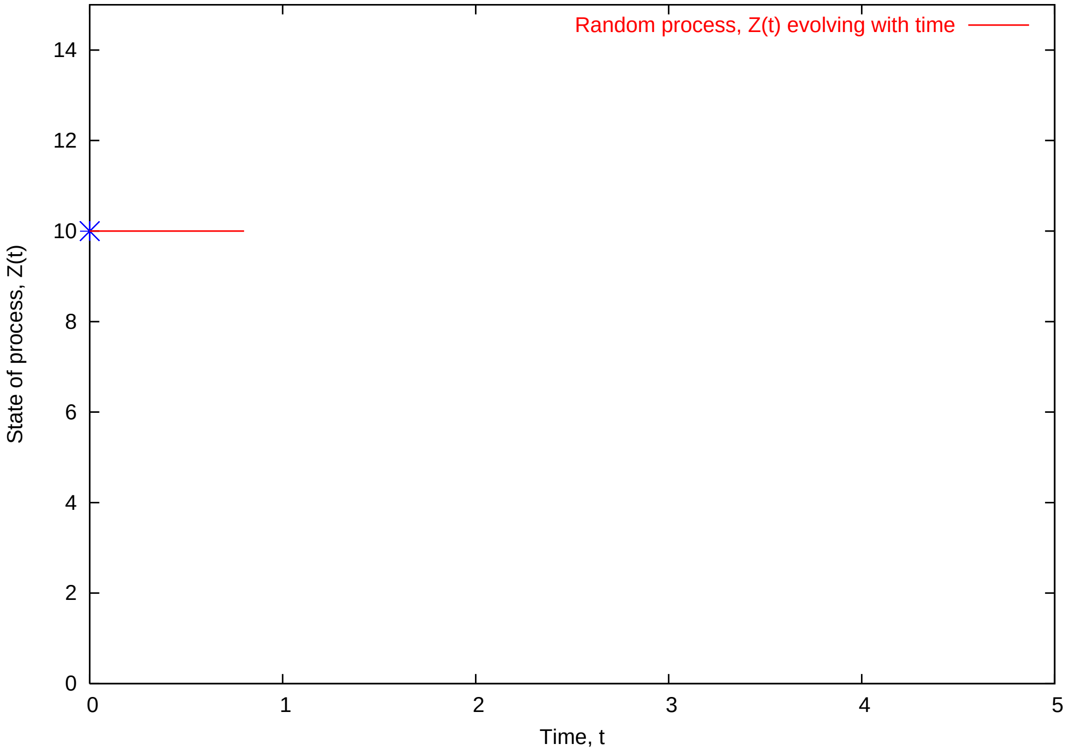

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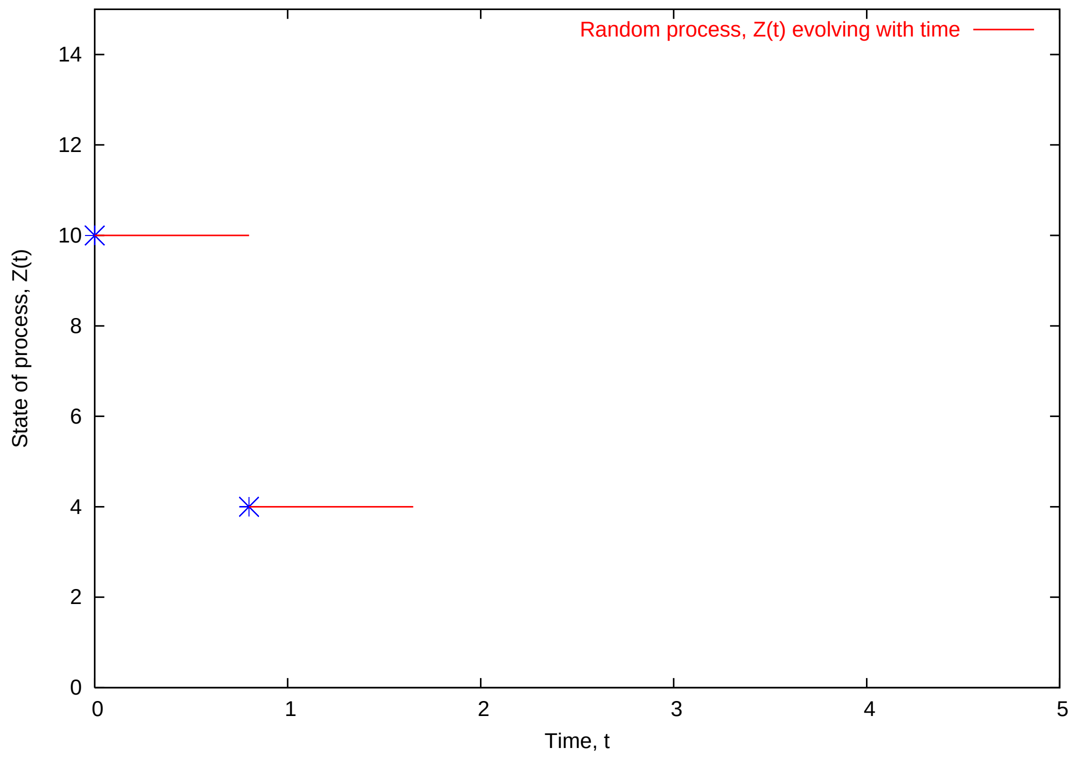
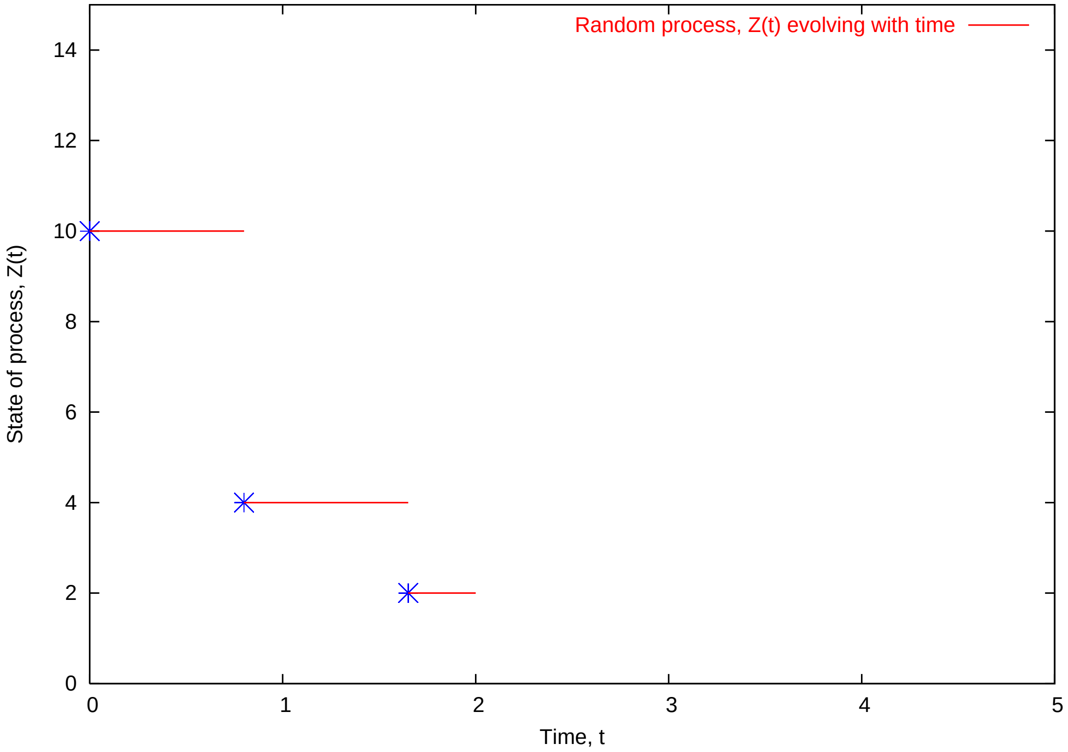
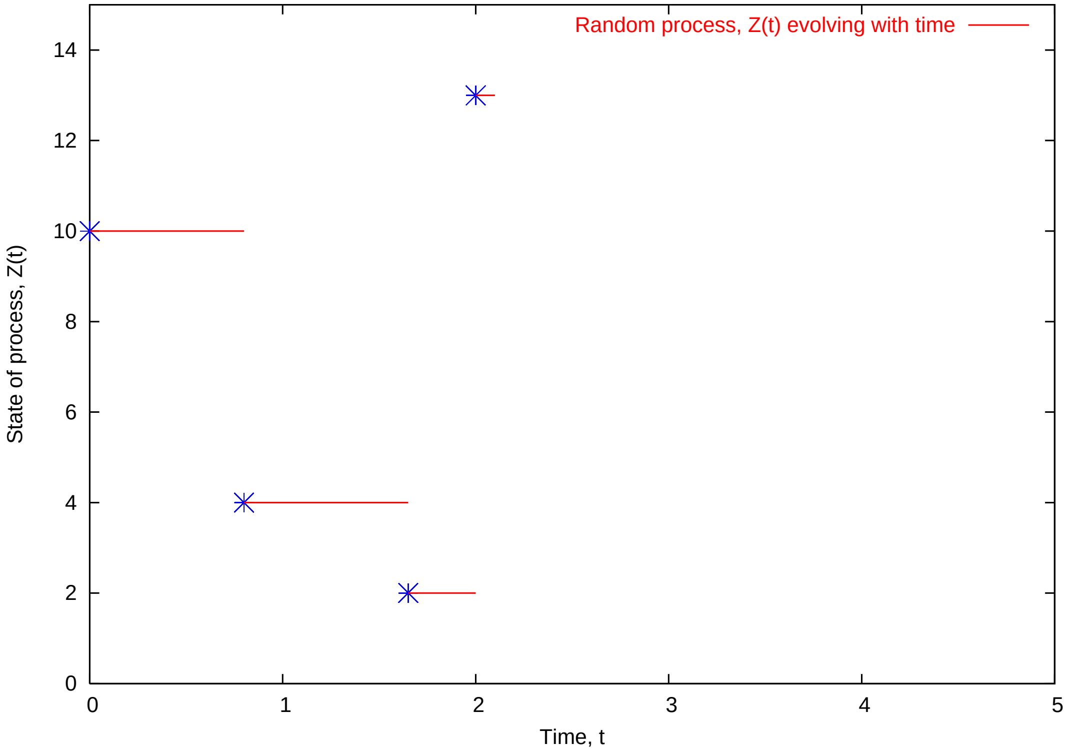


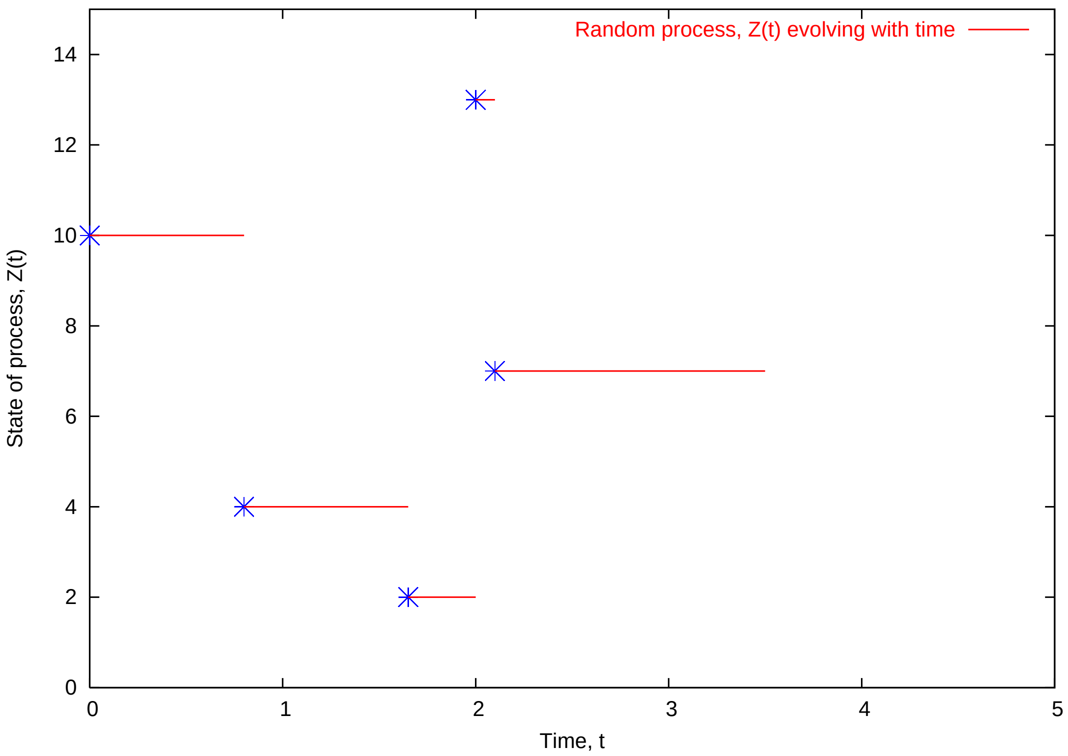
Random Process Evolution



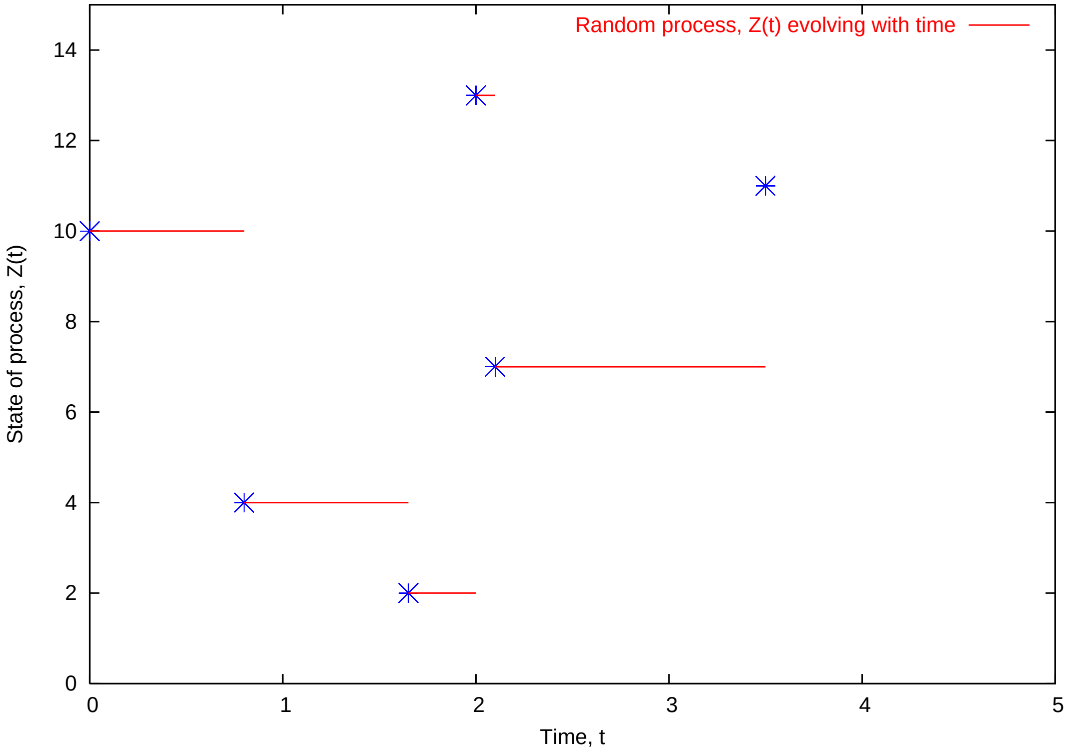
Random Process Evolution



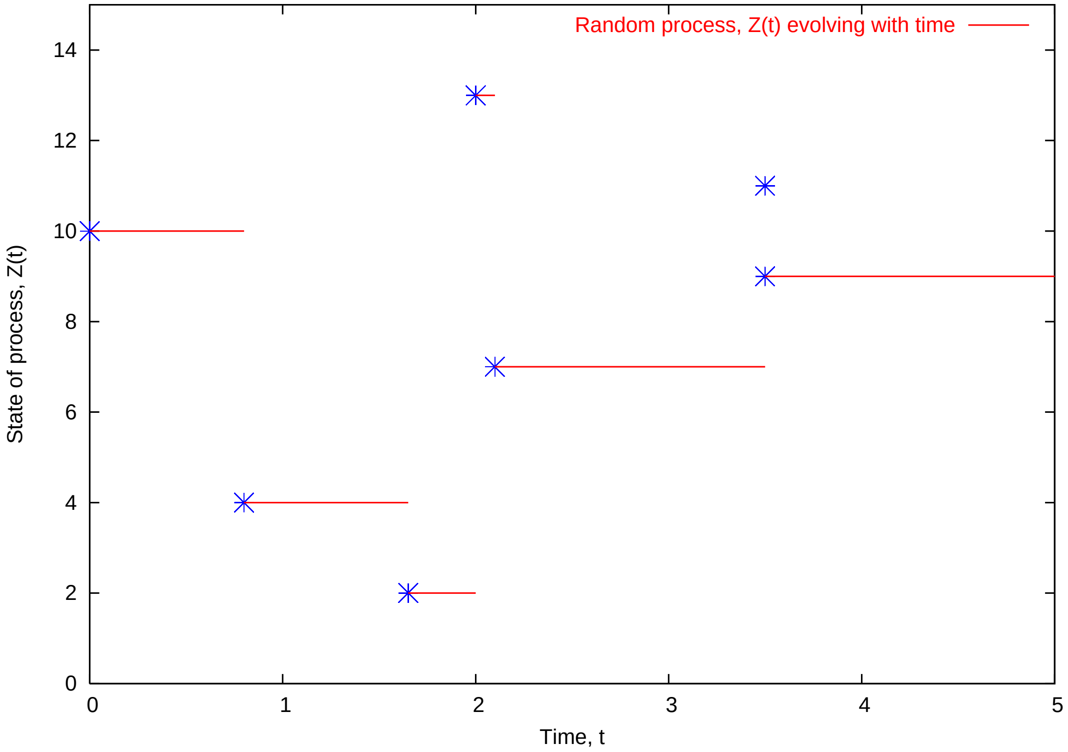
Random Process Evolution



Random Process Evolution



Random Process Evolution



What is an SMP?

- Represented by *discrete-time Markov chain*, P_{ij}, and state sojourn-time distribution matrix, $H_{ij}(t)$

- Kernel of time-homogeneous process:

$$\begin{aligned} R_{ij}(t) &= \mathbb{P}(X_{n+1} = j, T_{n+1} - T_n \leq t \mid X_n = i) \\ &= P_{ij} H_{ij}(t) \end{aligned}$$

- X_n is state at nth state change, T_n is time of nth state change

Passage-time Analysis

- Passage-time calculation from single start state to multiple end states:

$$L_{i\vec{j}}(s) = \sum_{k \notin \vec{j}} \hat{r}_{ik}(s) L_{k\vec{j}}(s) + \sum_{k \in \vec{j}} \hat{r}_{ik}(s) \qquad : \text{for } 1 \leq i \leq N$$

- Weighted passage-time to take into account many possible start states:

$$L_{\vec{i}\vec{j}}(s) = \sum_{k \in \vec{i}} \alpha_k L_{k\vec{j}}(s)$$

eCSL Syntax

- Extended Continuous Stochastic Logic
  - Performance level queries

$$\Psi \stackrel{\text{def}}{=} \text{tt} \;\Big|\; \Psi \wedge \Psi \;\Big|\; \neg\Psi \;\Big|\; \mathcal{S}_{\boldsymbol{\rho}}(\psi) \;\Big|\; \mathcal{T}_{\boldsymbol{\rho}}^{\boldsymbol{\tau}}(\psi, \psi) \;\Big|\; \mathcal{P}_{\boldsymbol{\rho}}^{\boldsymbol{\tau}}(\psi, \psi)$$

  - State level queries

$$\psi \stackrel{\text{def}}{=} \text{tt} \;\Big|\; p[N] \;\Big|\; \psi \wedge \psi \;\Big|\; \neg\psi$$

eCSL State Formulae

State formulae:

$$
\begin{array}{ll}
m \models \mathrm{tt} & \text{for all } m \\
m \models \psi_1 \wedge \psi_2 & \text{iff } m \models \psi_1 \wedge m \models \psi_2 \\
m \models \neg\psi & \text{iff } m \not\models \psi \\
m \models p[N] & \text{iff } m(p) \in N
\end{array}
$$

- m is a marking of the underlying SPN, p is a place label from the SPN, N is a subset of natural numbers

- a marking satisfies $p[N]$ iff the number of tokens in place p lies in the set N

eCSL Semantics I

Performance formulae:

$$
\begin{array}{ll}
\vec{m} \models \mathsf{tt} & \text{for all } \vec{m} \\
\vec{m} \models \Psi_1 \wedge \Psi_2 & \text{iff } \vec{m} \models \Psi_1 \wedge \vec{m} \models \Psi_2 \\
\vec{m} \models \neg\Psi & \text{iff } \vec{m} \not\models \Psi \\
 & \\
\vec{m} \models \mathcal{S}_{\boldsymbol{\rho}}(\psi) & \text{iff } \Pi_{\vec{j}} \in \boldsymbol{\rho} \text{ where } \vec{j} = \mathsf{Sat}(\psi)
\end{array}
$$

- $\boldsymbol{\rho}$ is a subset of $[0, 1]$, $\mathsf{Sat}(\psi) = \{m \in M \mid m \models \psi\}$.

- $\vec{m} \models \mathcal{S}_{\boldsymbol{\rho}}(\psi)$ says $\vec{m}$ satisfies $\mathcal{S}_{\boldsymbol{\rho}}(\psi)$ iff the accumulated steady state of all states states satisfying ψ lies in the set $\boldsymbol{\rho}$

eCSL Semantics II

- Passage-time performance formula:

$$\vec{m} \models \mathcal{P}_{\boldsymbol{\rho}}^{\boldsymbol{\tau}}(\psi_1, \psi_2) \text{ iff } \mathbb{P}(P_{\vec{m}\vec{j}}^{\vec{k}} \in \boldsymbol{\tau}) \in \boldsymbol{\rho}$$

- where $\vec{j} = \mathsf{Sat}(\psi_1), \vec{k} = \mathsf{Sat}(\psi_2)$

$$P_{\vec{i}\vec{j}}^{\vec{k}} = \sum_{i \in \vec{i}} \alpha_i \inf \left\{ u > 0 : Z(u) \in \vec{j} \mid Z(0) = i, \forall u' < u \,.\, Z(u') \notin \vec{k} \right\}$$

- i.e. does the random variable, $P_{\vec{i}\vec{j}}^{\vec{k}}$, lie in the time range $\boldsymbol{\tau}$ with probability in $\boldsymbol{\rho}$

eCSL Semantics III

- Transient performance formula:

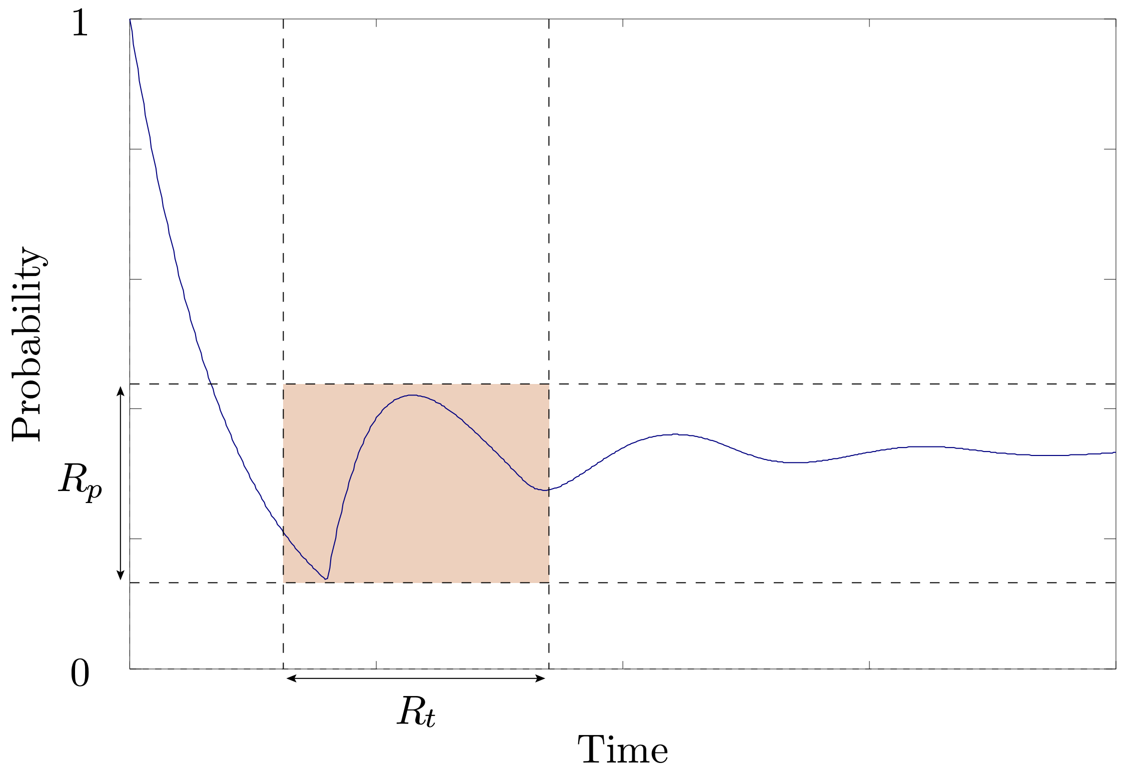
$$\vec{m} \models \mathcal{T}_{\boldsymbol{\rho}}^{\boldsymbol{\tau}}(\psi_1, \psi_2) \text{ iff } \forall t \in \boldsymbol{\tau}, T_{\vec{m}\vec{j}}^{\vec{k}}(t) \in \boldsymbol{\rho}$$

- where $\vec{j} = \mathsf{Sat}(\psi_1), \vec{k} = \mathsf{Sat}(\psi_2)$

$$T_{\vec{i}\vec{j}}^{\vec{k}}(t) = \sum_{i \in \vec{i}} \alpha_i \mathbb{P}\left(Z(t) \in \vec{j} \mid Z(0) = i, \forall t' < t \;.\; Z(t') \notin \vec{k}\right)$$

- i.e. does the transient dirstribution starting in $\vec{i}$, finishing in $\vec{j}$ and avoiding $\vec{k}$, stay within $\boldsymbol{\rho}$ for all $t \in \boldsymbol{\tau}$

Transient bound



eCSL Example

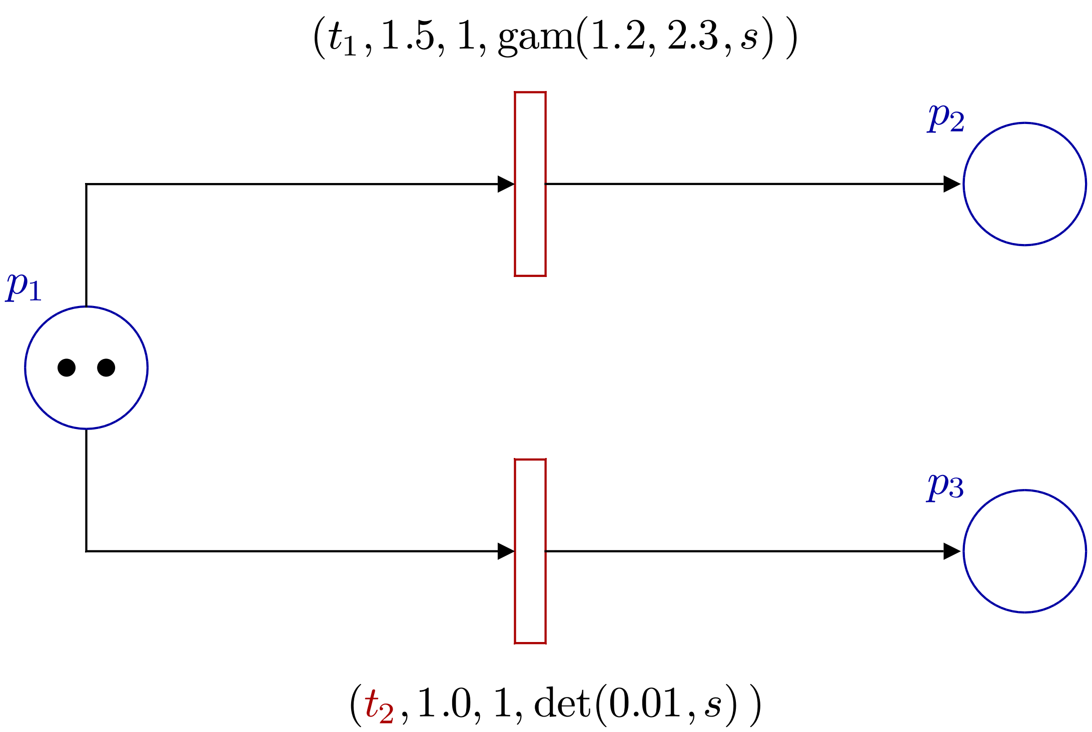
- Find value q such that:

$$\mathrm{Sat}(p_1[35] \wedge p_5[10]) \models \mathcal{P}_{\{q\}}^{[0,10)}(p_2[175], p_6[1])$$

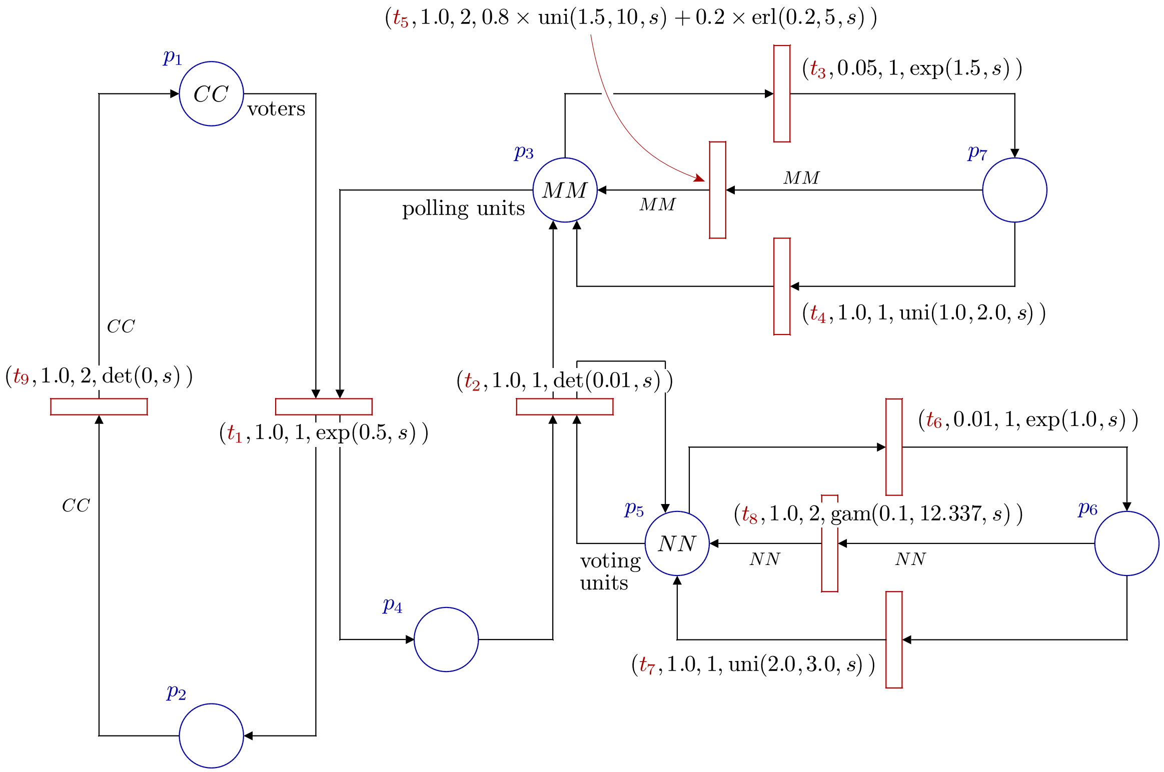
- i.e. find probability that defined passage occurs within 10 seconds?

Semi-Markov SPNs

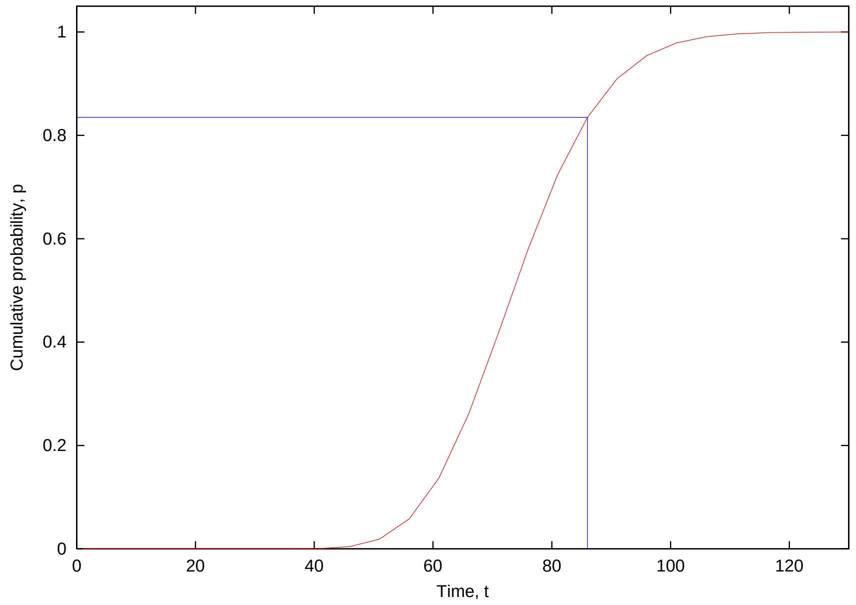
Use semi-Markov stochastic Petri nets to generate our SMP models



SM-SPN: Voting System

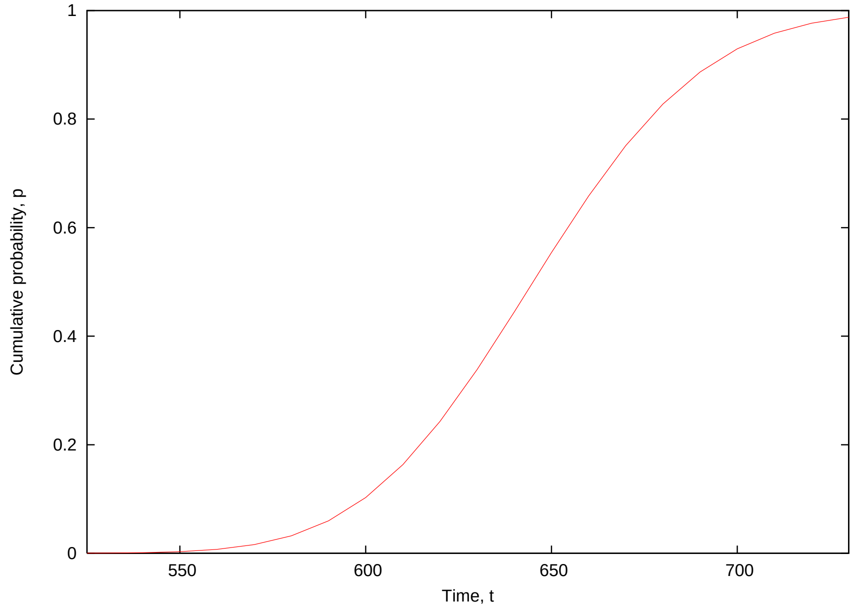


Passage-time (I): 1.14M



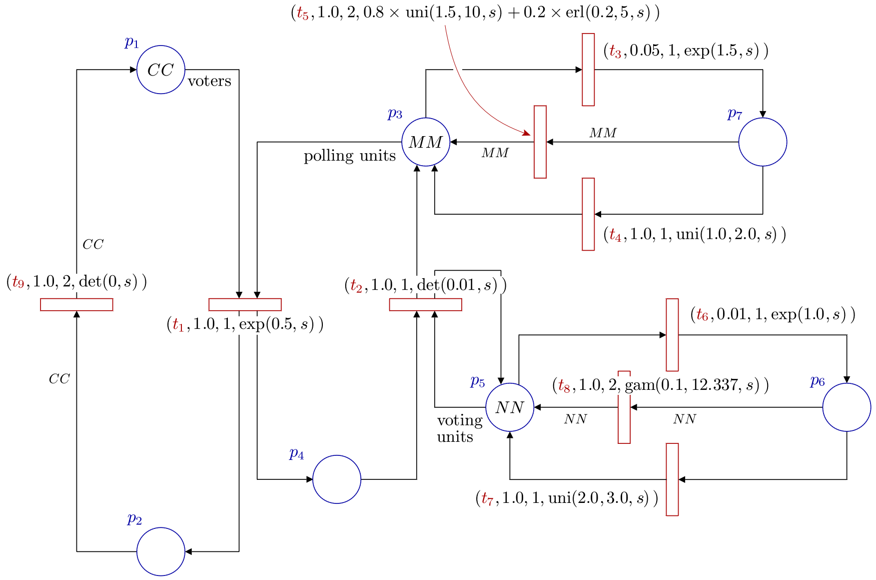
$$\mathrm{Sat}(p_1[35] \wedge p_5[10]) \models \mathcal{P}^{(0,86]}_{(0.8,1)}(p_2[175], p_6[1])$$

Passage Time (II): 10.9M



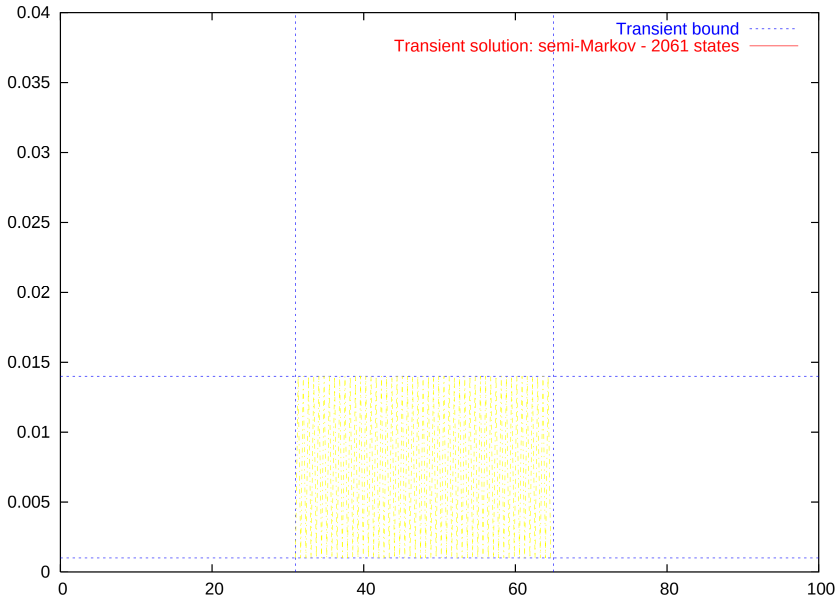
$$\mathsf{Sat}(p_1[300] \wedge p_3[80] \wedge p_5[10]) \models \mathcal{P}^{(0,t]}_{\{p\}}(p_2[300],\ \mathsf{ff}\ )$$

eCSL Example



$$\mathrm{Sat}(p_1[5]) \models \mathcal{T}^{(31,65)}_{(0.001,0.014)}(p_2[5],\ \mathrm{ff}\ )$$

Transient Constraint



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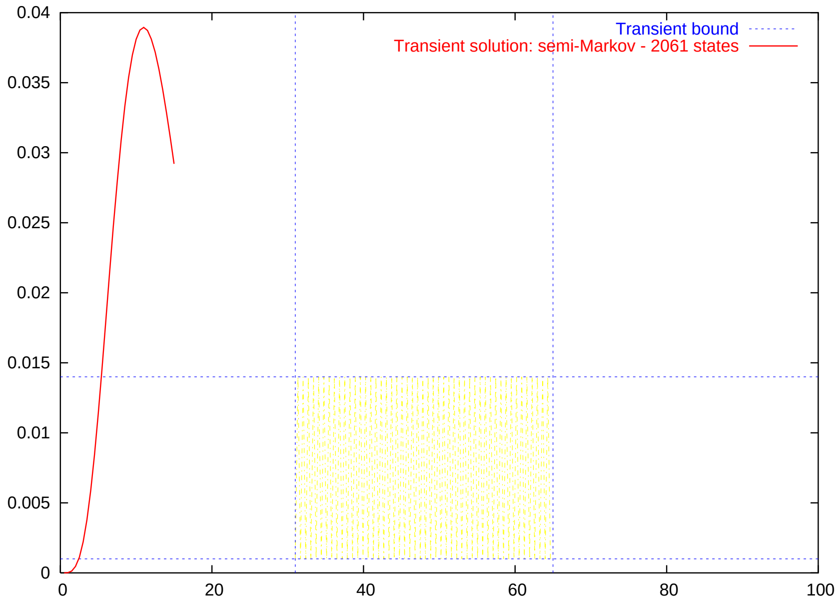

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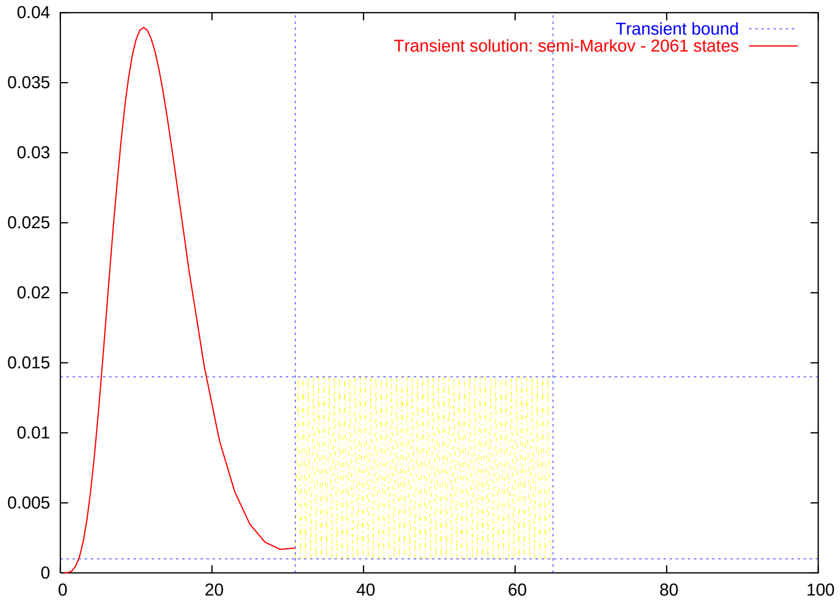

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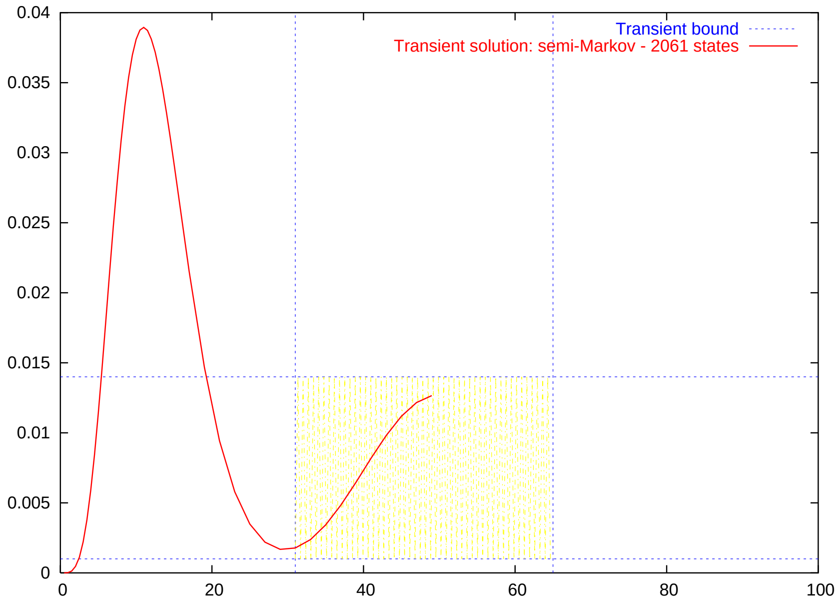

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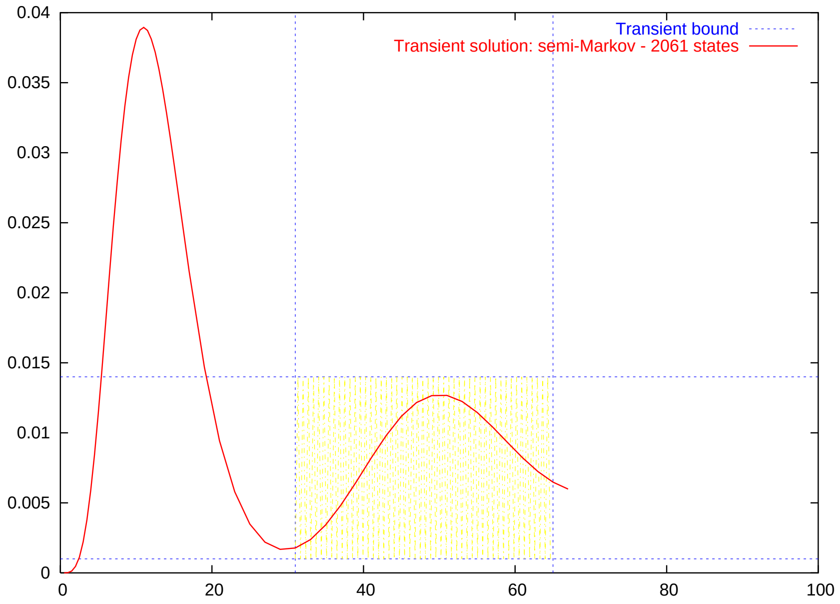

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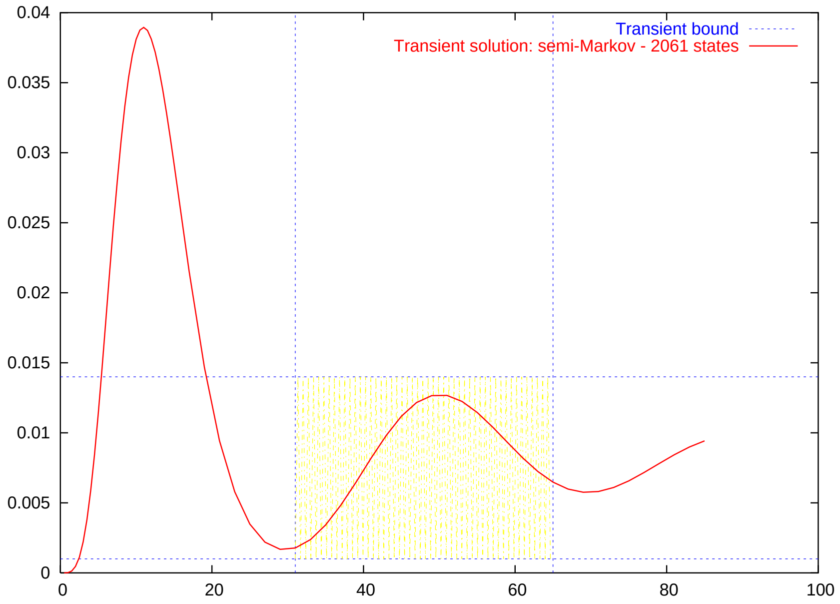

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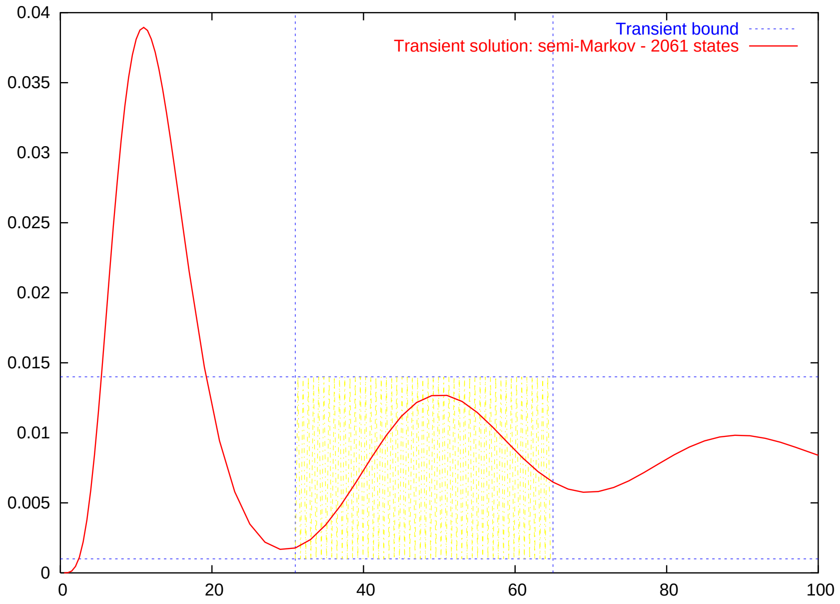

$$\mathrm{Sat}(p_1[5]) \models \mathcal{T}^{(31,65)}_{(0.001,0.014)}(p_2[5],\ \mathrm{ff}\ )$$

Conclusion

- Introduced extended continuous stochastic logic over SM-SPNs
- Added multiple start states to queries
- Added new bounded transient query

CSL Semantics

State formulae:

$$
\begin{array}{ll}
s \models \mathrm{tt} & \text{for all } s \\
s \models a & \text{iff } a \in \text{set of labels, } A(s) \\
s \models \Psi_1 \wedge \Psi_2 & \text{iff } s \models \Psi_1 \wedge s \models \Psi_2 \\
s \models \neg\Psi & \text{iff } s \not\models \Psi \\
s \models \mathcal{S}_{\boldsymbol{\rho}}(\Psi) & \text{iff } \Pi_{\vec{j}} \in \boldsymbol{\rho} \text{ where } \vec{j} = \mathrm{Sat}(\Psi) \\
s \models \mathcal{P}_{\boldsymbol{\rho}}(\psi) & \text{iff } \mathbb{P}(\sigma \in \mathrm{Path}(s) \mid \sigma \models \psi) \in \boldsymbol{\rho}
\end{array}
$$

Path formulae:

$$
\begin{array}{ll}
\sigma \models X\Psi & \text{iff } \exists \sigma[1] \models \Psi \\
\sigma \models \Psi_1 \cup^{\boldsymbol{\tau}} \Psi_2 & \text{iff } \exists u \in \boldsymbol{\tau} \;.\; (\sigma@u \models \Psi_2 \wedge \forall u' < u, \sigma@u' \models \Psi_1)
\end{array}
$$