Stochastic Probes

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Stochastic Probes

Stochastic Probes are:

- fragments of process algebra
- used to measure observed timed and probabilistic behaviour in stochastic process algebra models

Stochastic Probes

Stochastic Probes are:

- specified using action regular expressions
- currently implemented in PEPA, but are formalism independent
- not able to enforce behaviour – must be *behaviour neutral*

Stochastic Analysis

- Steady-state analysis
  - long-run average probability of ending up in set of states
- Transient-state analysis
  - Probability of being in a given set of states at a time, t
- Passage-time analysis
  - How long does it take to arrive at a set of states

PEPA: Process Algebra

- Syntax:

$$P \ ::= \ (a, \lambda).P \ \Big| \ P + P \ \Big| \ P \underset{S}{\bowtie} P \ \Big| \ P/L \ \Big| \ A$$

- $(a, \lambda).P$: prefix operation $P + P$: competitive choice

- $P \underset{S}{\bowtie} P$: component cooperation P/L: action hiding

PEPA: A Transmitter-Receiver

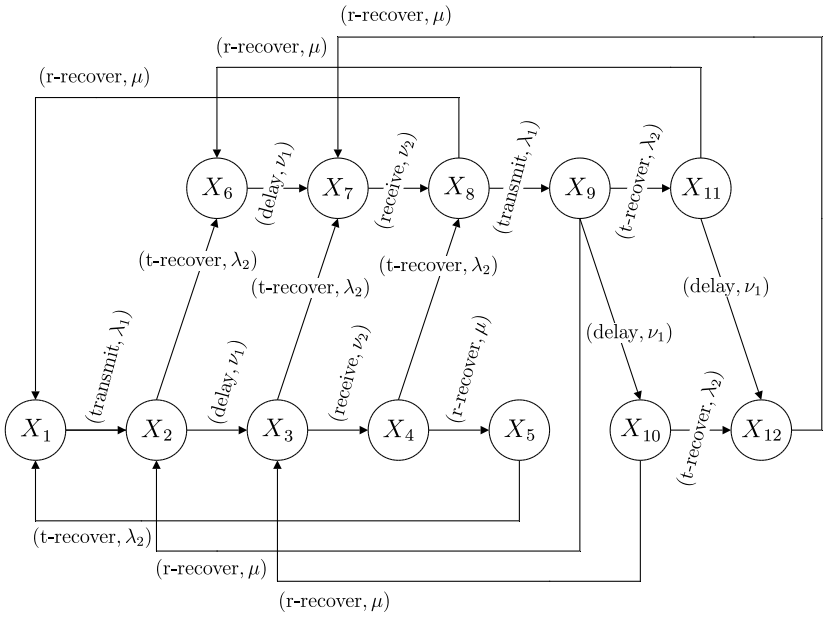
$$\text{Transmitter} \stackrel{\text{def}}{=} (\text{transmit}, \lambda_1).(\text{t_recover}, \lambda_2).\text{Transmitter}$$

$$\text{Receiver} \stackrel{\text{def}}{=} (\text{receive}, \top).(\text{r_recover}, \mu).\text{Receiver}$$

$$\text{Network} \stackrel{\text{def}}{=} (\text{transmit}, \top).(\text{delay}, \nu_1).(\text{receive}, \nu_2).\text{Network}$$

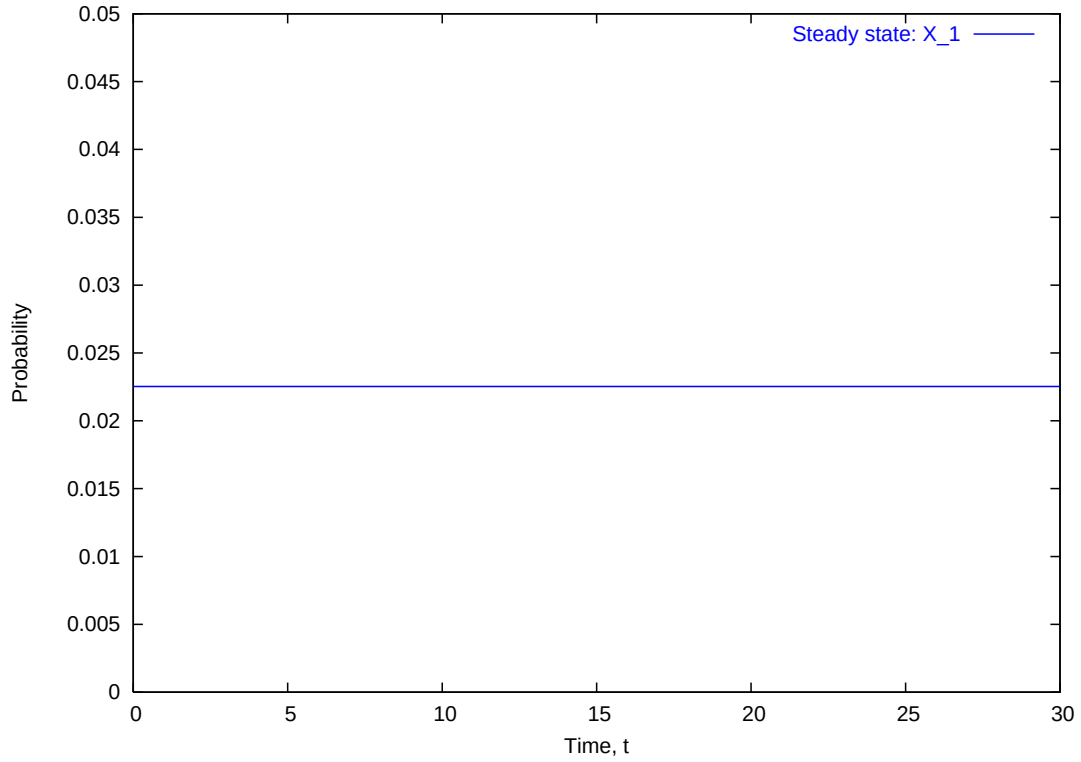
$$\text{System} \stackrel{\text{def}}{=} (\text{Transmitter} \underset{\emptyset}{\bowtie} \text{Receiver}) \underset{\{\text{transmit,receive}\}}{\bowtie} \text{Network}$$

Global state space

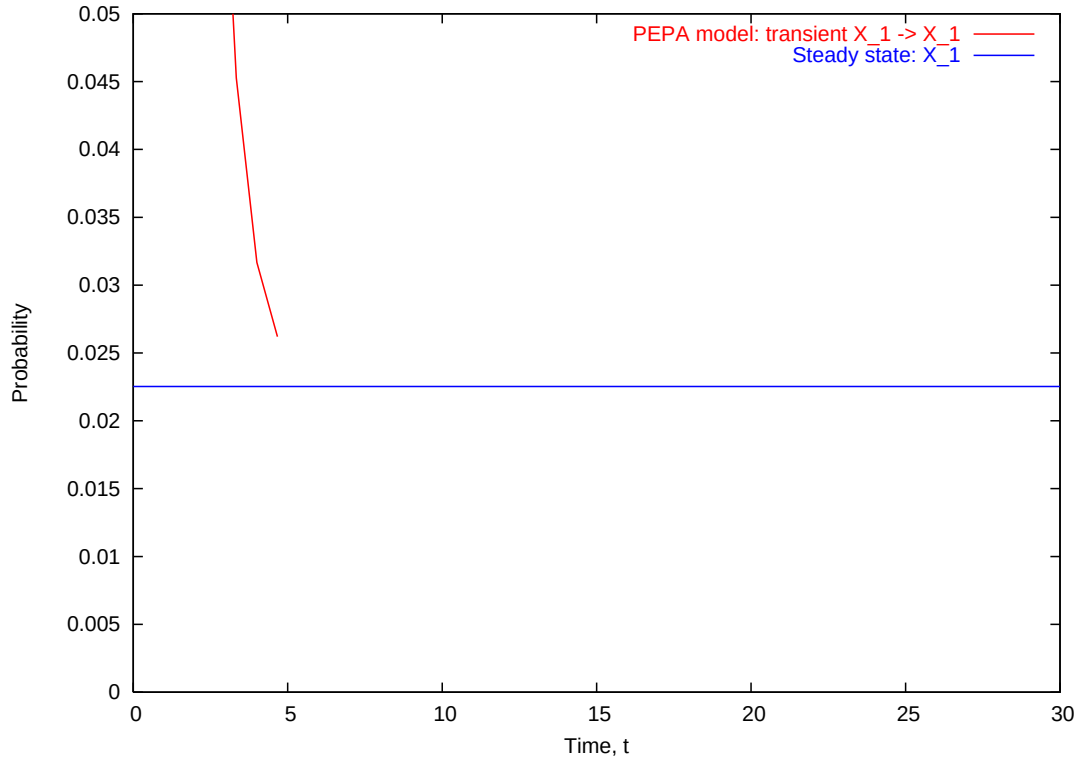


Look at probability of being in state X_1 at time, t

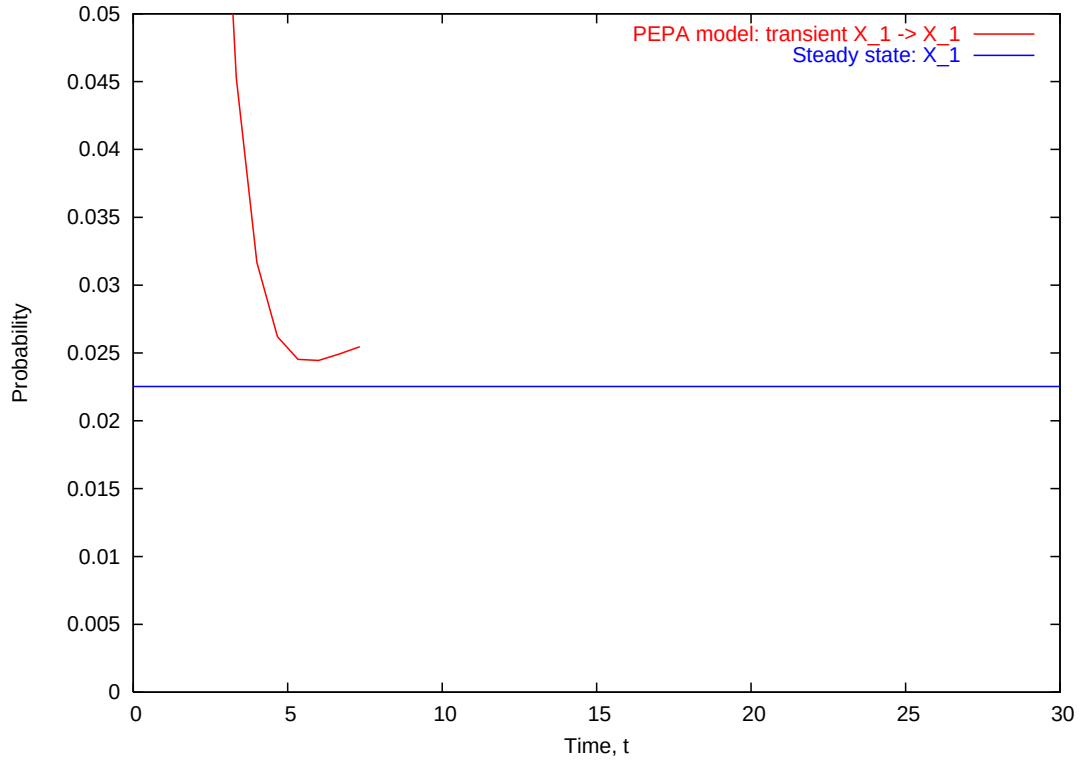
Transient distribution



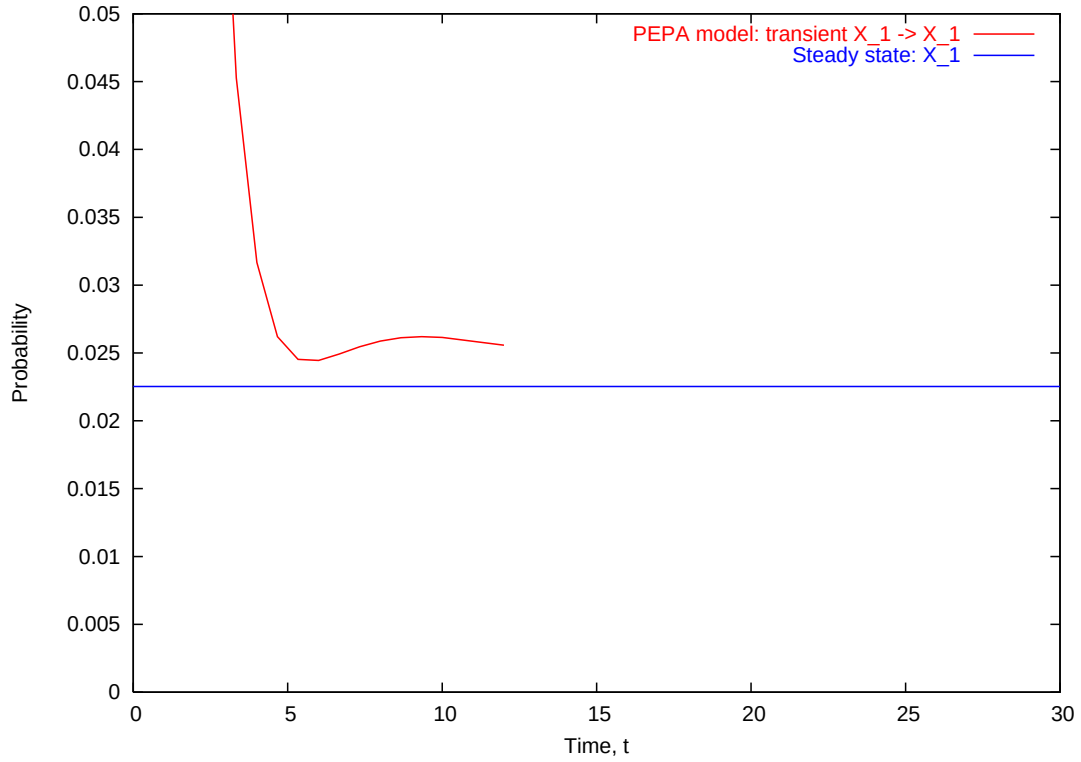
Transient distribution



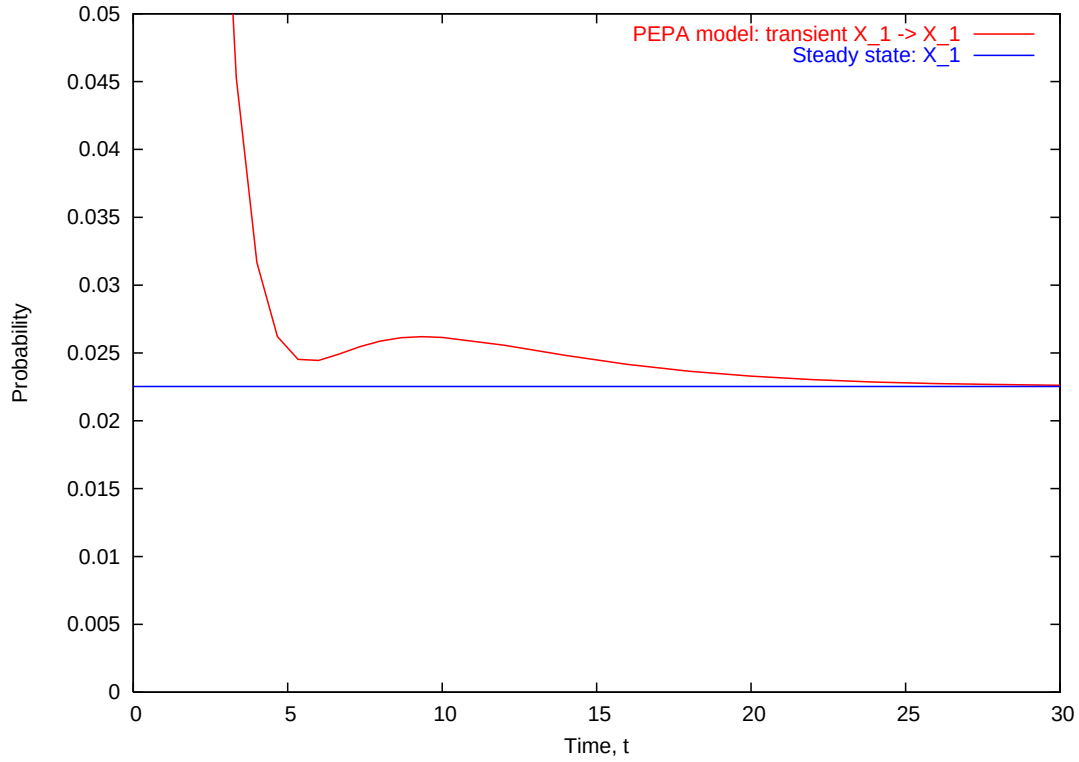
Transient distribution



Transient distribution



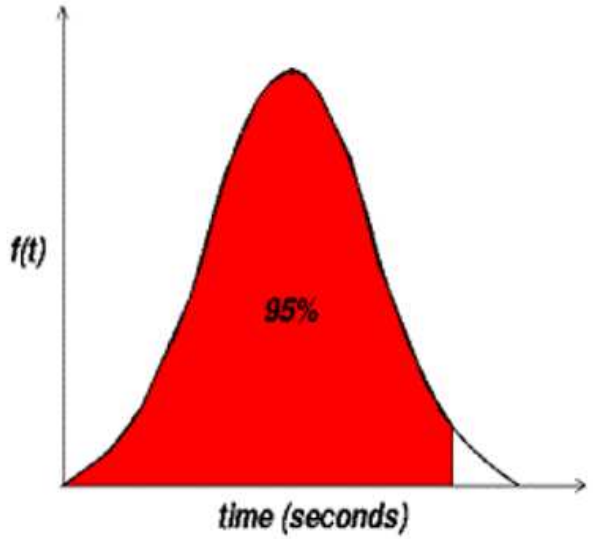
Transient distribution



Passage-time analysis

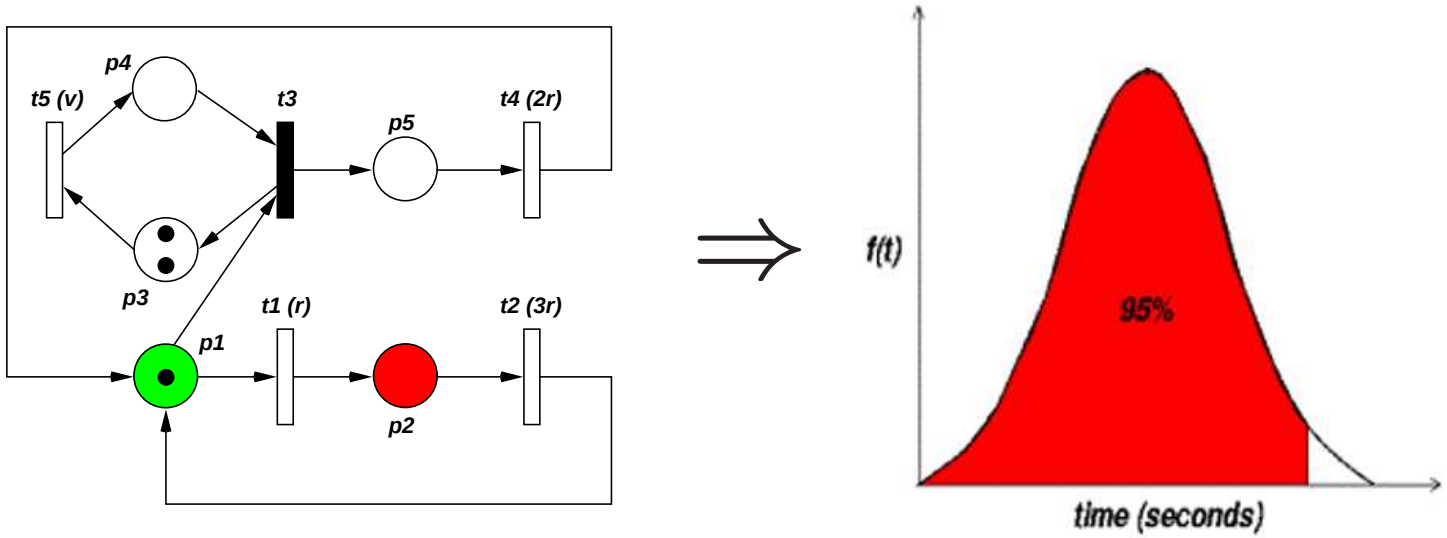
For example, allow us to extract passage-time density from a PEPA model:

$$
\begin{array}{lcl}
\mathrm{A1} & \stackrel{\mathrm{def}}{=} & (\mathrm{start}, r_1).\mathrm{A2} + (\mathrm{pause}, r_2).\mathrm{A3} \\
\mathrm{A2} & \stackrel{\mathrm{def}}{=} & (\mathrm{run}, r_3).\mathrm{A1} + (\mathrm{fail}, r_4).\mathrm{A3} \\
\mathrm{A3} & \stackrel{\mathrm{def}}{=} & (\mathrm{recover}, r_1).\mathrm{A1} \\
\mathrm{AA} & \stackrel{\mathrm{def}}{=} & (\mathrm{run}, \top).(\mathrm{alert}, r_5).\mathrm{AA} \\
\mathrm{Sys} & \stackrel{\mathrm{def}}{=} & \mathrm{AA} \underset{\{run\}}{\bowtie} \mathrm{A1}
\end{array}
\quad \Rightarrow \quad
$$



What is DNAmaca/HYDRA?

Extracting a passage-time density from a GSPN:



PEPA: Example

$$\begin{aligned}
\mathsf{A1} &\stackrel{\text{def}}{=} (\mathsf{start}, r_1).\mathsf{A2} + (\mathsf{pause}, r_2).\mathsf{A3} \\
\mathsf{A2} &\stackrel{\text{def}}{=} (\mathsf{run}, r_3).\mathsf{A1} + (\mathsf{fail}, r_4).\mathsf{A3} \\
\mathsf{A3} &\stackrel{\text{def}}{=} (\mathsf{recover}, r_1).\mathsf{A1} \\
\mathsf{AA} &\stackrel{\text{def}}{=} (\mathsf{run}, \top).(\mathsf{alert}, r_5).\mathsf{AA}
\end{aligned}$$

PEPA: Example

$$
\begin{aligned}
\mathsf{A1} &\stackrel{\text{def}}{=} (\mathsf{start}, r_1).\mathsf{A2} + (\mathsf{pause}, r_2).\mathsf{A3} \\
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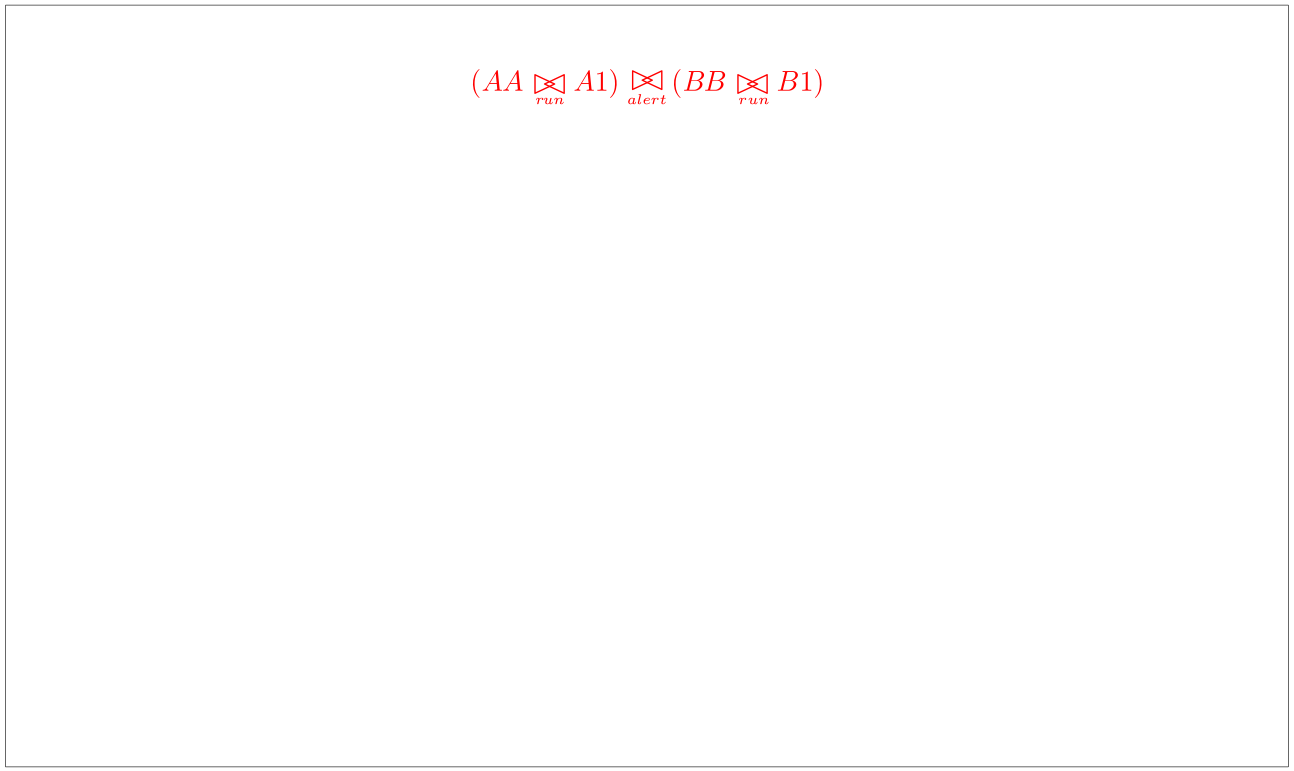
\mathsf{A3} &\stackrel{\text{def}}{=} (\mathsf{recover}, r_1).\mathsf{A1} \\
\mathsf{AA} &\stackrel{\text{def}}{=} (\mathsf{run}, \top).(\mathsf{alert}, r_5).\mathsf{AA}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{B1} &\stackrel{\text{def}}{=} (\mathsf{start}, r_1).\mathsf{B2} + (\mathsf{pause}, r_2).\mathsf{B1} \\
\mathsf{B2} &\stackrel{\text{def}}{=} (\mathsf{run}, r_3).\mathsf{B1} \\
\mathsf{BB} &\stackrel{\text{def}}{=} (\mathsf{run}, \top).(\mathsf{alert}, r_5).\mathsf{BB}
\end{aligned}
$$

PEPA: Example

$$\begin{aligned}
\mathsf{A1} &\stackrel{\text{def}}{=} (\mathsf{start}, r_1).\mathsf{A2} + (\mathsf{pause}, r_2).\mathsf{A3} \\
\mathsf{A2} &\stackrel{\text{def}}{=} (\mathsf{run}, r_3).\mathsf{A1} + (\mathsf{fail}, r_4).\mathsf{A3} \\
\mathsf{A3} &\stackrel{\text{def}}{=} (\mathsf{recover}, r_1).\mathsf{A1} \\
\mathsf{AA} &\stackrel{\text{def}}{=} (\mathsf{run}, \top).(\mathsf{alert}, r_5).\mathsf{AA} \\
\\
\mathsf{B1} &\stackrel{\text{def}}{=} (\mathsf{start}, r_1).\mathsf{B2} + (\mathsf{pause}, r_2).\mathsf{B1} \\
\mathsf{B2} &\stackrel{\text{def}}{=} (\mathsf{run}, r_3).\mathsf{B1} \\
\mathsf{BB} &\stackrel{\text{def}}{=} (\mathsf{run}, \top).(\mathsf{alert}, r_5).\mathsf{BB} \\
\\
\mathsf{Sys} &\stackrel{\text{def}}{=} (\mathsf{AA} \underset{\mathsf{run}}{\bowtie} \mathsf{A1}) \underset{\mathsf{alert}}{\bowtie} (\mathsf{BB} \underset{\mathsf{run}}{\bowtie} \mathsf{B1})
\end{aligned}$$

Example ipc Compilation



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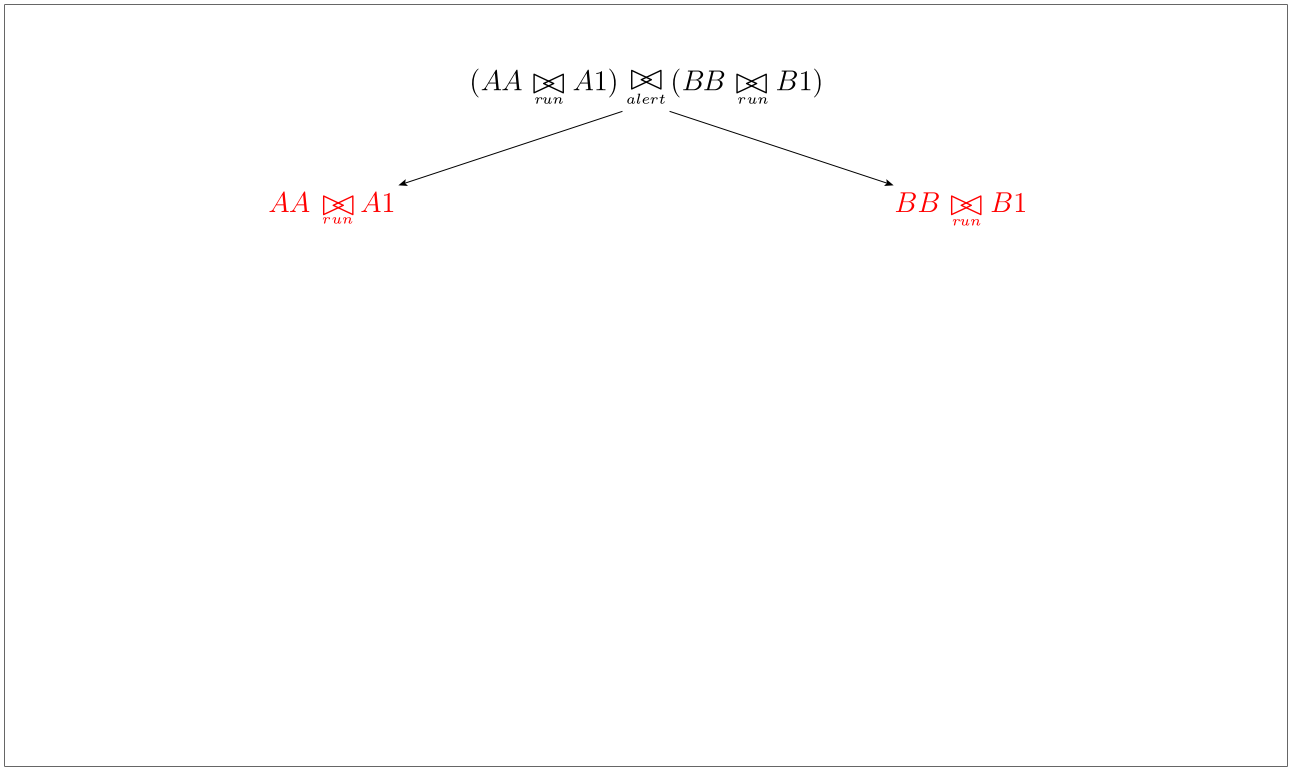

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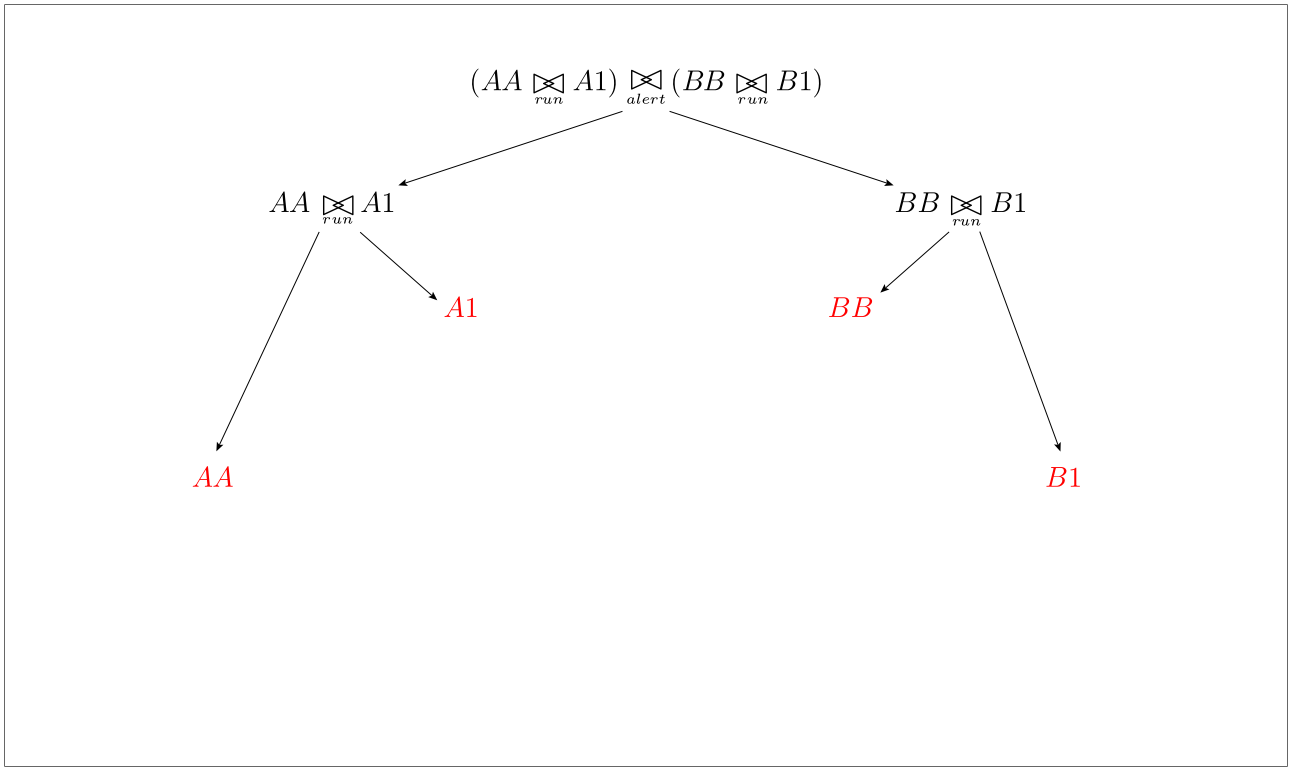

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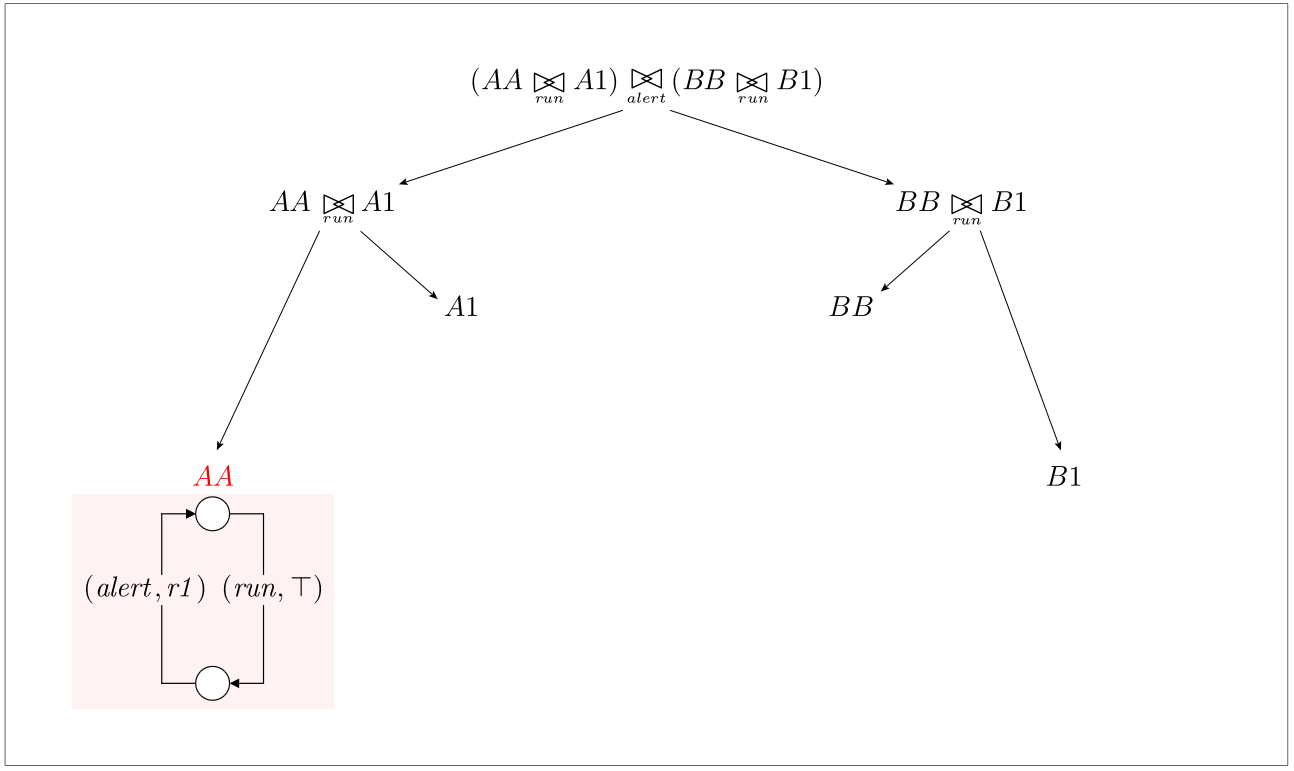

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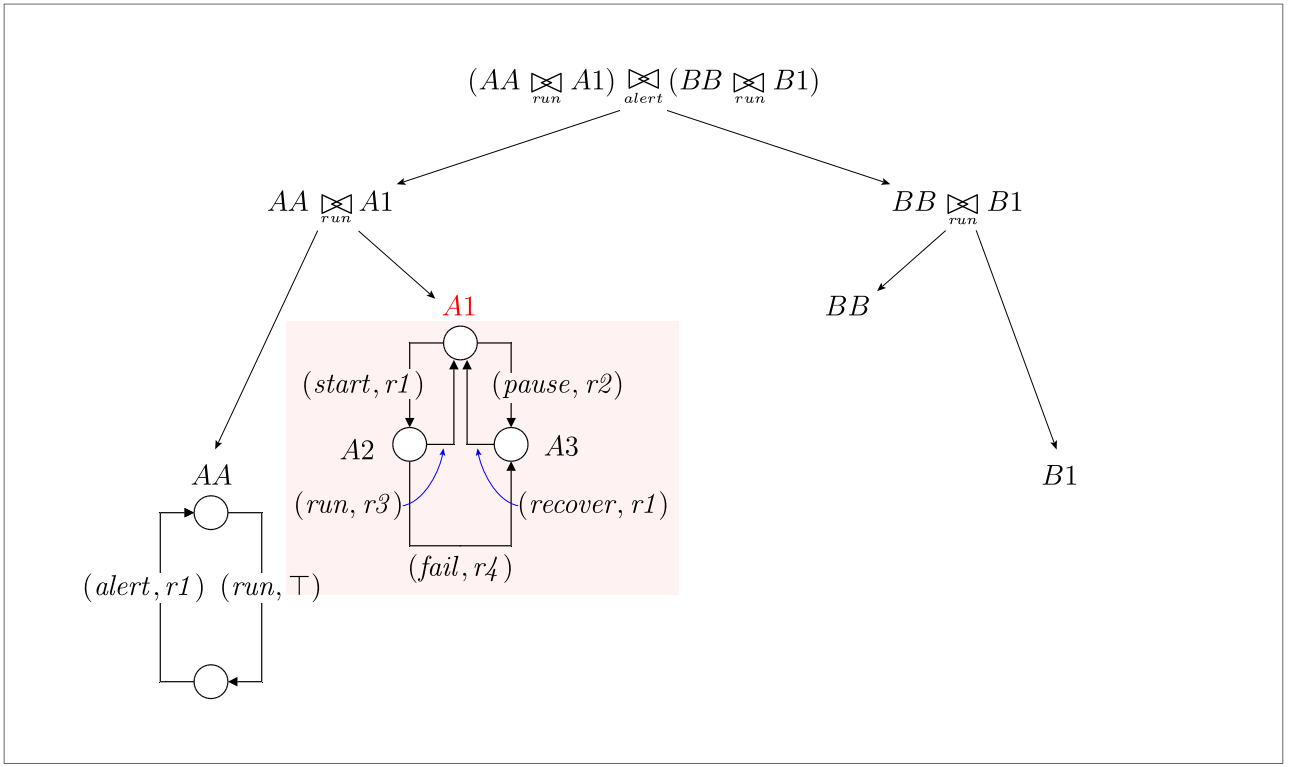

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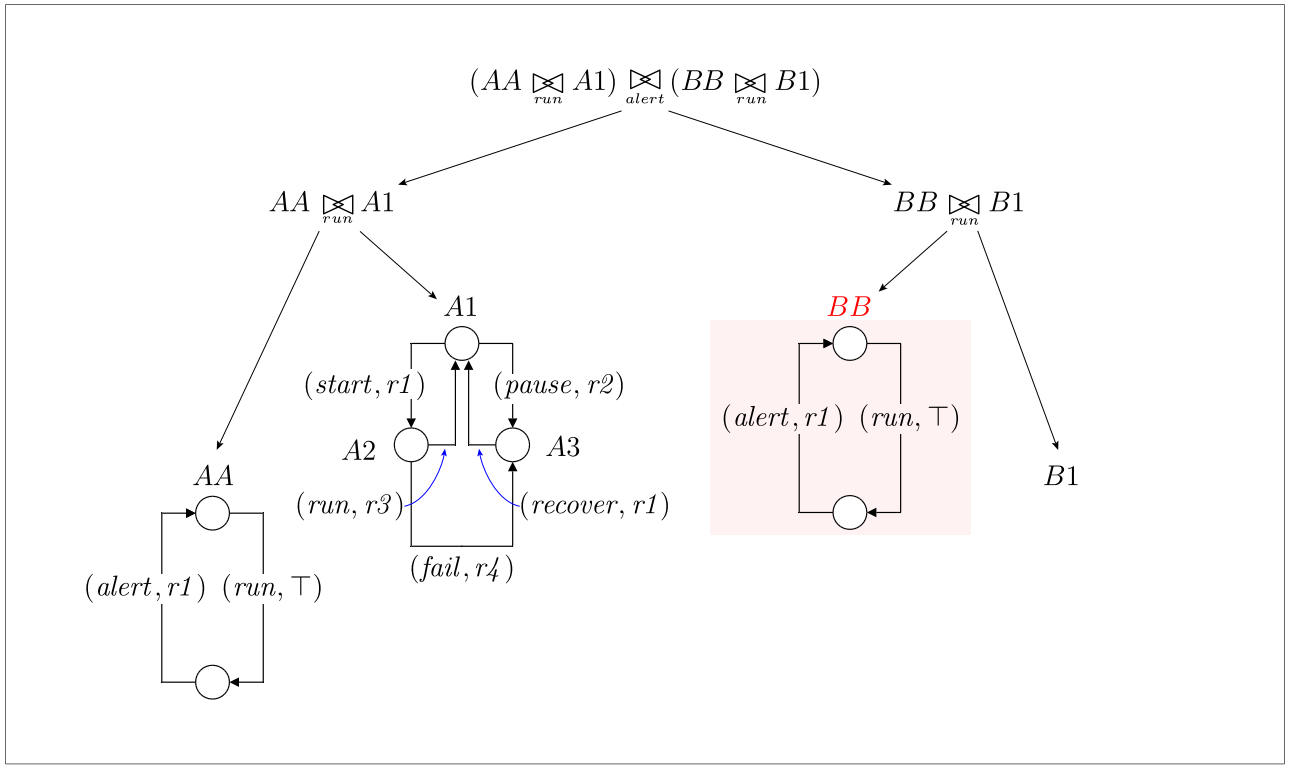

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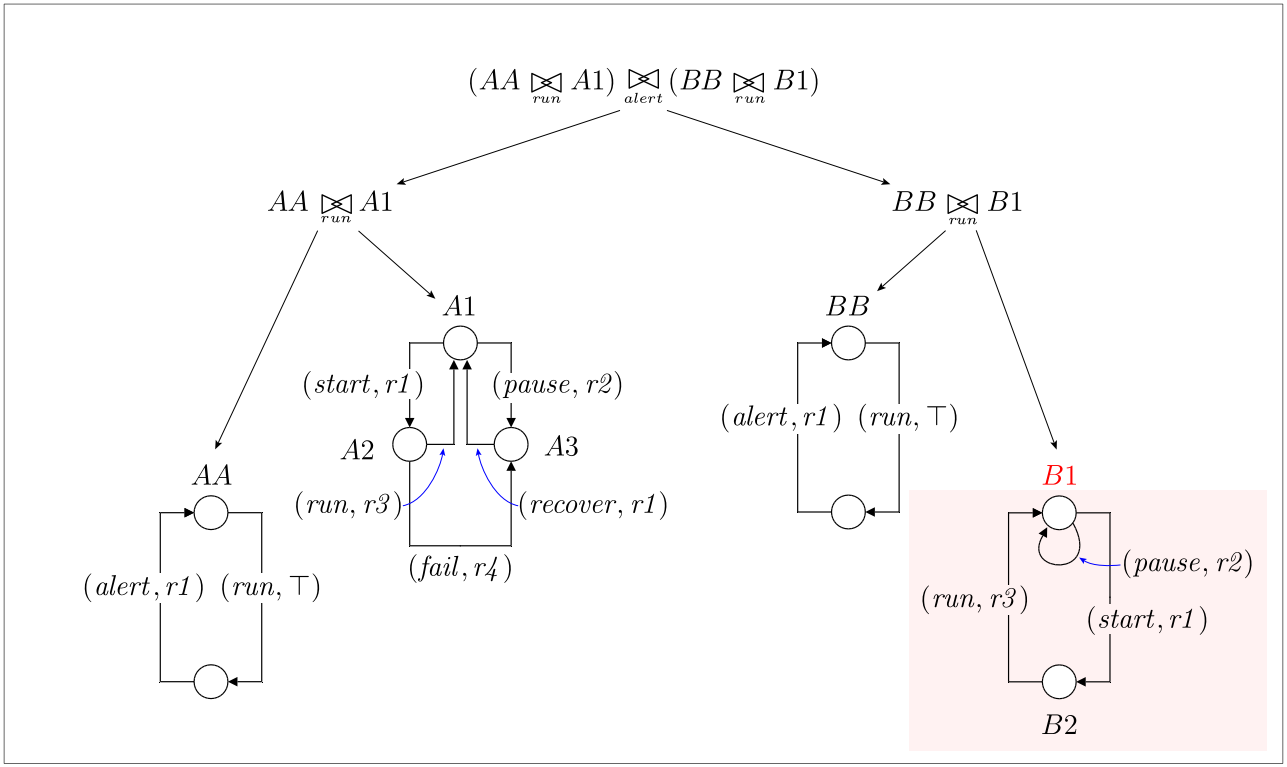

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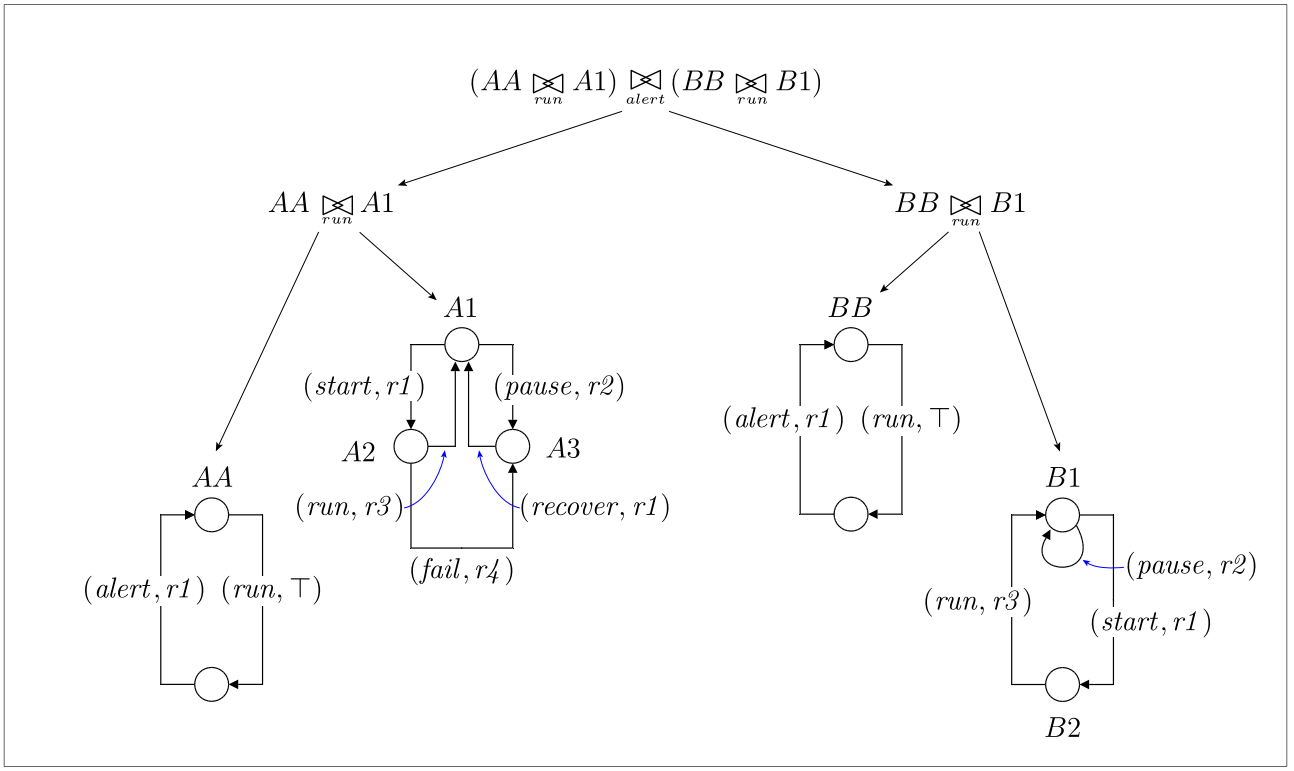

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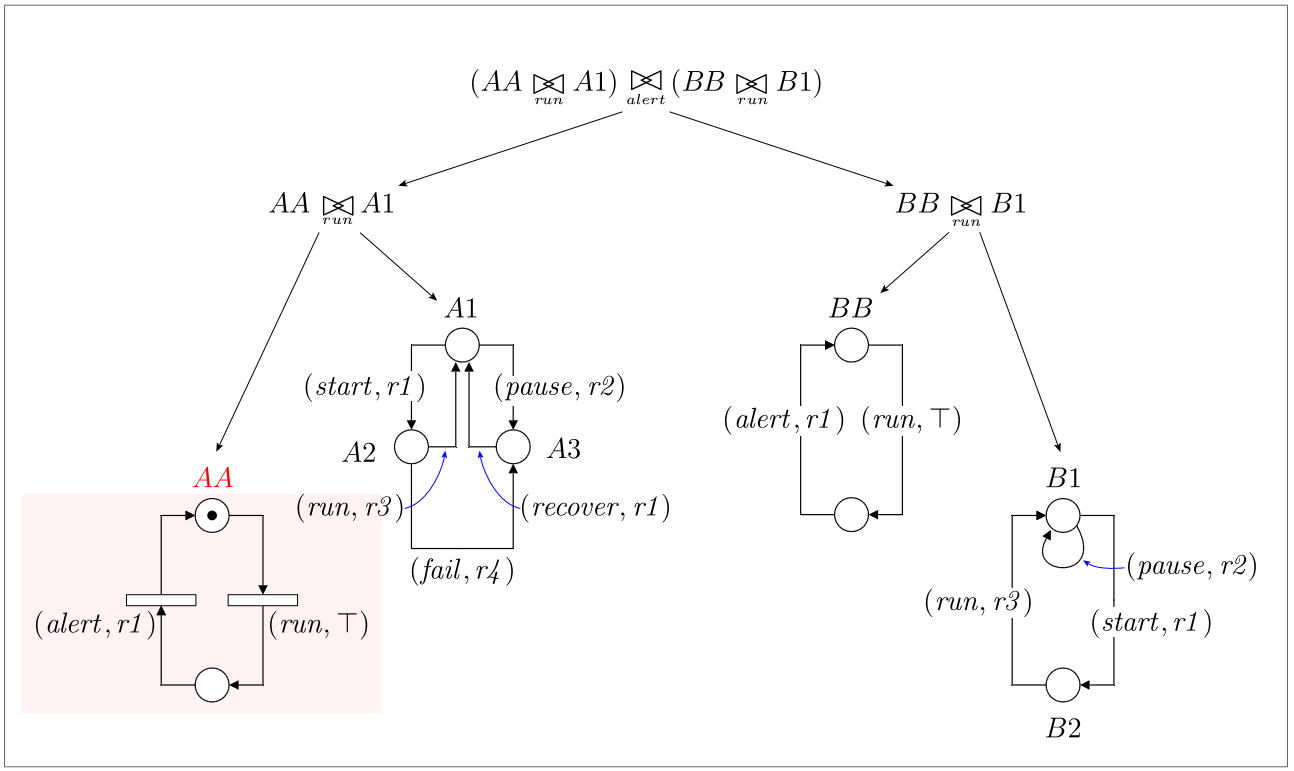

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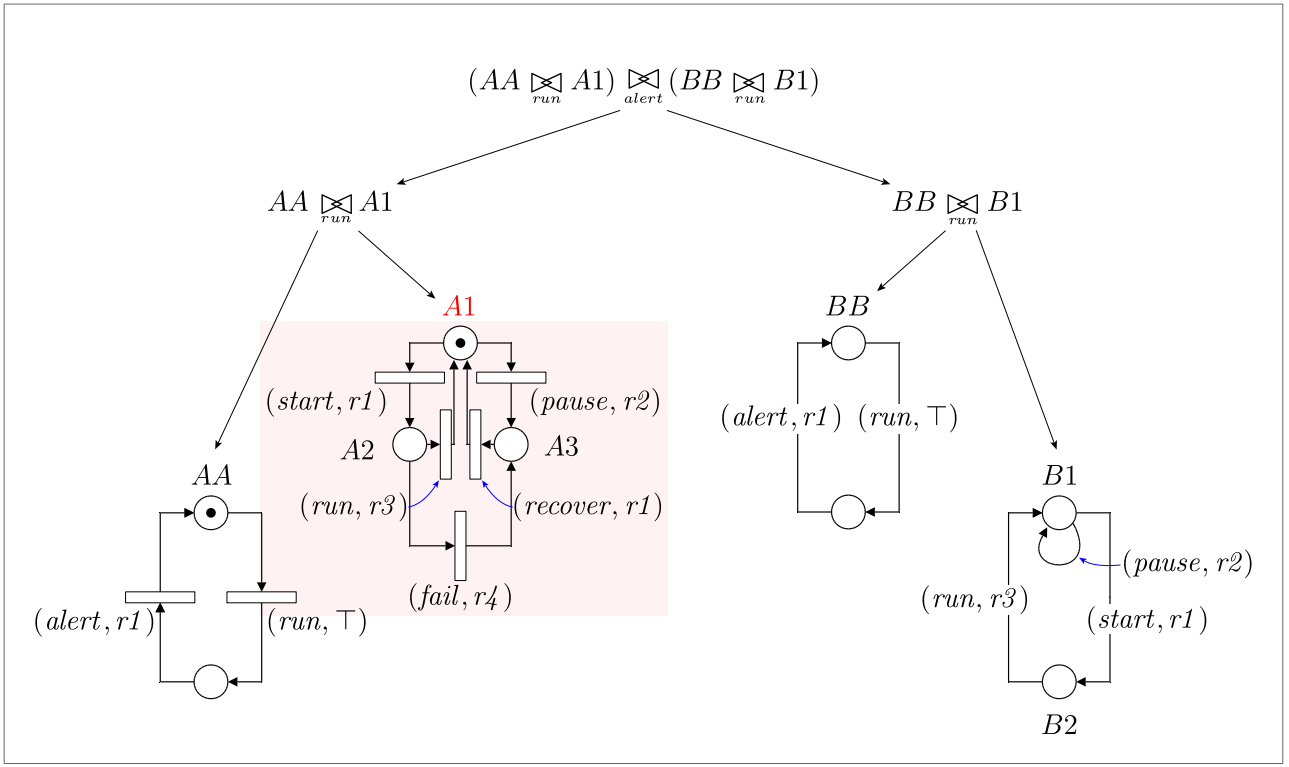

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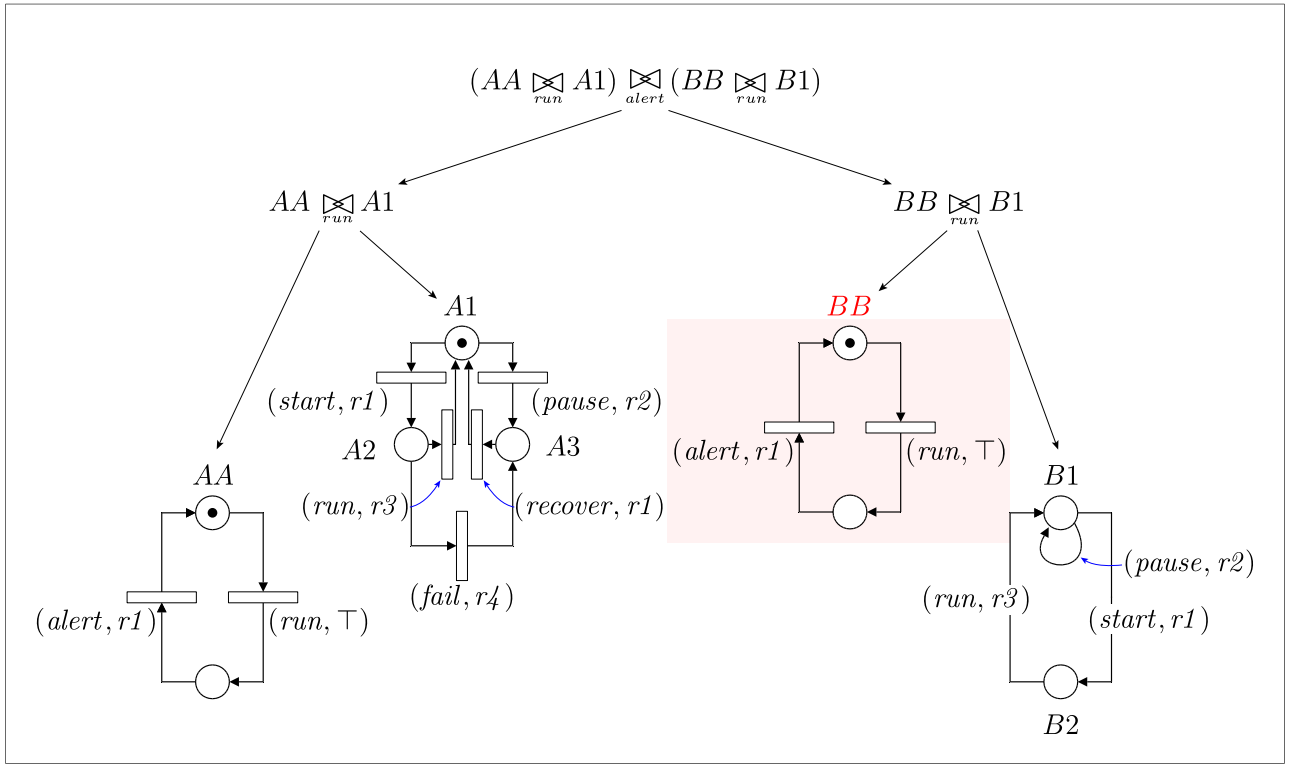

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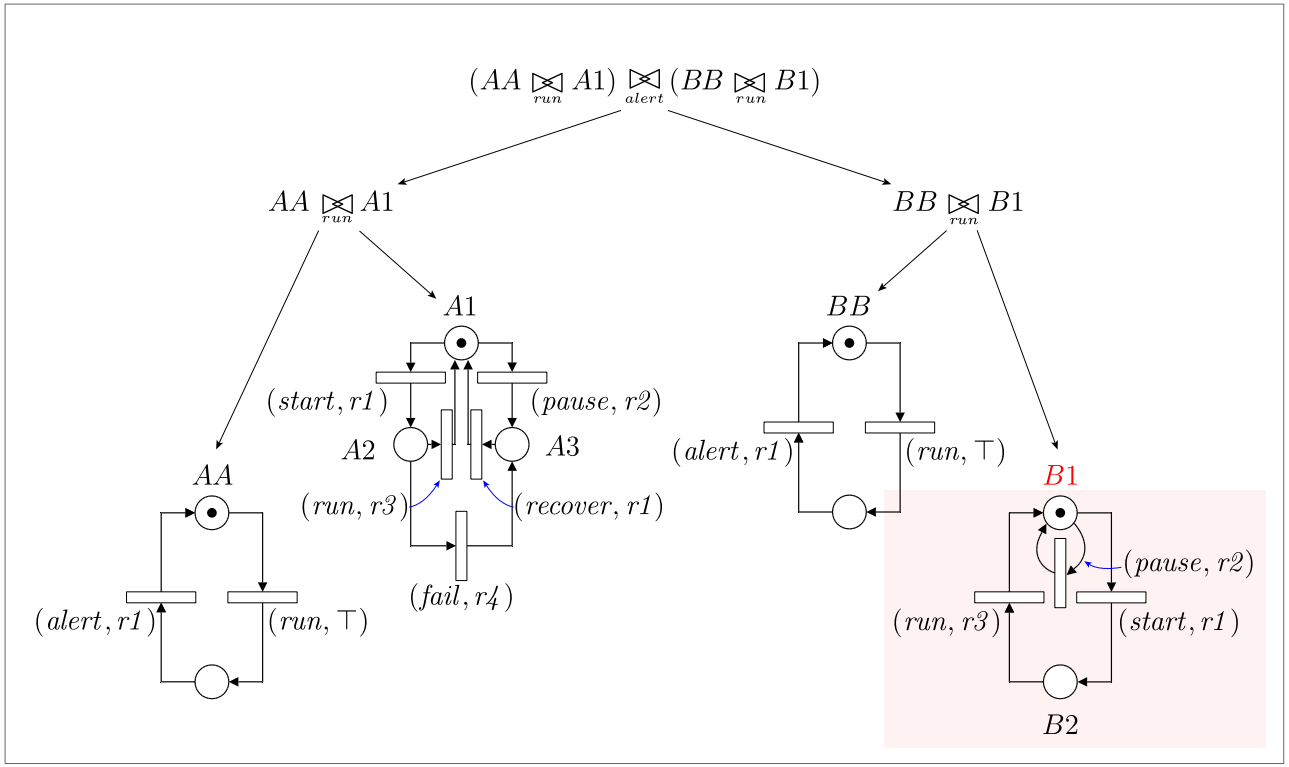

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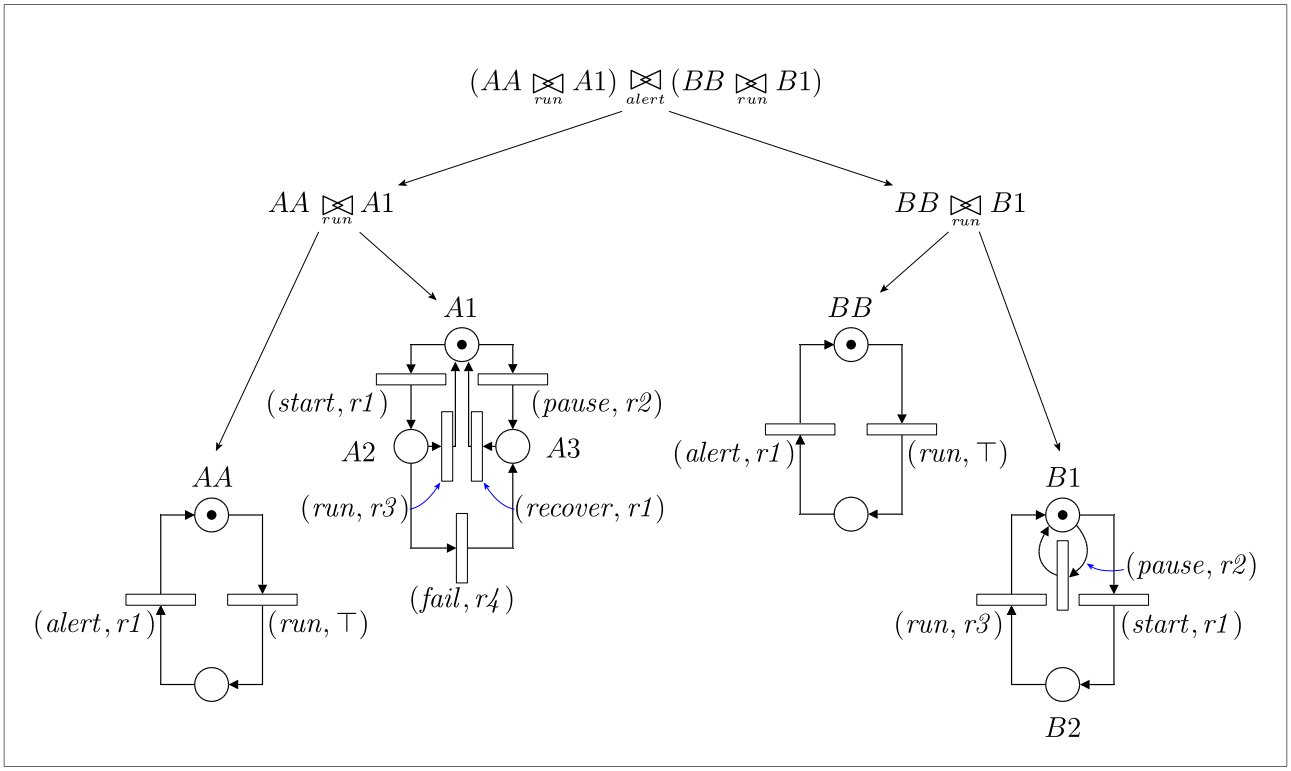

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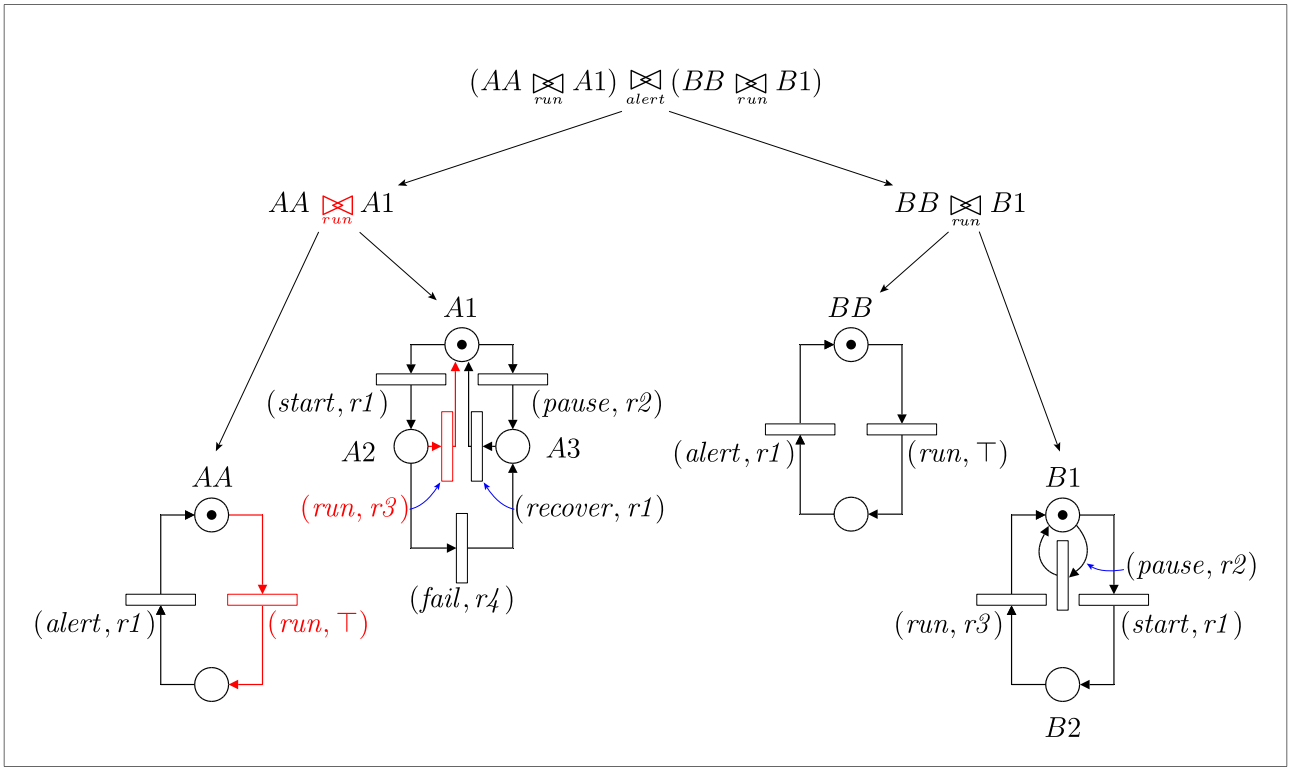

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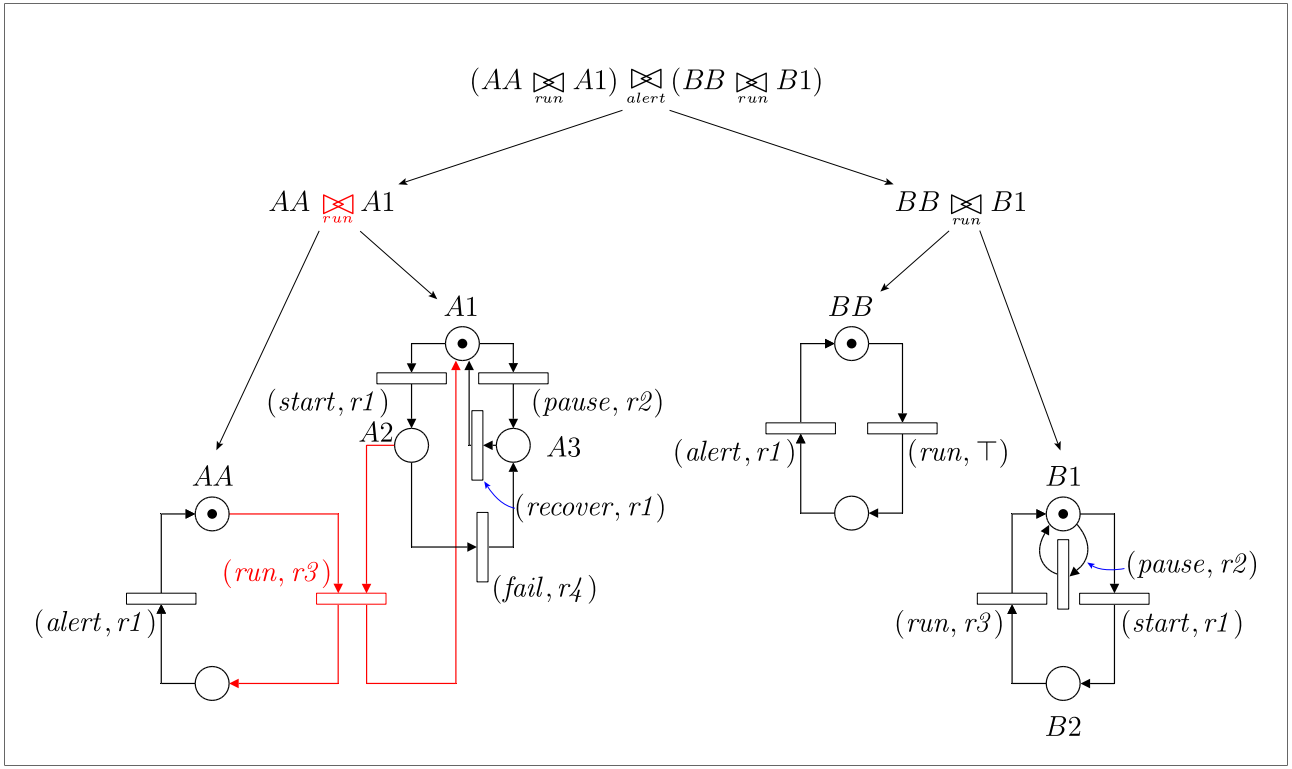

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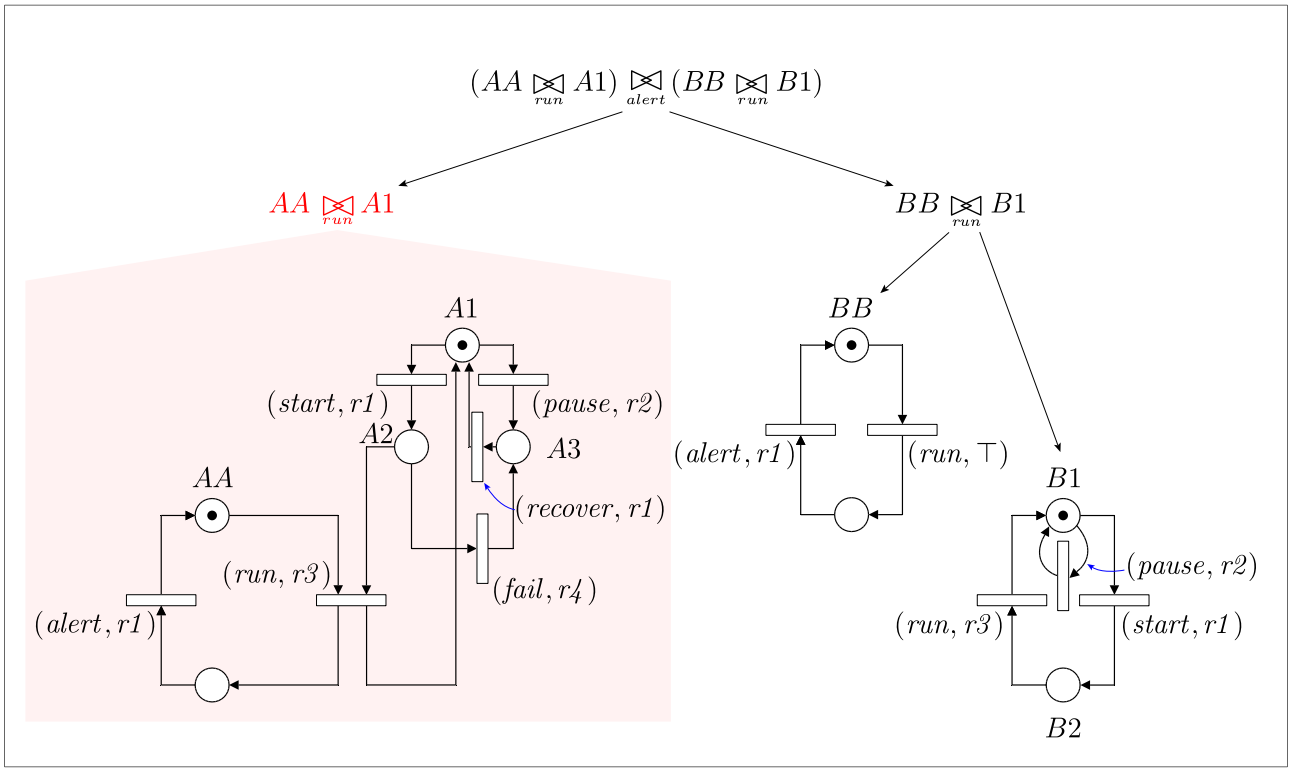

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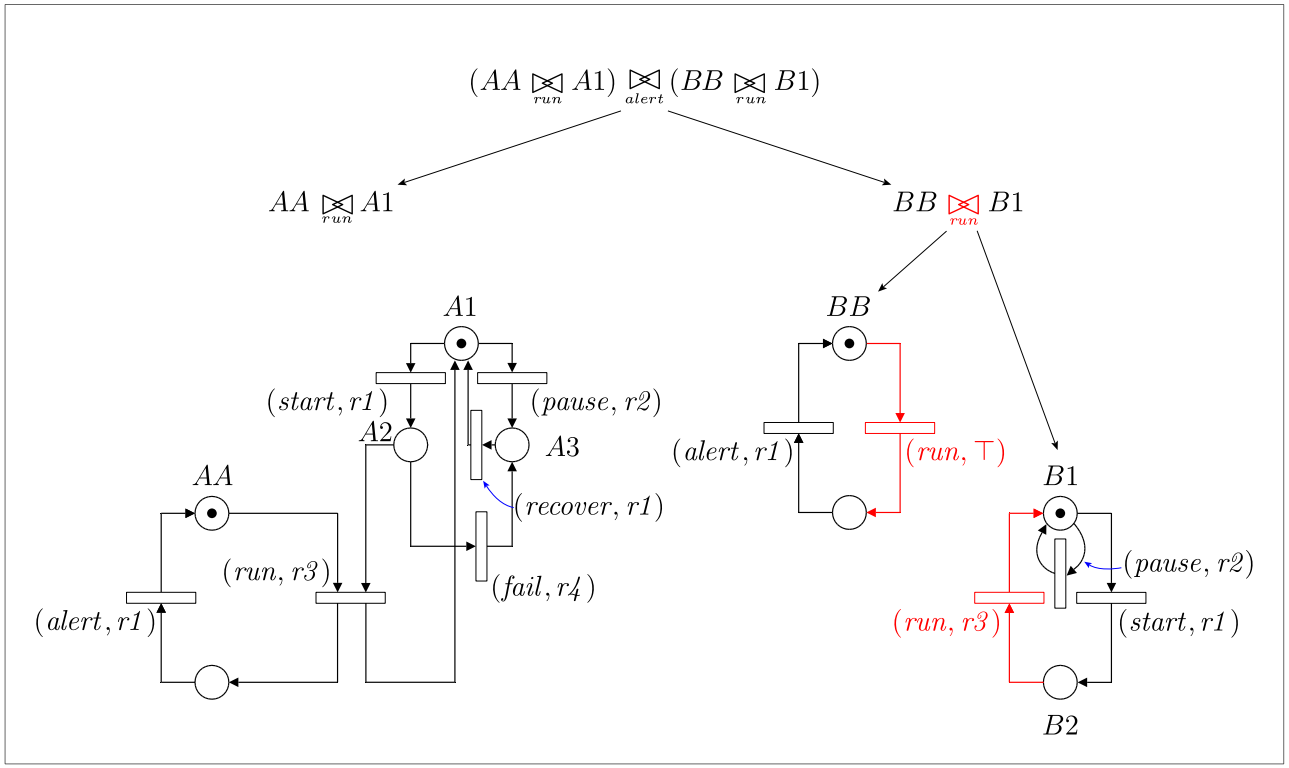

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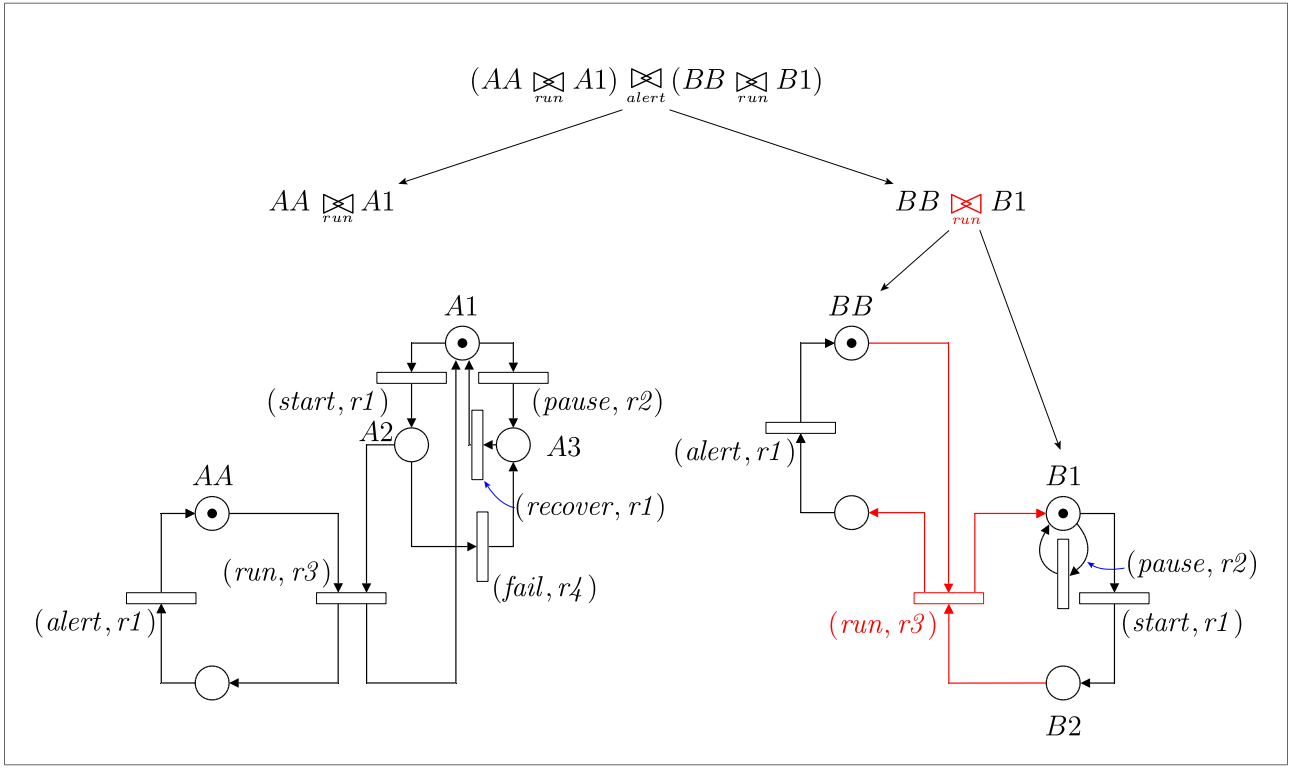

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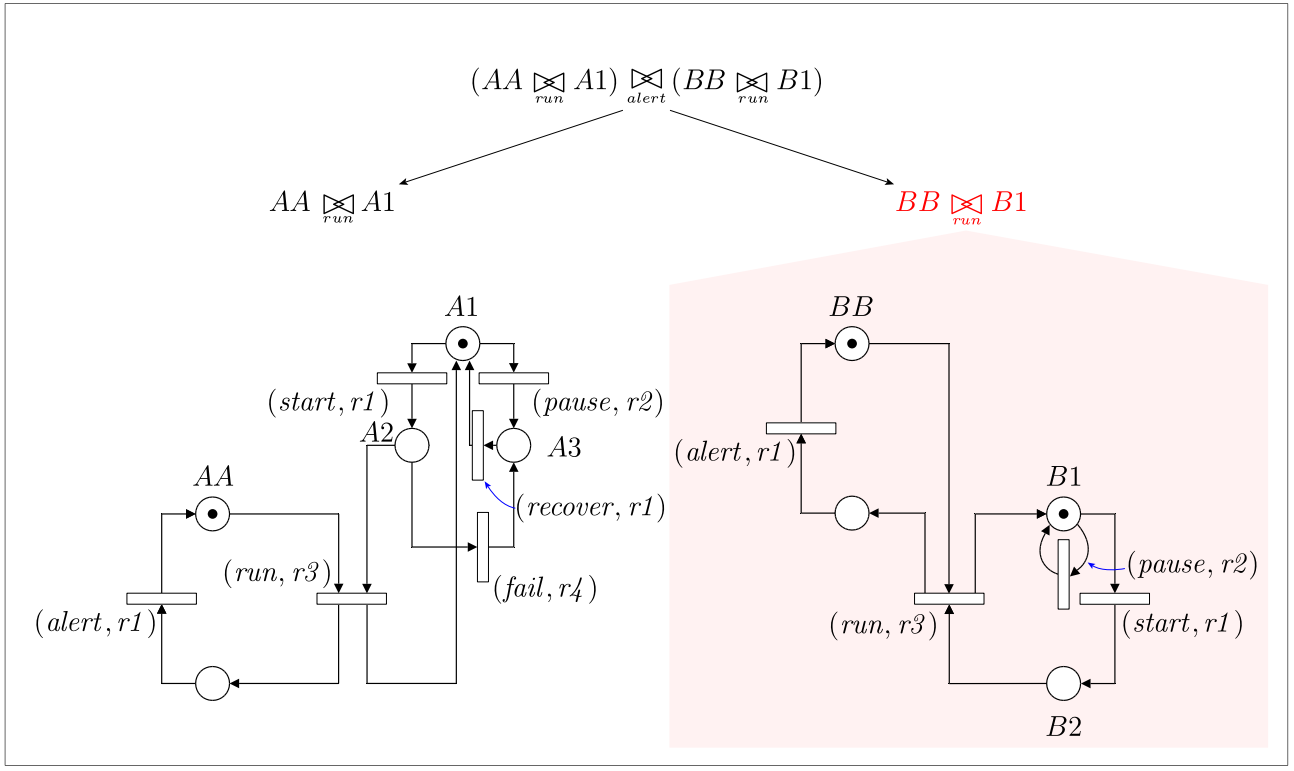

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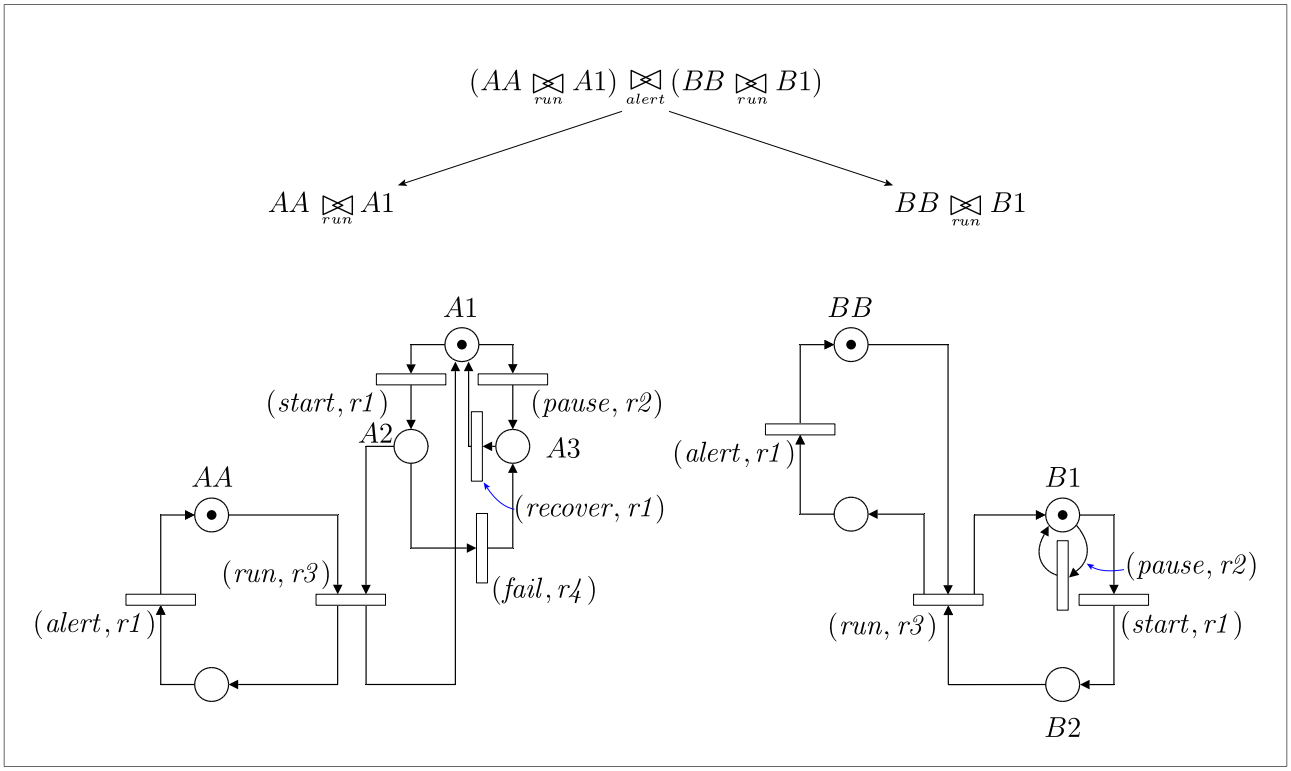

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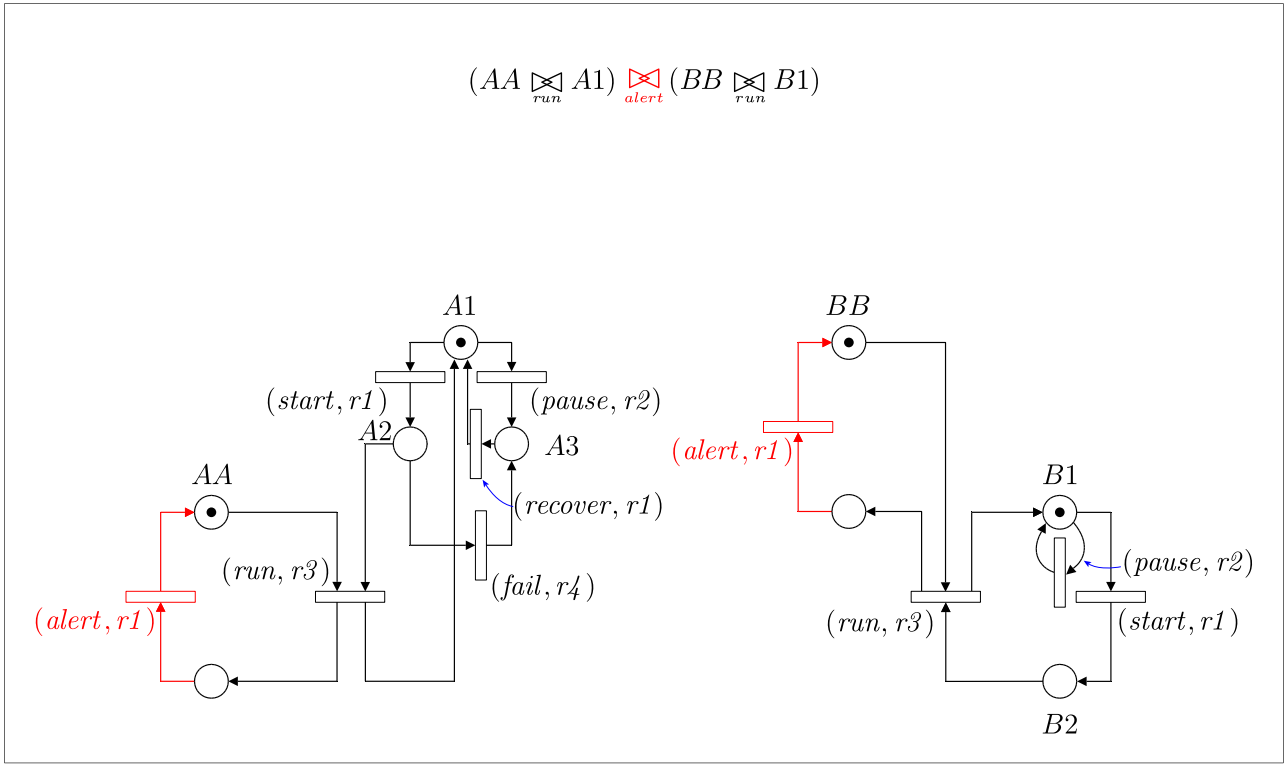

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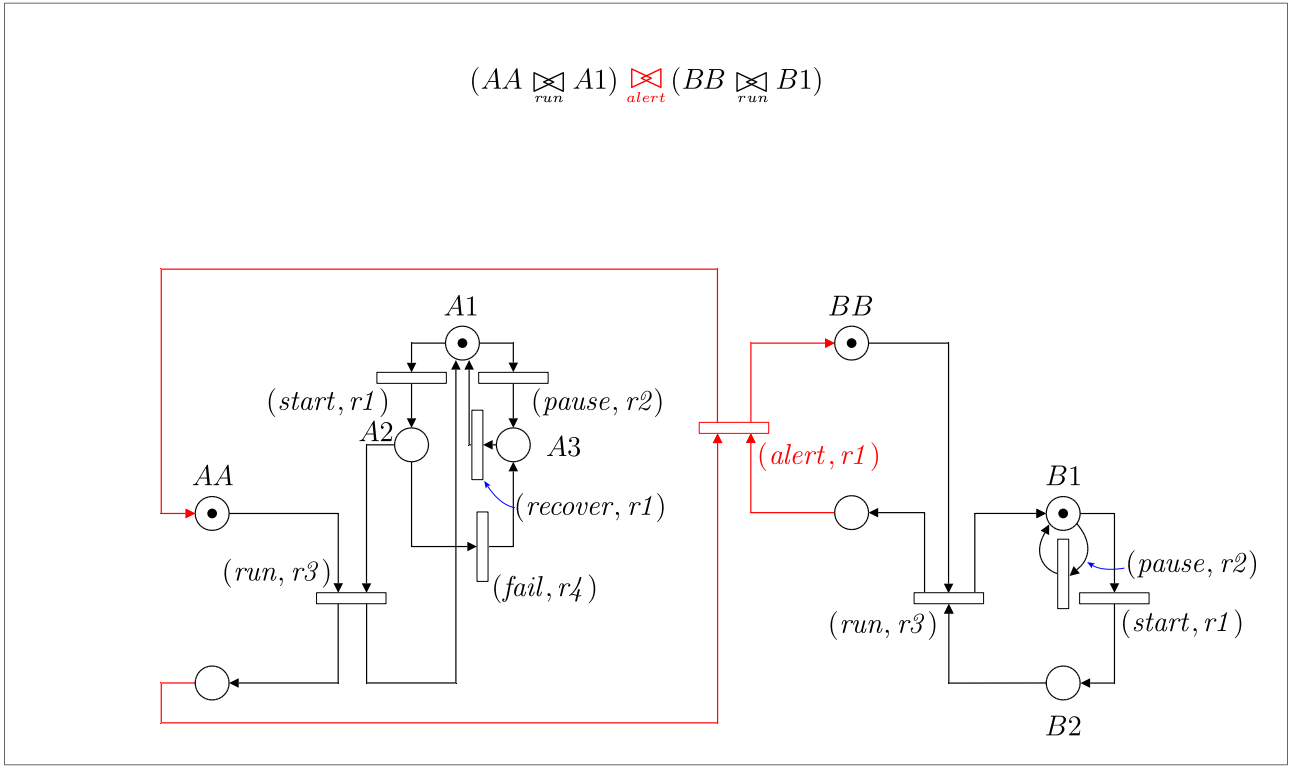

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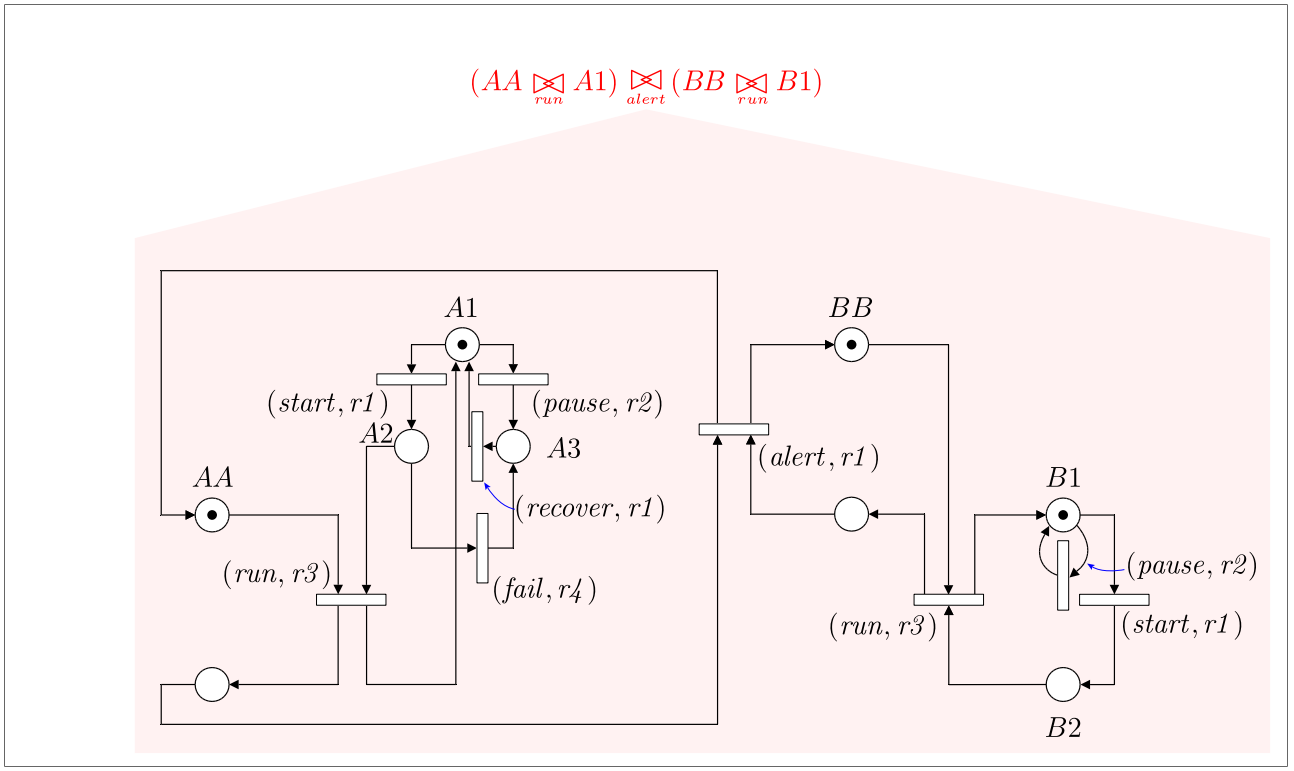
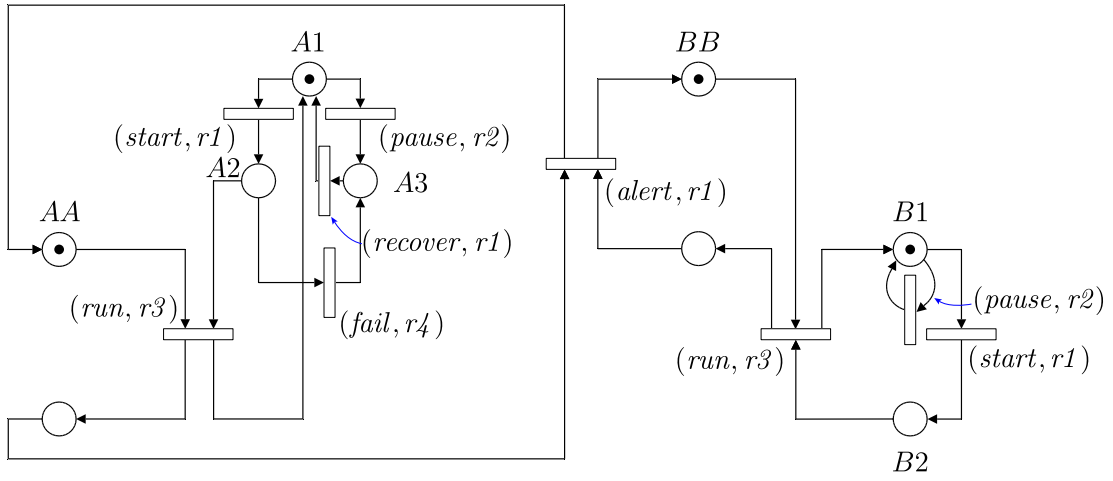


Example ipc Compilation

$$(AA \underset{run}{\bowtie} A1) \underset{alert}{\bowtie} (BB \underset{run}{\bowtie} B1)$$



DNAmaca: Example

A transition definition in DNAmaca:

```
\transition{A2_fail} {
        %% PEPAaction type { fail }
        \condition{ A2 > 0 }
        \action {
                next -> A2 = A2 - 1;
                next -> A3 = A3 + 1;
        }
        \priority{1}
        \rate{ PEPA_r4 }
}
```

Stochastic Probe Syntax

Probes consist of action regular expressions based on sequences, branching and ranges.

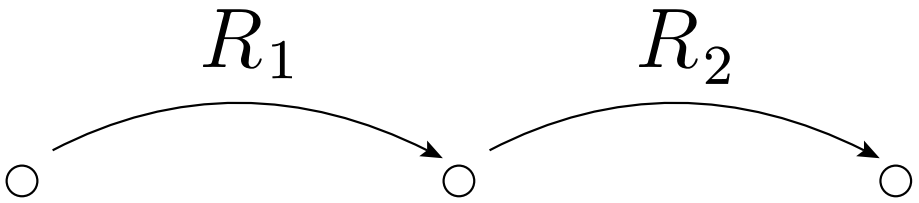
$$
\begin{array}{rcl}
R & ::= & a \mid T, T \mid (S) \\
S & ::= & T \mid T|S \\
T & ::= & R \mid R\{n\} \mid R\{m,n\} \mid R^{+} \mid R^{*} \mid R? \\
a & ::= & act \mid act : \mathsf{start} \mid act : \mathsf{stop}
\end{array}
$$

Require to specify start/stop actions to start and stop the measurement over the model

regexp Probes

Sequence operator:

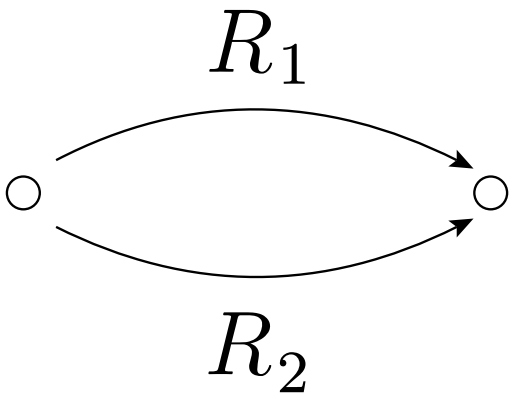
R_1, R_2



regexp Probes

Choice operator:

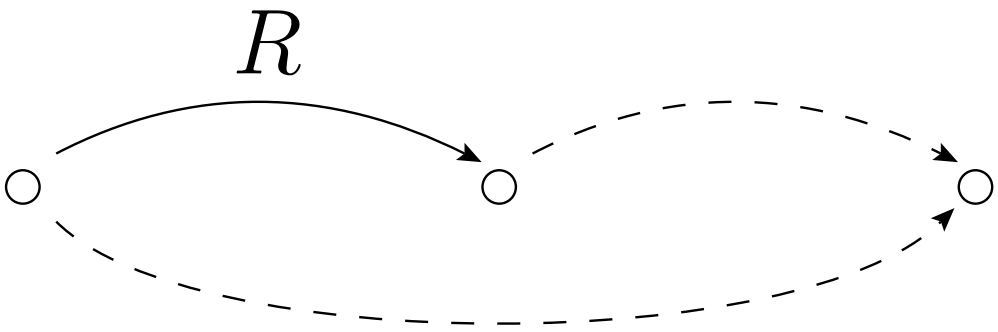
$$R_1 | R_2$$



regexp Probes

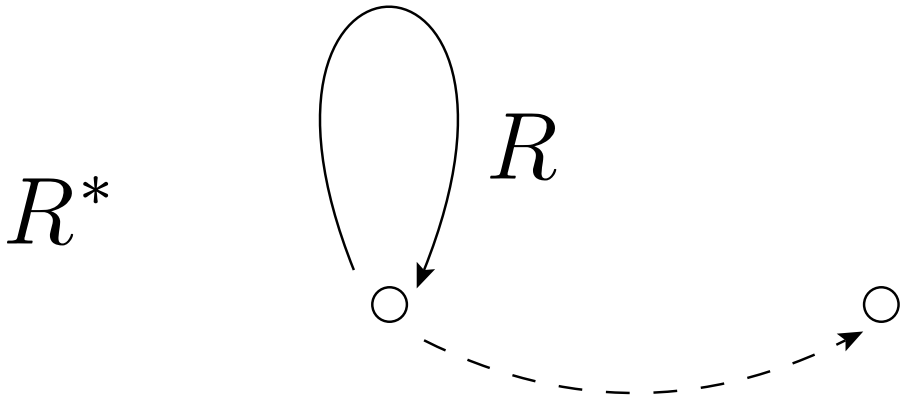
Zero-or-one operator:

$R?$



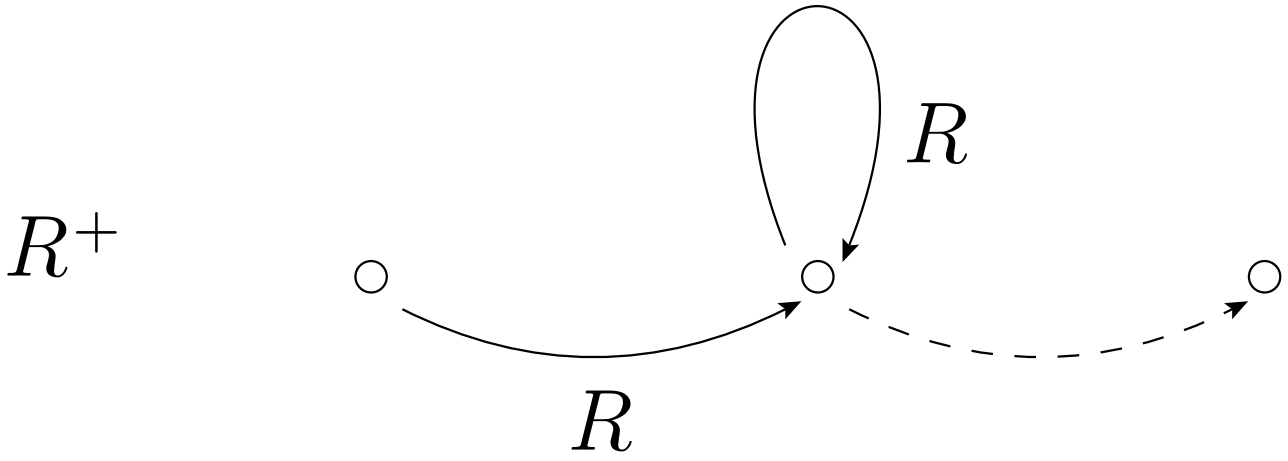
regexp Probes

Closure operator:



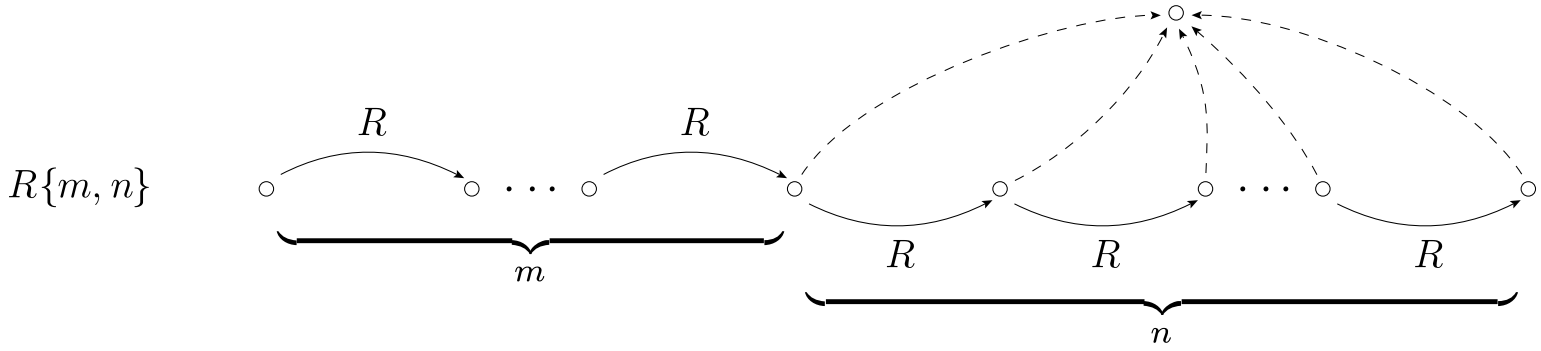
regexp Probes

Positive closure operator:



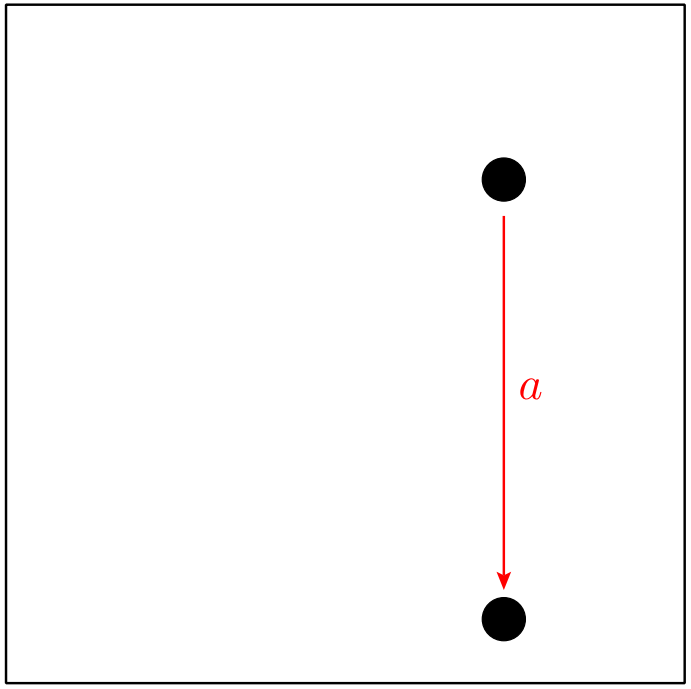
regexp Probes

Range operator:



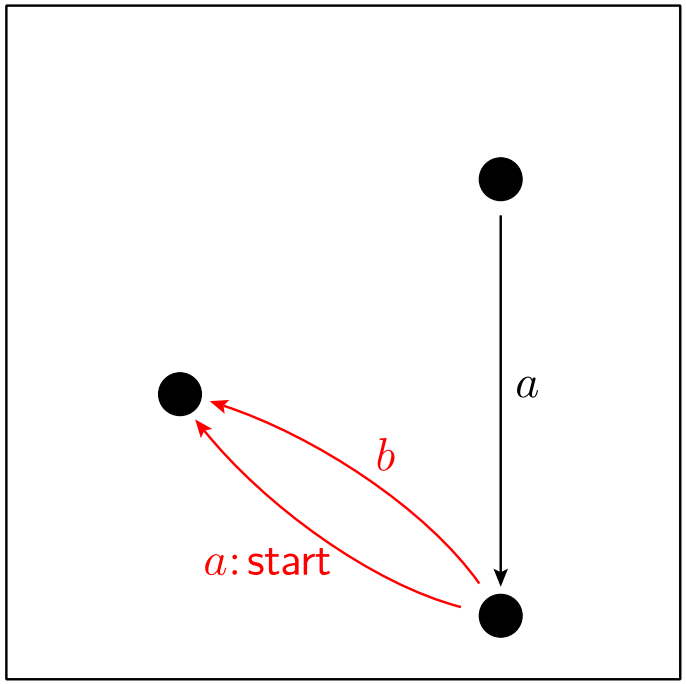
Probe Example

Probe definition: $(a, (a{:}\,\mathsf{start} \mid b), (a{:}\,\mathsf{stop} \mid b))$



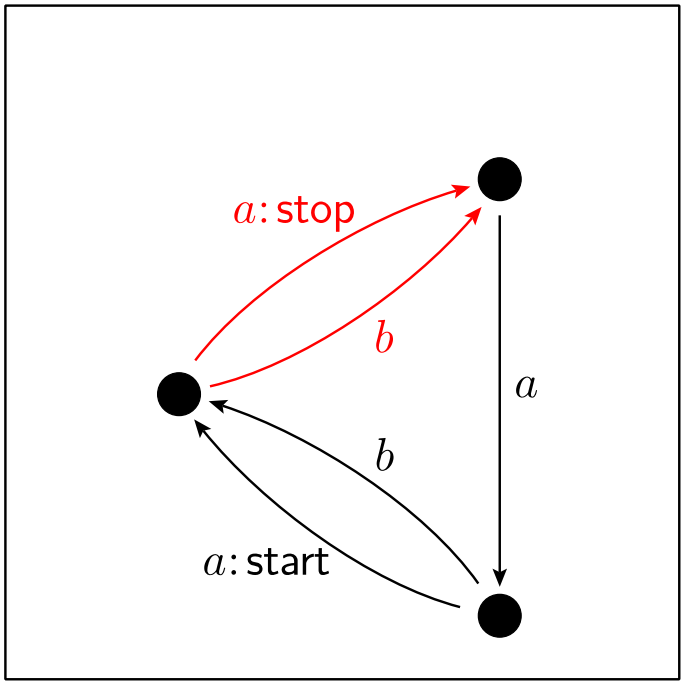
Probe Example

Probe definition: (a, (a: start | b), (a: stop | b))



Probe Example

Probe definition: (a, (a: start | b), (a: stop | b))



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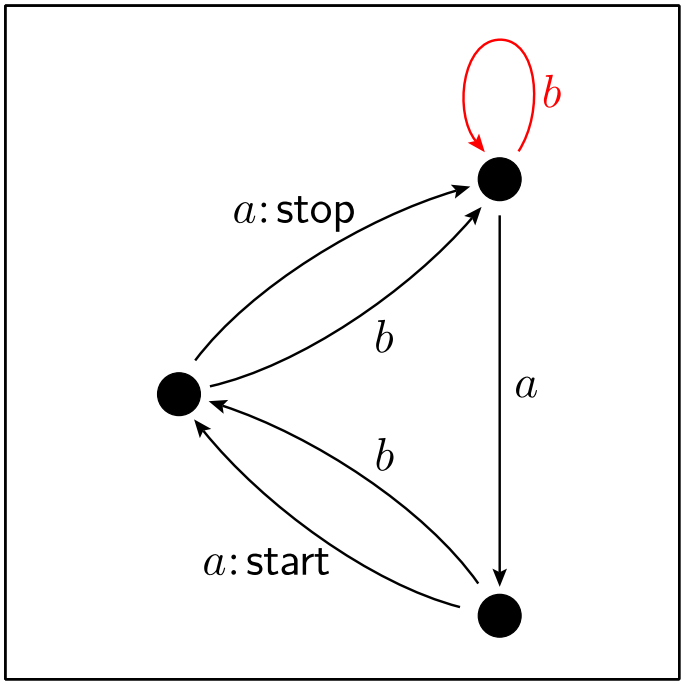
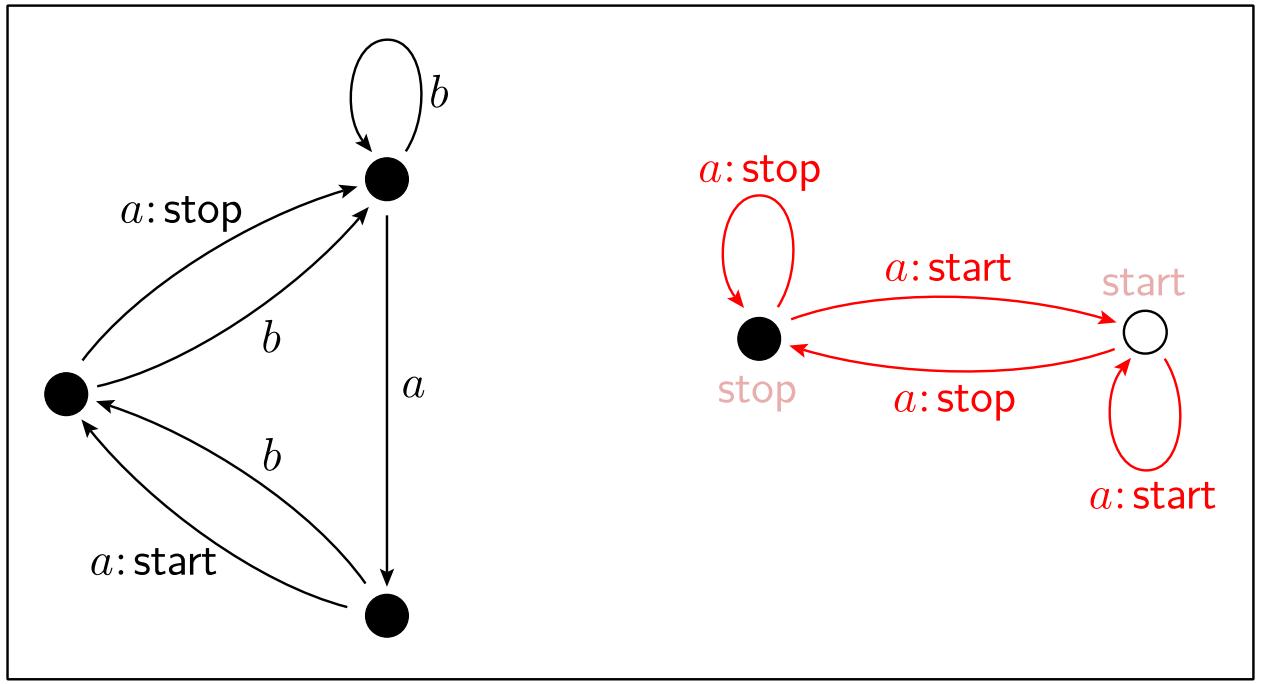
Probe definition: (a, (a:start | b), (a:stop | b))



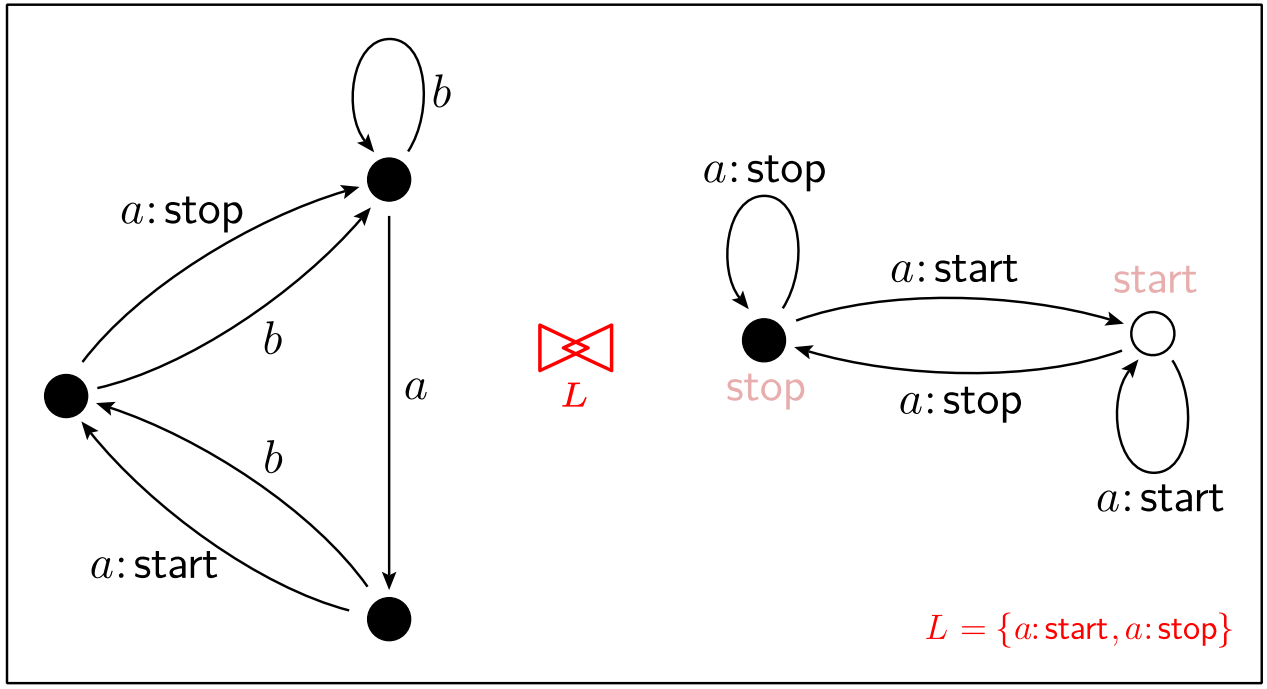
Probe Example

Probe definition: (a, (a: start | b), (a: stop | b))



Probe Example

Probe definition: (a, (a:start | b), (a:stop | b))



Probe Example

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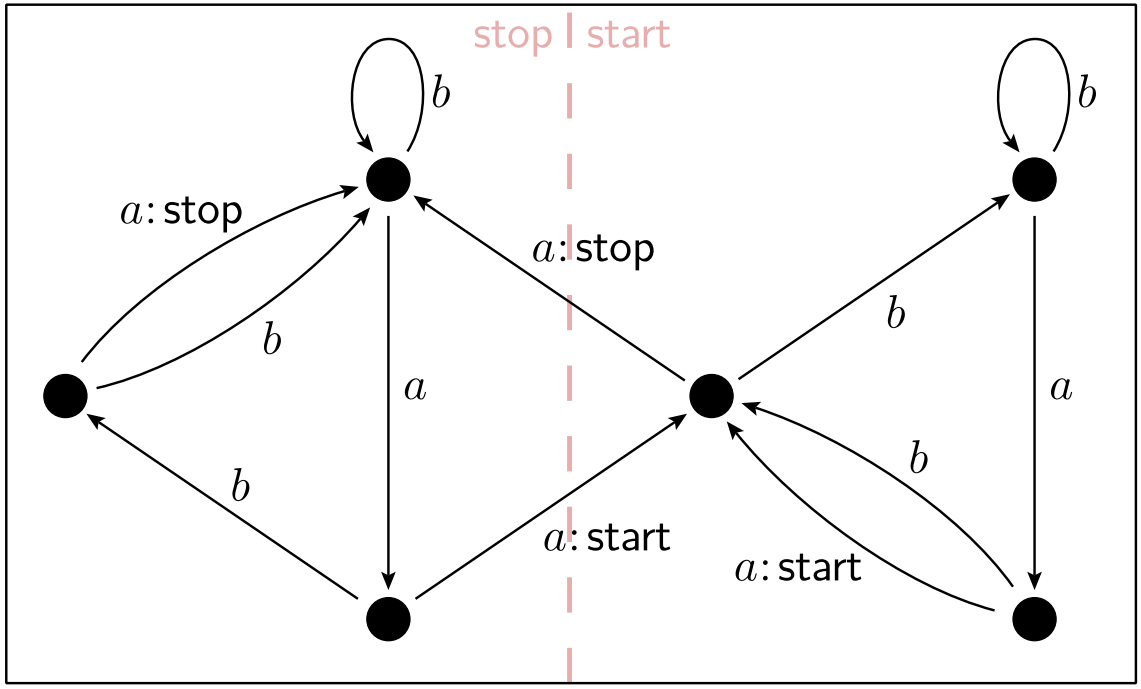
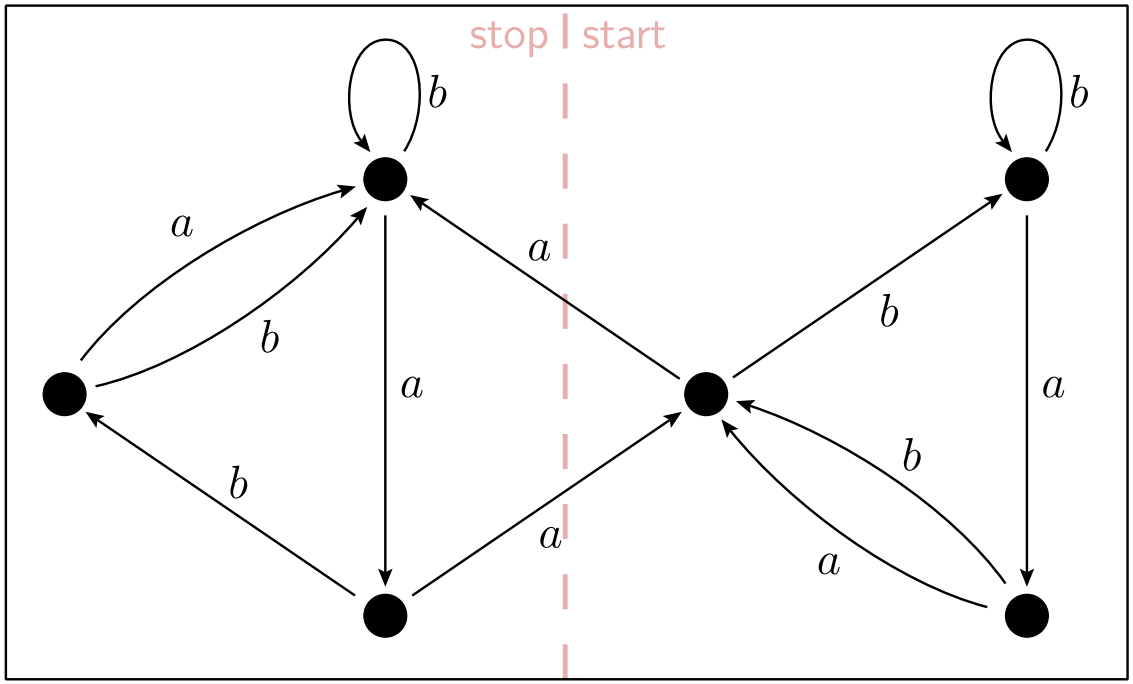


Probe Example

Probe definition: (a, (a:start | b), (a:stop | b))



Example: Web server

- Multiple redundant web servers
- Servers subject to failure and synchronised recovery if necessary
- Buffered web page updates
- Priority given to read access

Server definition

$$
\begin{aligned}
\mathrm{Server} &\stackrel{def}{=} (\mathrm{s_read_request}, \top).\mathrm{Server_read} \\
&+ (\mathrm{s_fail}, r_{s_f}).\mathrm{Server_fail} \\
&+ (\mathrm{s_write}, r_{s_w}).\mathrm{Server} \\
\mathrm{Server_read} &\stackrel{def}{=} (\mathrm{s_read_lookup}, r_{s_{rl}}).\mathrm{Server} \\
\mathrm{Server_fail} &\stackrel{def}{=} (\mathrm{s_fail_recover}, r_{s_{fr}}).\mathrm{Server} \\
&+ (\mathrm{s_fail_recover_all}, \top).\mathrm{Server} \\
&+ (\mathrm{s_write}, r_{s_{fsw}}).\mathrm{Server_fail}
\end{aligned}
$$

Server group definition

$$
\begin{aligned}
\mathsf{Server_group}_0 &\stackrel{def}{=} (\mathsf{s_fail}, \top).\mathsf{Server_group}_1 \\
\mathsf{Server_group}_i &\stackrel{def}{=} (\mathsf{s_fail}, \top).\mathsf{Server_group}_{i+1} \\
&\quad + \quad (\mathsf{s_fail_recover}, \top) \\
&\quad\quad .\mathsf{Server_group}_{i-1} : 1 \leq i < S \\
\mathsf{Server_group}_S &\stackrel{def}{=} (\mathsf{s_fail_recover_all}, r_{s_{gsfra}}) \\
&\quad\quad .\mathsf{Server_group}_0
\end{aligned}
$$

Server cluster definition

$$\text{Servers} \stackrel{def}{=} (\text{Server} \underset{\mathcal{L}}{\bowtie} \text{Server} \underset{\mathcal{L}}{\bowtie} \text{Server} \underset{\mathcal{L}}{\bowtie} \text{Server}) \underset{\mathcal{L}'}{\bowtie} \text{Server_group}_0$$

where:

- $\mathcal{L} = \{\text{s_write}, \text{s_fail_recover_all}\}$
- $\mathcal{L}' = \mathcal{L} \cup \{\text{s_fail_recover}\}$

Write buffer definition

$$
\begin{aligned}
\mathrm{Write_buffer}_0 &\stackrel{def}{=} (\mathrm{b_write}, \top).\mathrm{Write_buffer}_1 \\
\mathrm{Write_buffer}_i &\stackrel{def}{=} (\mathrm{b_write}, \top).\mathrm{Write_buffer}_{i+1} \\
&\quad + \quad (\mathrm{s_write}, \top).\mathrm{Write_buffer}_0 \\
&\qquad\qquad : 1 \leq i < B \\
\mathrm{Write_buffer}_B &\stackrel{def}{=} (\mathrm{s_write}, \top).\mathrm{Write_buffer}_0
\end{aligned}
$$

Writer definition

$$\text{Writer} \stackrel{def}{=} (\text{b_write}, r_{w_{bw}}).\text{Writer_writ}$$
$$\text{Writer_writ} \stackrel{def}{=} (\text{w_reset}, r_{w_r}).\text{Writer}$$

and an arbitrary group writers:

$$\text{Writers} \stackrel{def}{=} \text{Writer} \underset{\emptyset}{\bowtie} \text{Writer} \underset{\emptyset}{\bowtie} \text{Writer}$$

Reader defintion

$$\begin{aligned} \mathsf{Reader} &\stackrel{def}{=} (\mathsf{s_read_request}, r_{r_{srr}}) \\ &\quad .(\mathsf{s_read_lookup}, \top).\mathsf{Reader_read} \\ \mathsf{Reader_read} &\stackrel{def}{=} (\mathsf{r_reset}, r_{r_r}).\mathsf{Reader} \end{aligned}$$

and an arbitrary number or readers:

$$\mathsf{Readers} \stackrel{def}{=} \mathsf{Reader} \underset{\emptyset}{\bowtie} \mathsf{Reader} \underset{\emptyset}{\bowtie} \mathsf{Reader}$$

Overall System

$$\text{Environment} \stackrel{def}{=} \text{Writers} \underset{\emptyset}{\bowtie} \text{Readers}$$

$$\text{Web_cluster} \stackrel{def}{=} \text{Servers} \underset{\{\text{s_write}\}}{\bowtie} \text{Write_buffer}_0$$

$$\text{System} \stackrel{def}{=} \text{Environment} \underset{\mathcal{N}}{\bowtie} \text{Web_cluster}$$

where $\mathcal{N} = \{\text{b_write}, \text{s_read_request}, \text{s_read_lookup}\}$.

Probe Query A

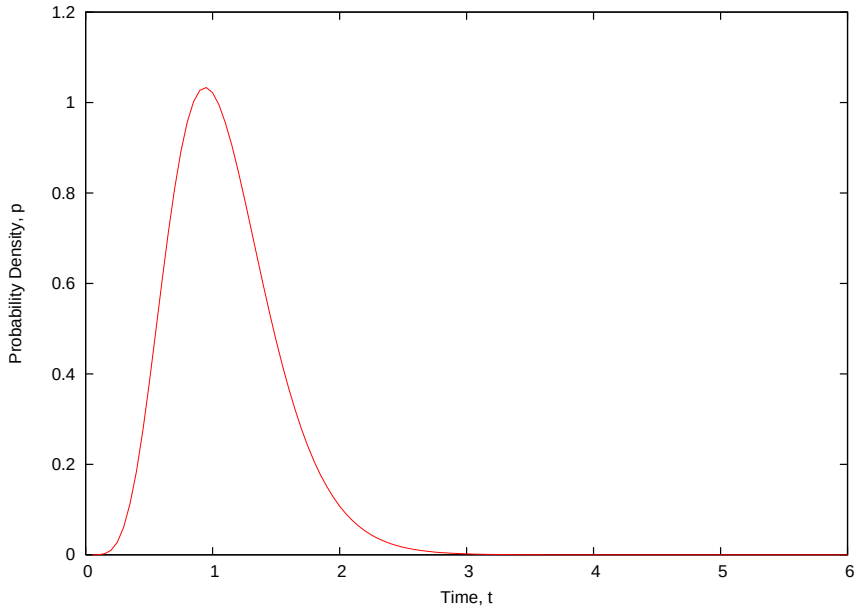
Probe A: measures the length of time to process at least 4 read events:.

$$\Pi_A = \text{s_read_request}: \text{start}, (\text{s_read_lookup}\{4\}), \text{r_reset}: \text{stop}$$

Model + probe stats: 362,240 states; 4,061,440 transitions

Probe Query A

Probability density function for probe A:



Probe Query B

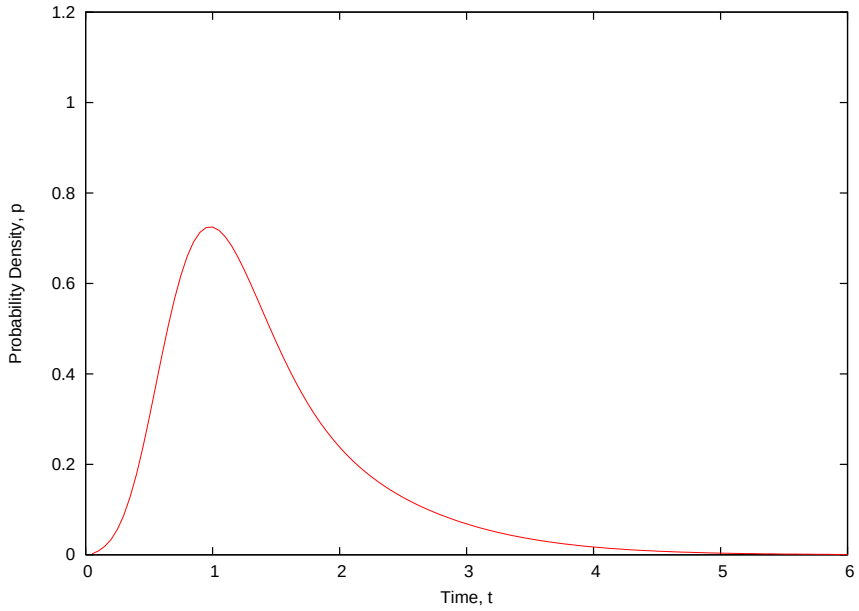
Probe B: observes the length of time it takes either to observe 4 completed reads or 2 completed writes:

$$\Pi_B = (\text{s_read_request}: \text{start}, (\text{s_read_lookup}\{4\}), \text{r_reset}: \text{stop}) \mid (\text{b_write}: \text{start}, \text{b_write}\{2\}, \text{w_reset}: \text{stop})$$

Model + probe stats: 581,200 states; 6,516,356 transitions

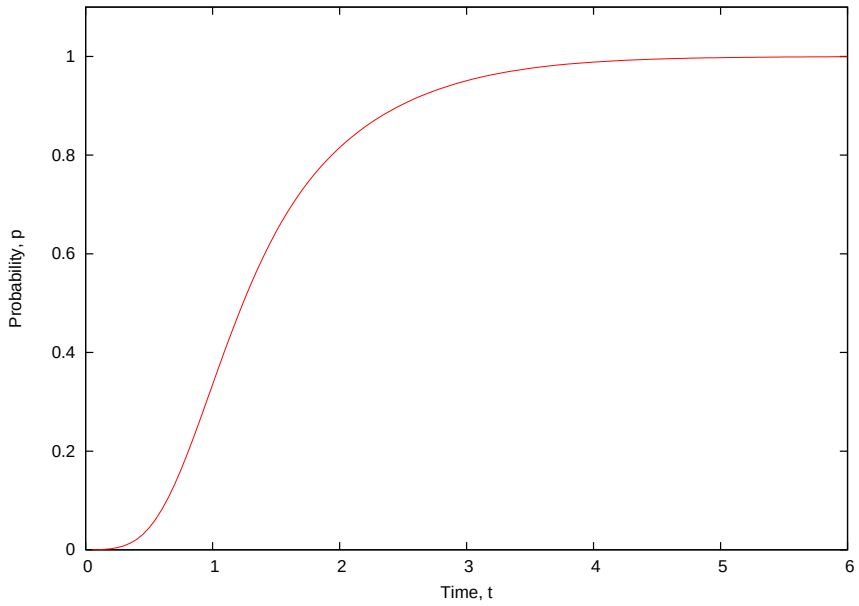
Probe Query B

Probability density function for probe B:



Probe Query B

Cumulative distribution function for probe B:



Probe Queries

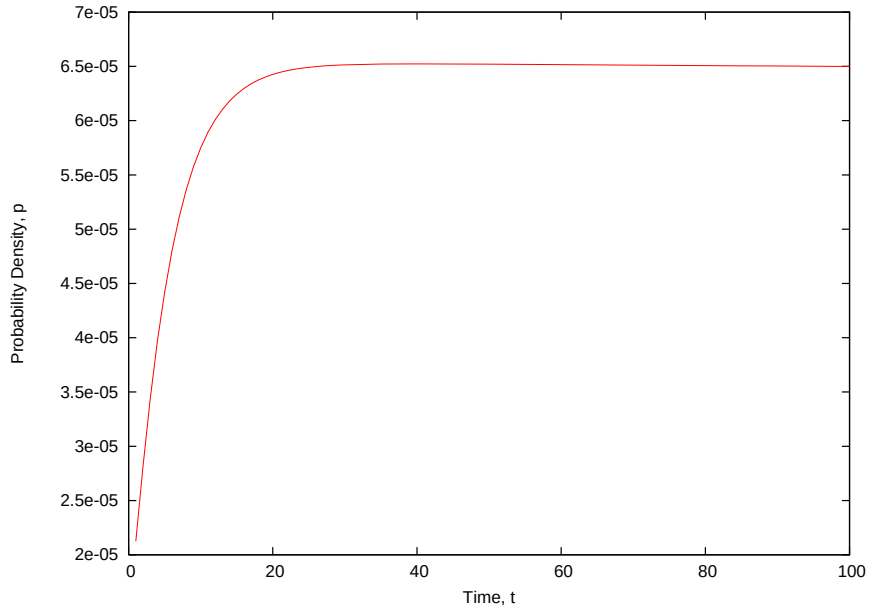
Probe C: extracts the passage-time to first full system failure:

$$\Pi_C = (\text{s_read_request}:\text{start} \mid \text{b_write}:\text{start}), \text{s_fail_recover}:\text{stop}$$

Model + probe stats: 661,960 states; 9,440,988 transitions

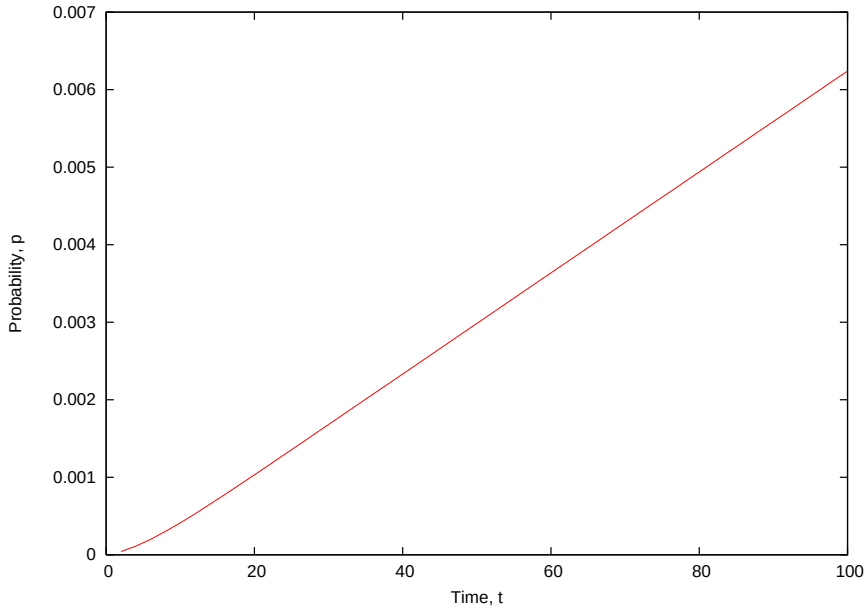
Probe Query C

Probability density function for probe C:



Probe Query C

Cumulative distribution function for probe C:



Conclusion

Stochastic probes:

- use additional SPA components to define performance measures
- unify three forms of performance analysis with a single specification mechanism
- use action regular expressions to specify probe behaviour
- cannot enforce behaviour – the next step?